大韓數學敎育學會 論文集 第 8 卷 第 1 號
Journal of the Korea Society of Educational Studies in Mathematics
Vol. Ⅷ, No.1, 31-39. July 1998

A Study of Selecting Problems for the Strategies Recognition of Problem Solving in School Mathematics

Choi Soon-Man*

Ⅰ. Introduction

Problem solving has been the research topic of educators of mathematics, psychologists of education, and philosophers since 1930s(Brannan & Oschaf, 1983), and it has been emphasized as a focus of school mathematics(An Agenda for Action, NCTM, 1983). It also is the ultimate goal of school mathematics across the times. It is because problem solving can be applied to every area of daily life, natural science, social science, occupational and technical skills as well as mathematics theory itself.

Problem solving is the first goal of school mathematics in *Curriculum and Evaluation Standards for School Mathematics*(1989) of NCTM and it should have been placed on a central focus of mathematics curriculum as the essential part of mathematics teaching-learning activities. Therefore, problem solving is not a part of mathematics but an integral part that is, and should be applied to the whole part of the mathematics curriculum.

But the process of problem solving is very complicated, so there has been little recognized agreement on the best method for the improvement of the problem solving abilities. In the end this fact leads to a dilemma. To solve this dilemma a generalized problem solving of process-oriented problem solving has been emphasized, not an individualized problem solving to be solved by case by case.

A general process of problem solving is presented as four or five stages by many writers. The widely known Polya's process comprise the four stages in the order of problem recognition, solution, execution and discussion(Lenchener, 1983; Suydan, 1980). The stage of solution is meant to select the appropriate strategy and is the most essential element(Musser & Others, 1980), and is the most important and difficult one(Krulik & Rudnik, 1982).

Therefore, strategies in problem solving have always been focused on and problem solving abilities can be developed by the instruction of various strategies(Blake, 1977). In this case students can utilize much more strategies and use the correct solution effectively(Kantowski, 1977). Various strategies should be developed and applied to develop problem solving abilities(Branca, 1980; Bruni, 1982; LeBlanc, 1977; Lenchner, 1983; Suydan, 1980). Various problems should be presented for the drill of

* 전주교육대학교

executing the strategies(Maletsky, 1982; Moses & Others, 1990), and in this respect it is necessary for the teachers to have this experience(Fisher, 1988).

In *Curriculum and Evaluation Standards for School Mathematics*(1989) of NCTM and Professional Standards for teaching Mathematics(1991) problem solving places an emphasis on the selection of the problem and the use of the various strategies. Therefore, the types of strategies, the reasonable method of the selection of the problem for the recognition of the strategies and the presentation of the problems based on the types of the strategies are included in this thesis.

II. Types of Strategies

The plan of behaviour to be executed for problem solving is the strategy(Lenchner, 1990). Problems can be solved by various strategies, but every strategy is not necessarily appropriate for solving all kinds of problems. Each strategy can be used in conjunction with one or more.

Branca(1980) states that it is more helpful that students solve the same problem through different methods or different problems through the same method rather than they solve the same problem through the same method. To improve problem solving abilities, recognition of the various strategies should be preceded and then the approach of the various strategies should be practiced for the given problem, and finally reasonable strategies should be applied.

The types of the strategies presented by many writers are as follows:

(1) Krulik & Rudnik(1982)

① finding patterns ② working backwards ③ reduction, guessing and reviewing ④ experimenting ⑤ simplifying ⑥ organizing a list ⑦ inference ⑧data expression(graph, equation, algebraic expression, table, chart, diagram)

These writers suggest that strategies(experiment, inference, hypothesis for solution) should be selected through the process of search(data expression, chart, diagram, experiment, finding patterns).

(2) LeBlanc & Others(1977)

The writers divide strategies into general strategies and supporting strategies.

<general strategy> : Planning to be employed to problem solving as a whole

① trial and error ② making a table ③ simplifying ④ finding patterns ⑤experimenting ⑥ inference ⑦ arithmetic ⑧ working backwards

<supporting strategies> : intermediate stage strategies for the general strategies

① diagram ② table ③ graph ④ list ⑤ equation

(3) Lenchner(1983)

① drawing a picture or diagram ② finding a pattern ③ making an organized list ④ making a table ⑤ solving a simpler problem ⑥ trial and error ⑦ experimenting ⑧ acting out the problem ⑨ working backwards ⑩ writing an equation ⑪ changing a viewpoint ⑫ using inference

These various strategies shown above can be used in elementary, middle or high school

mathematics. But it can't be expected that students use the strategies unfamiliar to them, so it is necessary to select the strategies applicable according to the degree of the students' affective and cognitive abilities.

