ENVIRONMENTAL SCIENCES Vol. 2, No. 1, 29~36
Bulletin of the Korean Environmental Sciences Society

Sediment Yield by Instantaneous Unit Sediment Graph

Yeong Hwa Lee

Department of Environmental Sciences, Kyungsan University, Kyungsan 712-715, Korea

(Manuscript received on 9 April, 1998)

An instantaneous unit sediment graph (IUSG) model is investigated for prediction of sediment yield from an upland watershed in Northwestern Mississippi. Sediment yields are predicted by convolving source runoff with an IUSG. The IUSG is the distribution of sediment from an instantaneous burst of rainfall producing one unit of runoff. The IUSG, defined as a product of the sediment concentration distribution (SCD) and the instantaneous unit hydrograph (IUH), is known to depend on the characteristics of the effective rainfall. The IUH is derived by the Nash model for each event. The SCD is assumed to be an exponential function for each event and its parameters were correlated with the effective rainfall characteristics. A sediment routing function, based on travel time and sediment particle size, is used to predict the SCD.

Key words : sediment yield, instantaneous unit sediment graph (IUSG), sediment concentration distribution (SCD), instantaneous unit hydrograph (IUH).

1. Introduction

Sediment prediction techniques were designed to estimate sediment yield for each storm, average annual sediment yield, or sediment transport. Estimates of watershed sediment yield are required for the solution of a number of problems. Some of the examples are design of dams and reservoirs, transport of pollutants, design of soil conservation practices, design of debris basins, depletion of reservoirs, lakes and wetlands, determination of the effects of basin management, and cost evaluation of a water project. Sediment is a pollutant or a carrier of pollutants such as radioactive material, pesticides, and nutrients. Increased awareness of environmental quality and the desire to control non-point source pollution have significantly increased the need to estimate sediment yield.

Rendon-Herrero (1974, 1978) derived a unit sediment graph (USG) and defined the USG as one unit of sediment for a given duration distributed over a watershed. The USG ordinates are obtained by dividing a sediment discharge graph by its total sediment load. Williams (1978) extended the concept of the instantaneous unit hydrograph (IUH), h, to determine sediment discharge from an agricultural watershed, using the IUSG, h_s. In the spirit of the instantaneous unit graph he defined the IUSG as the distribution of sediment from an instantaneous burst of rainfall producing one unit of runoff. The IUSG is the product of the IUH and the sediment concentration distribution (SCD). The SCD is assumed to be an exponential function for each event and its parameters were correlated with the effective rainfall characteristics. Sediment concentration of the IUSG is assumed to vary with the effective rainfall volume. A sediment routing function, using travel time and sediment particle size, was used to determine the SCD. The concept of IUSG has also been employed by

Singh et al. (1982), Chen and Kuo (1984), and Srivastava et al. (1984) among others.

Sharma et al. (1979a, b, 1980) developed input-output models for runoff-sediment yield processes for daily and monthly. They derived unit step and frequency response functions and studied the noise component in runoff-sediment yield processes.

An instantaneous unit sediment graph (IUSG) model is investigated for prediction of sediment yield from an upland watershed in Northwestern Mississippi. The IUH is derived by the Nash model (1957, 1958, 1959, 1960) for each event.

2. Instantaneous Unit Sediment Graph (IUSG)

Following Williams (1978) the IUSG can be defined as the distribution of sediment from an instantaneous burst of rainfall producing one unit of runoff and is considered to be the product of the IUH and the sediment concentration distribution:

$$h_s(t) = h(t)C(t) \qquad (1)$$

in which $h_s(t)$: the IUSG ordinate
$h(t)$: the IUH ordinate
$C(t)$: the sediment concentration

The IUSG assumes that C varies linearly with V_Q. Thus, the storm-sediment discharge Q_s can be obtained by convolving $h_s(t)$ with the incremental source runoff squared. Numerically,

$$Q_{si} = \sum_{j=1}^{i} v_j^2 h_{sk}, \quad k=i+1-j, \quad i=1, \; M \qquad (2)$$

in which Q_{si} : the storm-sediment discharge
v_i^2 : the effective rainfall

The sediment concentration distribution can be estimated by considering the sediment-routing equation (Williams, 1975b),

$$Y = Y_0 \exp(-aTd^{0.5}) \qquad (3)$$

in which Y : the sediment yield at a particular channel section
Y_0 : the sediment yield at an upstream section
a : the routing coefficient
T : the travel time between the two sections
d : the median sediment particle diameter

