JOURNAL OF THE KOREAN ASTRONOMICAL SOCIETY
31: 27 ~ 37, 1998

BINARY MICROLENSING EFFECTS
I. CAUSTICS AND THE FLUX FACTOR K

LEE, DONG WOOK [1], CHANG, KYONG AE [2] AND KIM, SANG JOON [1]
[1]Dept. of Astronomy & Space Science, Kyunghee University
[2]Dept. of Physics, Chongju University
(Received Feb. 23, 1998; Accepted Mar. 25, 1998)

ABSTRACT

We have made semi-analytical studies to investigate the configurations of caustics and the probability distribution of the flux factor K for the binary microlensing including external shears. A parametric equation of critical curve is derived in a 4th order complex polynomial. We present the topological dependencies of the caustics for selected gamma parameters (0, 0.3, 0.6, 1.3, 2.0, and 2.5) and convergence terms (0., 0.4, 0.8, 1.2, 1.6, and 2.0). For the purpose of analyzing the efficiency of High Amplification Event (HAE) on each caustics, we examine the probability distribution of the flux factor by a Monte Carlo method. Changing the separation of the binary system from 0.8 to 1.3 (in normalied unit), we examine the probability distribution of the K-values in various gamma parameters. The relationship between gamma parameters, seperations and their probabilties of the flux factor K have been studied. Our results show that the relatively higher K values (K>1.5) are increased as increasing the separation of the binary system. We therfore conclude that, in the N-body microlensing, the probabilities of higher HAEs are inversely proportional to the star density as well. We also point out that the present research might be used as a preliminary step toward investigating heavy N-body microlensing simulations.

Key Words : Microlensing, Binary, Caustics, HAE: Flux factor-K

I. INTRODUCTION

Gravitational microlensing is the lensing effects of the stars in a macro-lensing galaxy, which was first predicted by Chang and Refsdal (1979, 1984). It was named as *microlensing* by Paczynski (1986) because microlensing splits a macro-image into several sub-images on scales of micro arcseconds. Recentlly many astronomers have paid attention to the microlensing effect to use it as a searching tool for the dark matter in the Galaxy and extra-galaxies.

For the theoretical study of binary lensing, there were works only with various restrictions, such as no external shears and an identical mass ratio (Schneider and Weiss, 1986). So their general lensing behaviors depending on those parameters remain unclear. Since almost all stars are in binary or multiple systems, studies on the binary microlensing could be used for astronomical tools such as extra-planetary system searching, as pointed out by Mao and Paczynski(1991). Therefore we study binary microlensing with all possible free parameters, which could be applied to any situation including internal Galactic lensing or extra-galactic lensing.

Binary microlensing also have been detected by several dark matter observing groups (i.e. MACHO(Alcock et al., 1993), OGLE(Udalski et al., 1992), EROS(Aubourg et al., 1993), DUO(Alard, 1995). Up to now about 200 microlensing events have been observed, and the fraction of binary events is estimated about just above 10% in the total microlensing events (Paczyski, 1995). We have investigated the binary microlensing including all possible parameters (i.e. external shear, continuous matter and an arbitrary mass ratio), so our results could be applied to the Galactic microlensing and extra-galactic microlensing as well. We derive an anarytical expression of the critical curve. In this paper, we present some topologies of the critical curves and the caustics with various lens parameters. To understand the features of *HAEs* of binary microlens, we calculate the probability distribution of the flux factor K with a Monte carlo method.

II. THEORY OF MICROLENSING

The lens equation describes the mapping of the photon trajectories from the lens plane to the observer plane, which can be repesented by a conventional nomalized lens equation(Chang and Refsdal, 1979; Schneider et al. 1992),

$$\vec{z} = \begin{pmatrix} 1-\kappa_c+\gamma & 0 \\ 0 & 1-\kappa_c-\gamma \end{pmatrix} \vec{\zeta} - \frac{m_1}{|\vec{\zeta}-\vec{\zeta}_1|^2}(\vec{\zeta}-\vec{\zeta}_1) - \frac{m_2}{|\vec{\zeta}-\vec{\zeta}_2|^2}(\vec{\zeta}-\vec{\zeta}_2), \tag{1}$$

where $\vec{\zeta}$ (ξ,η) and $\vec{z}$ (x,y) are the two dimensional vectors of a light ray in the lens and the observer plane, respectively. κ_c is the continuous mass density of interstellar matter and/or dark matter in the unit of the critical surface density. γ is the external shear term induced by the mass distribution of a macrolens, which causes the astigmatic property, and m_i is the mass of the ith microlens in unit of an arbitrary reference mass (e.g. $M_\odot$). The critical surface density of which focal point locates on an observer plane is(Young, 1981; Paczynski, 1986),

$$\sum crit. = \frac{c^2 D_s}{4\pi G D_d D_{ds}}. \tag{2}$$

On the each plane, all length scales are normalized by following values:

$$\begin{aligned} &(\frac{D_s}{D_d})r_0 : source - plane, \\ &r_0 : deflector - plane, \\ &(\frac{D_s}{D_{ds}})r_0 : observer - plane. \end{aligned} \tag{3}$$

The Einstein radius, r_o, is given by Refsdal(1964),

$$r_0 = \sqrt{\frac{4GM}{c^2}D_{eff}}, \tag{4}$$

where M is the mass of a lens, and other symbols have their usual meaning. r_0 represents the Einstein radius of a arbitrary mass, r_o is also used as a reference scaling factor. D_{eff} is called effective distance that contains all cosmological parameters in the lens equation, which is,

$$D_{eff} = \frac{D_d D_{ds}}{D_s}, \tag{5}$$

where D_s and D_d are angular diameter distances from the observer to the source and the lens, and $D_{ds}$is the distance between the lens and the source.