Bruni(1982) suggests that teachers help the lower grades of elementary school students by having the readiness to utilize the strategies rather than the instruction of the specific strategies and that teachers enable students get different answers through open-ended questions. Worth(1982) suggests the strategies for the middle grades of the elementary school students as follows: ① arithmetic ② use of equation ③ utilization of diagram ④ guessing and check-up ⑤ simplifying ⑥ use of a table ⑦ use of a table ⑧ use of a graph ⑨experiment. *Curriculum and Evaluation Standards for School Mathematics*(1989) of NCTM already mentioned suggests the strategies for K-4 as follows: ① use of a manipulative data ② trial and error ③ use of a table ④ drawing a diagram ⑤ finding out patterns ⑥ acting out the problem.

III. Selection of the Problems

For recognition of the various strategies, appropriate questions should be selected for the drills to practice the strategies. And these skills of the selection of the problems needs creativity.

The types of the problems for the use of the strategies are as follow:

(1) Open-Search Problem (Butts, 1980)

These open problems can't include the practice of the strategies.

(2) Process Problem (LeBlanc & Others, 1980)

Though these problems need the knowledge of the past and algorithm, these problems involve interesting problems of daily life that can be solved by the search of the various strategies and the process through the approach of the nonalgorithm.

(3) Nonroutine Problem (Kantowski, 1981)

These problems don't involve known solution or well-formed algorithm from the past. These problems are never tried and never similar to the previous problems.

(4) Open-Ended problem (Bruni, 1982)

These problems can be solved many different ways, so the answers can vary according to the students.

The types of problems mentioned above show the same function and requires the use of various strategies through the approach of the nonalgorithm. The problem that "Six pitchers from six baseball teams got together and shook hands. What's the total number of handshaking?" can be solved by the five strategies such as making an experiment, making a diagram, making a list, finding patterns through tables and using inferences(Choi, 1984).

The progress of the ability for problem solving is important for all levels from kindergarten to graduate school(Malone & Others, 1983). *Curriculum and Evaluation Standards for School Mathematics*(1989) of NCTM also tries to develop and utilize various strategies focused on

the nonroutine problem. On the other hand, students can get frustration or satisfaction when they solve the problem. Therefore, this fact should be considered in the selection of reasonable problems. Open-ended problems are apt to make students get reinforcement to solve new problems(Moses & Others, 1990). Problems for facilitating problem solving activities should be nonroutine problems and open-ended problems(Bohan, 1993).

When you select a problem, it is necessary to consider students' interests, motivation(Branca, 1980; Lenchner, 1990; Worth, 1982), and the degree of the suitable difficulty. The problems to cause students' interest and challenge are generally familiar problems or problems focused on information even though they aren't familiar problems. For example, story problems are very effective(Fairbairn, 1993).

Therefore, the vocabulary should be simple and understood clearly though mathematical terms may be used. Because the length of the phrase or sentence causes difficulty in understanding, the sentence length of the problem should be short, if possible. The complex numbers should be simplified and the emphasis should be given on problem solving rather than arithmetics. The difficulty of the problem can be graded by the composition and expression of the problem, and the more procedure and arithmetic stages the solution of the problem needs, and the less similarity the problem has with previous problems, the more difficult is the problem.

The components of the difficulty of the problem are as follow:

① the selection of the vocabulary
② the length and the structure of the phrase or the sentence
③ the size and the complexity of the number
④ the form and the expression of the problem
⑤ the number of the procedures or the calculation for the solution
⑥ the similarity of the problem

Ⅳ. Problems for Recognizing the Types of Strategies

The next step in selection of the problems is followed by solving the mathematical problems. Students should decide on a plan of action to follow in solving the problem. Therefore it is necessary for teachers to help students choose the appropriate strategies to solve mathematical problems. The writer suggests that the following strategies be used based on the types of the problems: drawing a picture or diagram, finding a pattern, making an organized list, making a table, solving a simpler problem, trial and error, experimenting, acting out the problem, working backwards, writing an equation, using inference, and changing your point of view

1. Drawing a Picture or Diagram

When a problem is not illustrated, it is sometimes helpful to draw your own picture or diagram. If the situation is not easily pictured, a simple diagram using symbols to represent the situation may help to clarify the problem for you.

Problem: The lengths of the three rods are 6

cm, 9cm and 11cm respectively. How can you measure 14cm using these rods?

2. Finding a Pattern

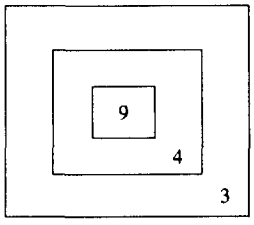
It is very useful to recognize and extend a pattern in solving the problems. Students can often solve some problems by identifying a pattern in given data and simply applying that pattern to the problem situation. This strategy can be used combined with other problem solving strategies.

