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Intrinsic bubbles in the case of stock prices : A note

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<요 약>

A simple general equilibrium model, where risk aversion and dividend process switching play a key role, shows that a stock price in a bubble-free economy can be observationally equivalent to that of the intrinsic bubble economy. Specifically, I seek a set of conditions under which the functional form of asset prices in the bubble-free economy is the same as that in the intrinsic bubble approach.

I. Introduction

In an important contribution, Froot and Obstfeld (1991) propose a specific type of rational bubbles in stock prices that depend exclusively on aggregate dividends. They argue that this type of bubbles, so-called intrinsic bubbles, provide a more plausible empirical account of deviations from present-value pricing than do the traditional examples of rational bubbles. Their parametric example of an intrinsic bubble appears to be capable of explaining long-term movements in stock prices.

It has been well known that the notion of rational bubbles is problematic. [1] One

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1) Tirole(1982) argues that in a representative agent model bubbles can not exist unless traders are irrational or myopic. Diba and Grossman(1988) claim that there is no possibility that rational

way around this problem is to suppose that equilibrium stock prices are determined in a bubble-free economy. The purpose of this paper is to provide a simple example economy in which a stock price can be observationally equivalent to that of the intrinsic bubble approach, even though the economy is bubble free. In the example economy, risk aversion of agents as well as dividend process switching plays a key role in the determination of equilibrium stock prices.

The paper is organized as follows. In Section 2, the model is constructed along the lines of Lucas (1978). Section 3 shows that a stock price in the bubble-free economy can be observationally equivalent to that of the instrinsic bubble approach. Section 4 concludes the paper.

Ⅱ. The model

The model is a simple version of Lucas (1978), where a representative consumer maximizes an additively separable utility of the form[2) :

$$E\left[\sum_{t=0}^{\infty} \beta^{t} u(C_t)\right], \quad 0 < \beta < 1, \tag{1}$$

where C_t is consumption of a single good at time t,

β is a discount factor,

$u(\cdot)$ is a current period utility function,

$E[\cdot]$ is an expectations operator.

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bubbles can crash and restart. Tirole(1985) shows that in an overlapping generations economy bubbles can be consistent with rationality. However,the model requires that stock prices grow slower than the expected growth rate of the economy,which is unrealistic as suggested by West(1988). These difficulties have led some theorists to abandon the assumption of rational behavior or complete information. One example is Lux(1995) who models the emergence of bubbles as a self-organizing process of infection among traders. The other is Allen and Gorton(1993) who assume that agents are rational but the stock market is characterized by asymmetric information.

2) The assumption is made for the closed form solution to stock prices.The more general approach is to use the non-expected utility developed by Epstein and Zin(1989) and Weil(1990).In an important contribution, Campbell(1993) uses this utility to derive the intertemporal asset pricing model without consumption data.

The utility function of the consumer exhibits constant relative risk aversion (CRRA) of the form :

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \quad \gamma > 0, \quad \gamma \neq 1 \quad \text{or} \quad u(C_t) = \ln C_t .$$

The consumption good is produced on one productive unit. The unit produces its nonstorable consumption good, y, according to the following process[3]:

$$\ln y' = \alpha \ln y + \varepsilon_t , \tag{2}$$

where $0 \leq \alpha \leq 1$ and $\varepsilon_t \sim N(0, \sigma^2)$.

Ownership in the productive unit is determined each period in a competitive stock market. The unit has outstanding one perfectly divisible equity share.
A share entitles its owner as of the beginning of t to all of the unit's output in period t. Shares are traded, after payments of real dividends, at a competitively determined equilibrium price. Let z and x be the consumer's beginning of period and end of period stock shares respectively. The consumer's budget constraint is given by

$$C + P(y) \cdot x \leq y \cdot z + P(y) \cdot z , \tag{3}$$

where $P(y)$ is the equilibrium stock price in terms of y.

The value function, $V(z, y)$, is defined by

$$V(x, y) = \max_{(c,\ x)} \left[u(C) + \beta \int V(x, y') \, dF(y' \mid y) \right] \tag{4}$$

subject to (3).