We can express Y as

$$Y = CV = C_0 V_0 \exp(-aTd^{0.5}) \qquad (4)$$

in which V : the volume of the effective rainfall
C_0 : the initial sediment concentration

If the channel losses are negligible, then $V = V_0$. Thus, the sediment concentration distribution, SCD, at any time t can be estimated as

$$C(t) = C_0 \exp(-atd^{0.5}) \qquad (5)$$

in which $C(t)$: the SCD at any time

The initial sediment concentration is produced by an instantaneous burst of one unit of the effective rainfall (or runoff). It is a function of the detachment caused by rainfall and runoff and can be approximated as a function of the shear stress τ on the watershed caused by the burst of runoff,

$$C_0 = b\tau = b\gamma rS \qquad (6)$$

in which b : the proportionality constant
γ : the water density
r : the hydraulic radius
S : the slope

For overland flow on a watershed, r can be approximated by the depth of flow y:

$$C_0 = b\gamma yS \qquad (7)$$

By the definition of the IUH, the flow depth is one unit of runoff over the watershed. Replacing y by v,

$$C_0 = b\gamma vS \qquad (8)$$

Letting $a = b\gamma S$.

$$C_0 = av \qquad (9)$$

Thus, it is constant for all increments of source runoff within a storm:

$$\frac{C_{0i}}{v_1} = \frac{C_{02}}{v_2} = \ldots\ldots = \frac{C_{0m}}{v_m} \qquad (10)$$

The sediment yield from each source runoff increment, U_i, can be expressed as

$$U_i = v_i^2 \int_o^\infty h_s(w)\,dw \qquad (11)$$

By definition of the IUSG, it represents the sediment yield for one unit of runoff. Inserting equation (1) in equation (11),

$$U_i = v_i^2 \int_o^\infty C(w)h(w)dw \qquad (12)$$

Substituting equation(5) into equation(12),

$$U_i = v_i^2 \int_o^\infty h(w)\exp(-awd^{0.5})\,dw \qquad (13)$$

C_0 is the initial concentration for one unit of runoff. The concentration for any amount of runoff can be determined from equation (10). Then, equation (13) can be written as

$$U_i = v_i C_{0i} \int_o^\infty h(w)\exp(-awd^{0.5})\,dw \qquad (14)$$

This can be further written as

$$U_i = v_i C_{0i} H \qquad (15)$$

where H represents the integral in equation (14) and is constant for all source runoff increments. Therefore,

$$H\sum_{i=1}^{m} v_i C_{0i} = Y \qquad (16)$$

Equation (16) simplifies to

$$C_{0i} = \frac{Yv_1}{\left[H\sum_{i=1}^{m} v_i^2\right]^{-1}} \qquad (17)$$

To use equation (17) v_i, Y, and H must be determined. Y is predicted with the modified universal soil loss equation, MUSLE (Williams, 1975a):

$$Y = 11.8(V_Q q_p)^{0.56} KCP(LS) \qquad (18)$$

in which V_Q : the volume of runoff
q_p : the peak flow rate
K : the soil factor
C : the crop management factor
P : the erosion control practice factor
LS : the slope length and gradient factor

To determine H requires knowledge of a and h. The routing coefficient a can be determined from equation (3) by replacing T by time to peak T_p and predicting Y and Y_0 with equation (18):

$$a = -\frac{\ln(q_p/Q_p)^{0.56}}{T_p d^{0.5}} \tag{19}$$

in which Q_p : the peak source runoff rate

T_p : the watershed time to peak

3. Instantaneous unit hydrograph (IUH)

The IUH can be specified by the Nash model. In a series of papers, Nash (1957, 1958, 1959, 1960) developed a model based on a cascade of equal linear reservoirs for derivation of the IUH from a natural watershed. This is one of the most popular models of IUH for determining the direct runoff hydrograph (Singh, 1988, 1989, 1992). The continuity equation for a reservoir can be expressed as

$$I - Q = k\frac{dQ}{dt} \tag{20}$$

in which I : the rate of inflow

Q : the rate of outflow

k : the storage parameter

When the instantaneous unit effective rainfall is fed into the first reservoir, then equation (20) becomes