Problem: Suppose that each of the following mathematical operation is true: 2 * 4 = 8, 5 * 3 = 13, 3 * 5 = 11, 9 * 7 = 25. What is the value of 7 * 3?

3. Making an Organized List

When students should deal with a large amount of data, it is suggested that they organize information into some type of list. It enables students account for all possibilities and avoid repetitions. The ways to make organized lists are as follows: a classified table, a distribution chart and a tree diagram. The tree diagram is preferred by some people because the lines of "tree" visually help them account for all possibilities.

Problem:



Three arrows are thrown at the target shown below. Assume that each of the arrows lands within one of the rectangulars. How many different point totals are possible?

4. Making a Table

Organizing a data into a table can be an effective problem solving strategy when the data has more than one characteristic. A table is an excellent device for recording what you have done. Students have only to make a table using the given data, and then detect significant patterns between the data on the table.

Problem: There are a total of 18 hens and calves in the yard. If the legs of the animals are 50, how many hens and calves are there respectively?

5. Solving a Simpler Problem

If a problem appears difficult or complicated, it is helpful to solve similar problems with simpler conditions at first. It is also suggested that you divide the original problem into parts easy to deal with. Solving simpler problems eventually enables students to solve the original problem without difficulty.

Problem: The cards are numbered consecutively from 1 to 150. How many card numbers contain at least one digit 7?

6. Trial and Error

An effective way for problem-solving is hypothesis testing. That is, students can make a reasonable guess of the answer, then check the guess against the conditions of the problem. When a problem seems difficult, it is advised that students do risk-taking without hesitation. By continuous trial and error, students can solve the problem by eliminating one possible answer and obtain other information to the right answer.

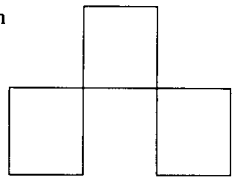
Problem: Arrange the counting numbers from

1 to 6 in the circles at the left so that the sum of the numbers along each side of the triangle is 10.

7. Experimenting

Problems can often be solved by experiments. Students can solve the problems with geometric configurations or spatial relationships by experimenting with a physical model in which concrete objects may be manipulated.

Problem



Three squares of the same size are made using twelve matches of the same size as shown in the picture at the left. Move three matches and then make five squares.

8. Acting Out the Problem

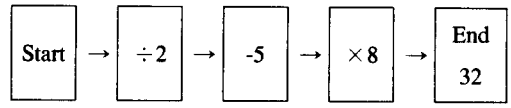
When it is very difficult to visualize a problem or the procedure necessary for its solution, it is helpful to act out the problem situation physically. Acting out the problem itself may lead you to the answer, or it may lead you to find another strategy that will help you find the answer. It is very effective for young children.

Problem: Suppose that you buy a stamp for 150, sell it for 200, buy it back for 250, and finally sell it for 300. How much money did you make or lose in buying and selling this stamp?

9. Working Backwards

Some problems involve a sequence of actions: the final result of the actions is known, and you are asked to determine the beginning conditions of the problem. An effective way to solve this type of problem is to consider the actions in reverse order.

Problem: In the following flow chart what is the appropriate number for the 'Start'?



10. Writing an Equation

The conditions of the problem are represented by an equation or inequality. Students can find the solution of the problem by solving the equation or inequality. When they write an equation, letters of the alphabet are usually used as variables to represent the unknown quantities in the problem.

Problem: Two apples weigh the same as a banana and a strawberry. A banana weigh as much as nine strawberries. How many strawberries weigh the same as an apple?

11. Using Inference

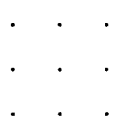
Inference is a frequently-used strategy in mathematics. Students can reach a conclusion through deduction and induction.

Problem: Each of six baseball players shook hands with the other baseball players. What's the total number of handshaking?

12. Changing Your Point of View

It is often the case that you are blocked in your attempts to solve a problem. It is because you may have decided that there is only one way

to approach the solution, or perhaps you made an incorrect assumption about the given information. To correct the frozen or wrong assumption, it is helpful to read the problem once again and try to change your point of view.

Problem:  Draw four line segments through the nine dots shown at the left without lifting your pencil from the paper.

V. Conclusion

The objectives of school mathematics are viewed as having three aspects such as the accomplishment of concept, the master of arithmetic skills and the development of problem-solving ability. Each of the aspects deserves emphasis but the development of students' problem-solving ability is the ultimate objective.