In this economy, it is trivial to determine the equilibrium price function, $P(y)$, and the value function, $V(z, y)$. In equilibrium, all output will be consumed and all shares will be held, i. e., $C_t = y_t \cdot z_t$ and $z_t = x_t = 1$ for all t. However,

3) For notational convenience, I define y_t , y_{t+1} , y_{t+2} , $\cdots$ as y, y' , y'', $\cdots$. Note also that $P \equiv P_t$, $C \equiv C_t$, $x \equiv x_t$ and $z \equiv z_t$.

sufficient care should be taken in determining equilibrium since CRRA utility is, in general, not bounded.

Assuming the existence and uniqueness of equilibrium[4], I focus on the derivation of the equilibrium price function, $P(y)$. In equilibrium, $P(y)$ must satisfy the following stochastic Euler equation :

$$u'(y)P(y) = \beta \int u'(y')(y' + P(y')dF(y' \mid y) . \tag{5}$$

Define

$$g(y) \equiv \beta \int u'(y')y'\, dF(y' \mid y), \quad f(y) \equiv u'(y)P(y) . \tag{6}$$

Then, $f(y) = g(y) + \beta \int f(y')\, dF(y' \mid y)$, (7)

$$P(y) = \frac{f(y)}{u'(y)} , \tag{8}$$

where $u'(y) = y^{-\gamma}$, $\gamma > 0$,

$\ln y' \mid \ln y \sim N(\alpha \ln y, \sigma^2), \quad 0 \leq \alpha \leq 1$.

My objective is to find $P(y)$ satisfying the above functional equations.

Successive substitution of function $f(\cdot)$ yields

$$f(y) = g(y) + \beta\ E\,[g(y') \mid y] + \beta^2\ E\{E\,[g(y'') \mid y'] \mid y\} + \cdots\cdots . \tag{9}$$

From the definition of $g(y)$ and properties of lognormal distributions ,

$$g(y) = \beta \int u'(y')y'\, dF(y' \mid y) = \beta\ \exp[\frac{1}{2}(1-\gamma)^2\sigma^2]y^{\alpha(1-\gamma)} ,$$

$$E\,[g(y') \mid y] = \beta\ \exp[\frac{1}{2}(1+\alpha^2)(1-\gamma)^2\ \sigma^2]y^{\alpha^2(1-\gamma)} ,$$

$$E\,\{E\,[g(y'') \mid y'] \mid y\} = \beta\ \exp[\frac{1}{2}(1+\alpha^2+\alpha^4)(1-\gamma)^2\ \sigma^2]y^{\alpha^3(1-\gamma)} . \tag{10}$$

Therefore,

$$f(y) = \beta \exp[\frac{1}{2}(1-\gamma)^2\sigma^2]y^{\alpha(1-\gamma)} + \beta^2\exp[\frac{1}{2}(1+\alpha^2)(1-\gamma)^2\sigma^2]y^{\alpha^3(1-\gamma)}$$

4) For the existence and uniqueness of equilibrium, see Lucas (1978).

$$+\beta^3\exp[\frac{1}{2}(1+\alpha^2+\alpha^4)(1-\gamma)^2\sigma^2]y^{\alpha^3(1-\gamma)}+\cdots\cdots.$$

$$=\beta\exp[\frac{1}{2}(1-\gamma)^2\sigma^2]y^{\alpha(1-\gamma)}\{1+\beta\exp[\frac{1}{2}\alpha^2(1-\gamma)^2\sigma^2]y^{-\alpha(1-\alpha)(1-\gamma)}$$

$$+\beta^2\exp[\frac{1}{2}(\alpha^2+\alpha^4)(1-\gamma)^2\sigma^2]y^{\alpha(1-\alpha)(1+\alpha)(1-\gamma)}+\cdots\}. \quad (11)$$

III. Risk aversion, dividend process switching, and stock prices

To highlight the role of risk aversion and the dividend process in the determination of equilibrium stock prices in the one-asset exchange economy described in section 2, assume that the representative consumer has CRRA utility, and that natural logarithms of dividends follow either the IID process ($\alpha=0$) or the random walk process ($\alpha=1$).

Certainly, the agent's preference and the dividend process are the market fundamentals in this economy, therefore equilibrium stock prices are just determined by the above two factors. Note that the economy is bubble free by construction.

Suppose as a first example that the consumer has CRRA utility and that natural logarithms of dividends follow the IID process. In this case, it follows from (8) and (11) that

$$P(y)=\frac{\beta}{(1-\beta)}\exp[\frac{1}{2}(1-\gamma)^2\sigma^2]y^{\gamma}. \quad (12)$$

It is not difficult to verify that $P(y)$ is the equilibrium price function and the value function (4) is well-defined, as in Kim (1986). Note that in this example economy $P(y)$ is a nonlinear function of y if γ is not equal to one.