$$h + k\frac{dh}{dt} = \delta(t) \tag{21}$$

in which h : the instantaneous unit hydrograph

$\delta(t)$: the instantaneous unit effective rainfall

Taking the Laplace transform,

$$L\left[h + k\frac{dh}{dt}\right] = L[\delta(t)]$$

$$h(s) + ks \cdot h(s) = 1 \tag{22}$$

$$h(s) = \frac{1}{1+ks} = \frac{1}{k\left(\frac{1}{k}+s\right)}$$

in which L : Laplace transform

The outflow from the first reservoir due to an instantaneous inflow is

$$h_1(s) = \frac{1}{k\left(\frac{1}{k}+s\right)} \tag{23}$$

For the second reservoir,

$$h_2 + k\frac{dh_2}{dt} = h_1 \tag{24}$$

Then

$$\begin{aligned} L\left[h_2 + k\frac{dh_2}{dt}\right] &= L[h_1] \\ h_2(s)(1+ks) &= \frac{1}{k\left(\frac{1}{k}+s\right)} \end{aligned} \tag{25}$$

Thus

$$h_2(s) = \frac{1}{k^2\left(\frac{1}{k}+s\right)^2} \tag{26}$$

Similarly, the nth reservoir can be written as

$$h_n(s) = \frac{1}{k^n\left(\frac{1}{k}+s\right)^n} \tag{27}$$

By taking the inverse of the Laplace transform, the Nash model can be expressed as

$$h_n(t) = \frac{1}{k}\left(\frac{t}{k}\right)^{n-1}\frac{e^{-\frac{t}{k}}}{\Gamma(n)} \tag{28}$$

The two parameters of Nash model, n and k, can be estimated by using the method of moments. The first two moments of IUH expressed by equation (28) can be written as

$$\begin{aligned} M_1 &= nk \\ M_2 &= nk^2(n+1) \end{aligned} \tag{29}$$

n and k can be determined from the first two moments of the IUH.
Thus, the IUSG procedure consists in specification of the SCD and the IUH.

4. Application and analysis

4.1. Study basin

A small upland watershed, W-5, a part of Pigeon Roost basin located near Oxford, Marshall County, Mississippi, was selected for the test of the IUSG using Kalman filter. It has an area of approximately 4.04 km^2, which is 1288 m long and 128.8 m wide. The watershed consists of a rather flat flood plain with natural channels and rolling severely dissected interfluvial areas. The channels have few straight reaches, and most have banks that scour easily. The average channel width-depth ratio is approximately 2:1 at the gaging station.

4.2. IUH

The IUH was determined by Nash model for each event. The parameters of Nash model are the number of reservoirs n and the storage parameter k, and they were estimated using the method of moments. The estimated parameter values are given in Table 1, and the IUH are illustrated Fig. 1.

Table 1. Parameters of the Nash model and the characteristic values of IUH.

storm	n	k (hrs)	peak (m^2/sec)	peak time (hrs)
No.1(72.12. 9)	4.21	0.256	0.793	0.822
No.2(73. 3.14)	1.41	0.472	1.027	0.195
No.3(73. 5.27)	3.95	0.141	1.497	0.416
No.4(74. 7.21)	4.74	0.195	0.968	0.729
No.5(74.11.19)	3.33	0.265	0.888	0.619
No.6(75. 1.10)	2.90	0.405	0.640	0.770
No.7(75. 3.12)	2.13	0.554	0.590	0.624
No.8(76. 2.17)	2.45	0.435	0.673	0.631
No.9(76. 3.20)	1.52	0.340	1.310	0.178

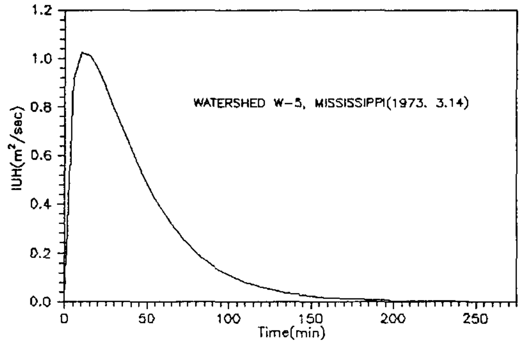

Fig. 1. IUH for watershed W-5, Oxford, Mississippi.