Problem solving is focused on the cultivation of the ability to apply the various strategies, for which the identification of the types of strategies should be preceded. The various types of strategies for school mathematics are presented in general, but it is necessary to examine the study of reasonable strategy suitable to the learner's affective and cognitive style. And it is desirable to select the problems for the recognition of the strategy such as nonroutine problems and open-ended problems, but more study is needed in these areas.

References

Blake, R. N. (1977). Th Effect of Problem Contest upon the problem Solving Process Used by Field Dependent and Independent and Independent Students(Dissertation, Univ of Columbia). Dissertation Abstract International 37A: 1491-92.

Bohan, H. & Bohan, S. (1993). Extending the Regular Curriculum Through Creative Problem Solving. At, 41(2), NCTM, 83-87.

Branca, N. A. (1980). Problem Solving as a Goal, Process, and Basic Skill. 1980 Year Book, NCTM, 4-6.

Brannan, R. & Schaef, O. (1983). An Introductional Approach to Problem Solving. 1983 Year Book, NCTM, 41.

Bruni, J. V. (1982). Problem Solving for the Primary Grades. Arithmetic Teacher, 29, NCTM, 10-15.

Butts, T. (1980). Posing problems Properly. 1980 Year Book, NCTM, 24-46.

Carpenter, T. O. & Others (1993). Models of Problem Solving. *Journal for Research in Mathematics Education*, 24(5), NCTM, 428-441.

Choi, S. M. (1984). A Strategy on Teaching Problem-Solving in School Math. Journal of Science Education of Chonju National University of Education.

Engen, H. V. (1993). Twentieth Century Mathematics for the Elementary School. AT, 41(2), NCTM, 92-96.

Fairbairn, D. M. (1993). Creating Story Problems. AT, 41(1), NCTM, 140-142.

Fisher, L. C. (1988). Strategies Used by

Secondary Mathematics Teachers to Solve Proportion Problems. *Journal for Research in Mathematics Education*, 19(2), NCTM, 158-168.

Jacobson, M. & Others(1980). Making Problem Solving Come Alive In the Intermediate Grades. 1980 Year Book, NCTM, 127-131.

Kantowski, M. G. (1977). Processes Involved Mathematical Problem Solving. *Journal for Research in Mathematics Education*, 8, NCTM, 163-180.

Kantowski, M. G. (1980). Some Thoughs on Teaching for Problem Solving. 1980 Year Book, NCTM, 195.

Krulik, S. & Rudnik, J. A. (1982). Teaching Problem Solving to Preservice Teachers. AT, 29, NCTM, 43.

Leblanc, J. F. (1977). You can Teach Problem Solving. AT, 25, NCTM, 16-20.

Leblanc, J. F. & Others (1980). Problem Solving in School Mathematics. 1980 Year Book, NCTM, 104-116.

Lenchner, G. (1983). Creative Problem Solving in School Mathematics. Houghton Mifflin Company, 8-44.

Lenchner, G. (1990). Mathematical Olympiad Contest Problems for Children. Glenwood Publications Inc., 9-17.

Maletsky, E. M. (1982). Problem Solving for the Junior High School. AT, 29, NCTM, 20-24.

Malone, J. A. & Others (1983). Measuring Problem-Solving Ability. 1983 Year Book, NCTM, 204.

Mcleod, D. B. (1988). Affective Issues in Mathematical Problem Solving. *Journal for Research in Mathematics Education*, NCTM, 19(2), 134-141.

Moses, B. & Others (1990). Beyond Problem Solving. Teaching and Learning Mathematics in the 1990s, NCTM, 82-91.

Muser, G. L. & Others (1980). Problem-Solving Strategies in School Mathematics. 1980 Year Book, NCTM, 136-137.

NCTM (1989). Curriculum and Evaluation Standards for School Mathematics. 23-24, 75-77, 137.

NCTM (1991). Professional Standard for Teaching Mathematics. 95-103.

Suydam, M. N. (1980). Untangling Clues from Research in Problem Solving. 1980 Year Book, NCTM, 38-40.

Worth, J. (1982). Problem Solving in the Intermediate Grades. AT, 29, NCTM, 16-19.

A Study of Selecting Problems for the Strategies Recognition of Problem Solving in School Mathematics

Choi Soon-Man

In this study the types of strategies, the reasonable method of the selection of the problem for the recognition of the strategies and the presentation of the problems based on the types of the strategies are explored.

The strategies can be used in elementary, middle or high school mathematics. But it can't be expected that students use the strategies unfamiliar to them, so it is necessary to select the strategies applicable according to the degree of the student's affective and cognitive abilities. And it is desirable to select the problems for the recognition of the strategy such as nonroutine problems and open-ended problems, but more study is needed in these areas.

Key Words: Strategies (types of); Problems (types of)