Suppose as a second example that the consumer has CRRA utility and natural logarithms of dividends follow the random walk process. Then it follows from (8) and (11) that

$$P(y)=\frac{\beta^*}{(1-\beta^*)}y, \quad \beta^*=\beta\cdot\exp[\frac{1}{2}(1-\gamma)^2\sigma^2], \quad 0<\beta^*<1. \quad (13)$$

$P(y)$ is linear in y regardless of γ. It can also be shown that $P(y)$ is the equilibrium price and the value function is well-defined.

Now I am in a position to state the following proposition that gives a set of conditions under which a stock price in the bubble-free economy can be observationally equivalent to that of the intrinsic bubble approach.

Proposition 1. Consider a one-asset exchange economy in which the representative consumer has CRRA utility. Suppose that the consumer anticipates switching of the dividend process some time in the future, i. e., natural logarithms of dividends follow the random walk process for t, but it switches to the IID process for t+T , where t=1, 2, 3, ⋯ and T is an arbitrary natural number.

Then $P(y) = k_0\, y + k_1\, y^{\gamma}$,

$$\text{where} \quad k_0 = \beta^{*} \frac{1-\beta^{*T}}{1-\beta^{*}}, \quad \beta^{*} = \beta \cdot \exp[\tfrac{1}{2}(1-\gamma)^2\sigma^2],$$

$$k_1 = \beta^{T} \frac{\beta}{1-\beta} \exp[\tfrac{1}{2}(1-\gamma)^2\sigma^2], \quad 0 < \beta < 1 .$$

<proof> The proposition can be proved easily by using (8), (11), (12) and (13). Q. E. D.

Note that the equilibrium stock prices in the proposition have the same functional form[5] as that in the intrinsic bubble approach by Froot and Obstfeld (1991), if γ is greater than unity[6]. However, in this example economy the stock prices are driven by market fundamentals only, and thus bubbles are not allowed. The example might be interpreted as evidence of possible future changes in regime, [7] i. e., dividend process switching, as shown by Flood and Hodrick(1986).

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5) Note that this is true, even if the sequence of the dividend processes is reversed. In this case, however, it is required that β^* be between zero and one, which is unnecessarily restrictive.

6) The case that the risk aversion measure is below unity is ruled out to generate high asset price volatility enough to match up the intrinsic bubble model.

7) Hamilton and Whiteman (1985) were the first to show that self-fulfilling expectations are empirically indistinguishable from the misspecification of the fundamentals. This paper can be regarded as a simple extension of their results using the consumption capital asset pricing model. I thank an anonymous referee for having pointed it out.

However, that is not all in the example. Note that it follows from (8) and (11) that $P(y) = [\beta/(1-\beta)]\ y$ when γ is equal to unity, regardless of the dividend process switching. In the example, risk aversion of agents as well as the dividend process is a fundamental determinant of equilibrium stock prices.

The difference between the example economy and the intrinsic bubble model is as follows. The intrinsic bubble model is a continuous time one, and the stock prices are derived from the no arbitrage condition. Intrinsic bubbles are possible, because agents can be compensated sufficiently if bubbles grow enough. However the example economy model is based on discrete time, and the stock prices are determined by competitive equilibrium.

Therefore, bubbles are not allowed in the model by construction.

IV. Conclusion

The purpose of the paper is to seek a set of conditions under which an asset price in a bubble-free economy can be observationally equivalent to that of the intrinsic bubble approach introduced by Froot and Obstfeld (1991). This is a meaningful task in that the notion of intrinsic bubbles is problematic.

The set of conditions derived from a simple general equilibrium model is that the relative risk aversion measure of the representative agent is greater than unity , and natural logarithms of dividends follow the random walk process for t, but it switches to the IID normal process for t+T, where t=1, 2, 3, ⋯ and T is an arbitrary natural number.

The intuition behind this result is quite simple. The result is simply based on the key observation that the risk averse and truly rational agent in the dynamic sense is trying to maximize his or her own lifetime utility given the anticipated dividend process switching. In other words, it is just an optimal response on the part of the risk averse agent anticipating the dividend process switching some time in the future that the equilibrium stock prices include nonlinear components as well as linear ones in the bubble-free economy.

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