4.3. IUSG

The IUSG was determined by equation (1) and the IUH was determined by Nash model for each event. The parameters for the sediment yield estimated by MUSLE in equation (18) for watershed W-5 are as follows: The soils factor, K, is 0.26, the crop management factor, C, is 0.07, the erosion control practice factor, P, is 0.47 and the slope length and gradient factor, LS, is 0.34. The routing coefficient a, for estimating H is estimated by equation (19) for each event and is given in Table 2. The initial concentration for one unit of runoff, C_{01}, the sediment yield, Y, estimated by MUSLE and H in equation (17) for each event are given in Table 2. The IUSG estimated by equation (1) is illustrated Fig. 2.

Table 2. Characteristic values for the determination of the IUSG.

storm	a	H	C_{01}(mg/l)	Y(t/h)
No.1	0.256	1596.27	195595.1	91.30
No.2	1.735	577.88	257880.3	110.08
No.3	0.328	1893.15	135140.3	136.07
No.4	7.333	833.13	404593.1	18.57
No.5	7.536	878.87	239309.7	57.74
No.6	0.592	888.48	245075.4	62.83
No.7	0.837	697.05	210107.8	70.75
No.8	0.599	970.45	275864.3	59.77
No.9	1.695	690.28	239137.4	124.06

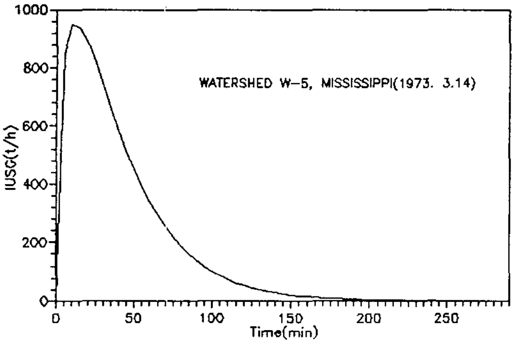

Fig. 2. IUSG for watershed W-5, Oxford, Mississippi.

4.4. Determination of runoff hydrograph

The runoff hydrographs were computed for each event by convolving an observed rainfall with the IUH of Fig. 1. The observed and computed hydrographs were nearly the same as shown in Fig. 3. In each case the obseved and computed runoff peak were occured as almost similar peak flow at the same time.

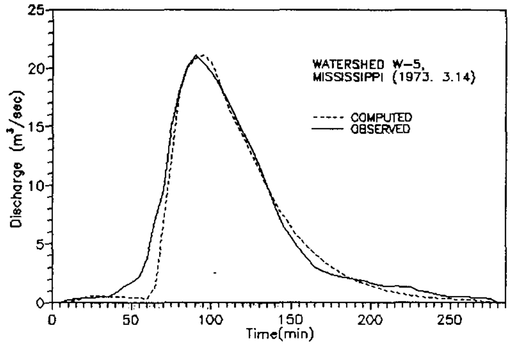

Fig. 3. Comparison of the observed and computed hydrographs.

4.5. Determination of sediment yields

The sediment yields were computed for each event by convolving source runoff with the IUSG of Fig. 2. The observed and computed sediment yield graphs were compared in Fig. 4. The sediment yield and runoff hydrographs possess similar shapes, and have the same duration in Fig. 3 and 4.

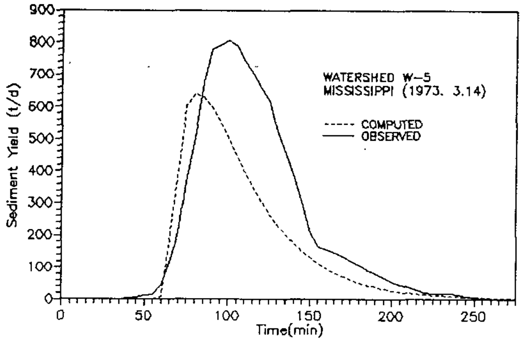

Fig. 4. Comparison of observed and computed sediment yield graphs.

In order to make a quantitative comparison of the computed sediment yield by the IUSG and the observed sediment yield, the predicted result was evaluated and based on: (1) model efficiency, ME; (2) mean square error, MSE; (3) Bias; (4) volume error, VER; (5) peak sediment yield error, PER; and (6) peak time error, TER.

The calculated error indices for the sediment yield by the IUSG are given in Table 3.

As shown in Table 3, the model efficiency for each event is between 0.68 and 0.85. The above results show that the IUSG is a suitable model to predict sediment yields.

Table 3. Error indices for the sediment yield by the IUSG.

storm	ME	MSE	Bias	VER (%)	QER (%)	TER (min)
No.1	0.69	51.17	31.16	37.10	39.87	15
No.2	0.71	143.8	62.33	31.64	20.49	20
No.3	0.70	70.12	46.91	27.86	27.31	20
No.4	0.85	11.20	2.99	12.24	16.91	0
No.5	0.71	35.60	1.35	2.54	41.13	-15
No.6	0.70	97.12	44.06	30.68	46.20	0
No.7	0.75	2.46	0.84	16.06	31.65	-5
No.8	0.68	77.29	48.25	32.49	40.60	-10
No.9	0.76	112.30	56.15	26.33	22.58	-5

5. Conclusions

The following conclusions can be drawn from this study. (1) The sediment yields by IUSG obtain a reasonable results. (2) The IUH and IUSG have the same shape and the same duration for each event. (3) The sediment yield graphs and runoff hydrographs possess similar shapes and the same duration. (4) The parameters of SCD are correlated with the effective rainfall characteristics. (5) The model efficiency for each event is shown between 0.68 and 0.85. (6) The IUSG is a suitable model to predict sediment yields.

References

Chen, V. J., and C. Y. Kuo, 1986, *J. Hydrology*, 84, 35-54.

Nash, J. E., 1957, The form of the Instantaneous unit hydrograph, Publication 42. Wallingford, England, International Association of Scientific Hydrology, pp 114-121.

Nash, J. E., 1958, Determining runoff from rainfall, Proceeding of the Institution of Civil Engineers (Ireland) vol. 10, pp 163-184.

Nash, J. E., 1959, Systematic determination of unit hydrograph parameters, Journal of Geophysical research vol. 64(1), pp 111-115.

Nash, J. E., 1960, A unit hydrograph study with particular reference to British catchments, Proceeding of the Institution of Civil Engineers (London) vol. 17, pp 249-282.

Rendon-Herrero, O., 1974, Estimation of wash load produced on certain small watersheds, Journal of the Hydraulics Division, Proceedings of the ASCE, vol. 100, pp 835-848.

Rendon-Herrero, O., 1978, *Water Resources Research*, 14, 889-901.

Sharma, T. C. and W. T. Dickinson, 1980, *Water Resources Research*, 16, 501-506.

Sharma, T. C. and W. G. S. Hines and W. T. Dickinson, *J. Hydrology*, 40, 299-322.

Singh, V. P., ed., 1982, Applied modeling in catchment hydrology, Littleton, Colo.: Water Resources Publications.

Singh, V. P. and A. Baniukiwicz and V. J. Chen, 1982, An instantaneous unit sediment graph study for small upland watersheds, Littieton, Colo.: Water Resources Publications.

Singh, V. P., 1988, Hydrologic Systems: vol. 1, Rainfall-Runoff Modeling, Prentice Hall, Englewood Cliffs, New Jersey.

Singh, V. P., 1989, Hydrologic Systems: vol. 2, Watershed Modeling, Prentice Hall, Englewood Cliffs, New Jersey.

Singh, V. P., 1992, Elementary Hydrology, Prentice-Hall, pp 585-593.

Srivastava, P. K., R. A. Rastogi and H. S. Chanhan, 1984, *J. Agricultural Engineering, Indian Society of Agricultural Engineers*, 21, 121-126.

Williams, J. R., 1975a, Sediment yield prediction with universal equation using runoff energy factor, In Present and Prospective technology for Predicting Sediment Yields and Sources, 244-52, ARS-S-40, Agricultural Research Service, U. S. department of Agriculture, Washington, D. C.

Williams, J. R., 1975b, *Water Resources Bulletin* 11, 965-974.

Williams, J. R., 1978a, A sediment yield routing model, Proceedings of the Specialty Conference on Verification of Mathematical and Physical Models in Hydraulic Engineering, ASCE, 662-670, August 9-11, College Park, Md.

Wiliams, J. R., 1978b, *Water Resources Resarch* 14, 659-664.