III. CRITICAL CURVES AND CAUSTICS

Critical curves are defined by closed curves where Jacobian of the lens equation are vanished (Bourussa and Kantowski, 1975; Chang, 1979) in the lens plane, and the caustics are the mapping of the critical curves to the observer plane. That is,

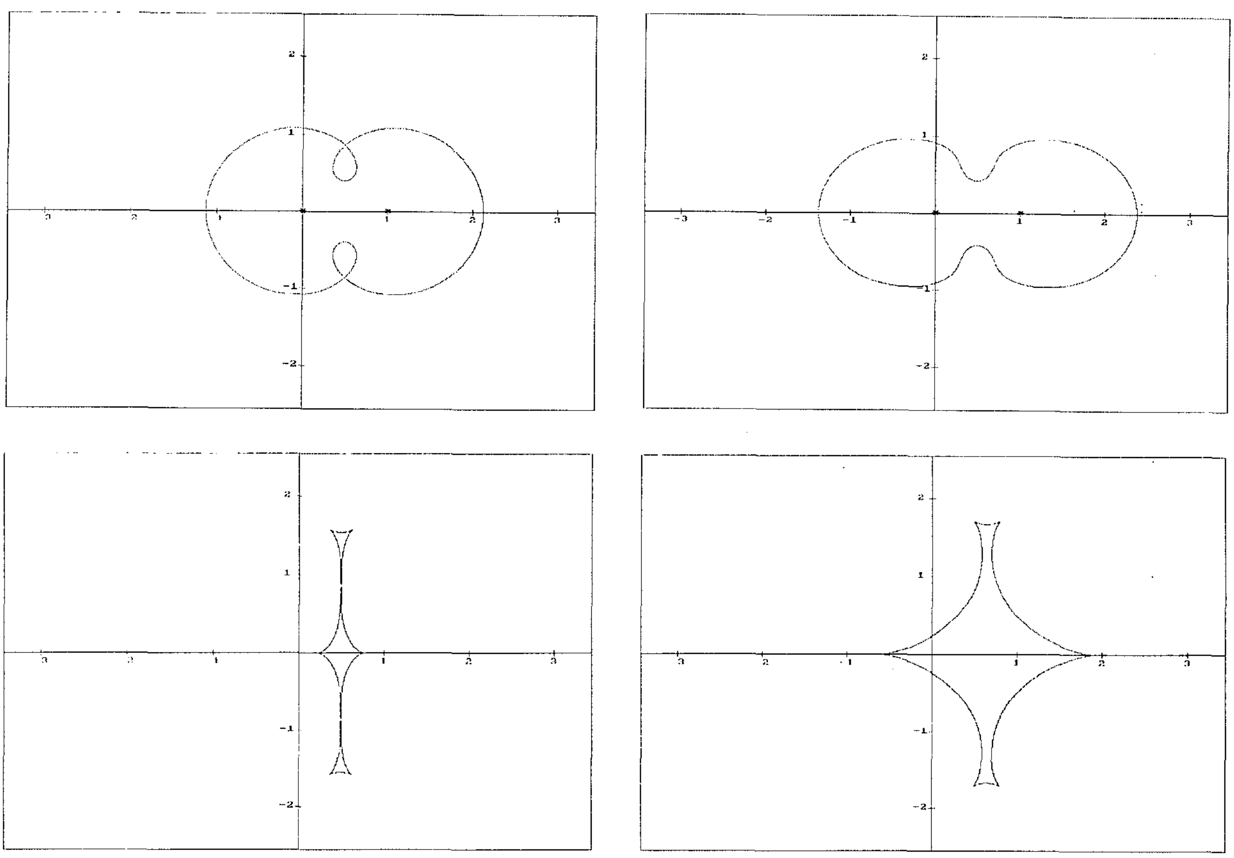
$$J = |\frac{\partial(x,y)}{\partial(\xi,\eta)}| \equiv 0. \tag{6}$$

The Jacobian of the lens equation is directly related to the amplification event of a light bundle. The amplification is inversely proportional to the determinant of Jacobian matrix. Whenever a source crosses the critical line, the number of images changes by ± 2, which is always hold. Since this effect relates with the appearing or disappearing of images, it would have an important role in gravitational lens theory, so called *High Amplification Events* (hereafter, HAEs), especially in the microlensing. For mathematical convenience, we use complex notations to derive a parametric equation of critical curve, but it is not indispensable. We derive the parametric critical curve equation including all free parameters (external shear, arbitrary mass ratio and positions of the masses). The complex polynomial of degree 4 in $re^{i\theta}$ is,

$$
\begin{aligned}
F_{crit.}(X) =& [(1-\kappa_c)e^{-i\theta}+\gamma]\times X^4 \\
&-2[(1-\kappa_c)e^{-i\theta}+\gamma](X_1+X_2)\times X^3 \\
&+[((1-\kappa_c)e^{i}+\gamma(X_1^2+X_2^2)+m_1+m_2]\times X^2\,, \\
&+[-2(m_1X_2+m_2X_1)]\times X \\
&+(X_1^2X_2^2+m_1X_2^2+m_2X_1^2)
\end{aligned}
\qquad (7)
$$

where X_1 and X_2 are the positions of mass components, m_1 and m_2 are the normalized masses of microlenses. Above parametric equation can be readily solved by a simple numerical solver. Solving this equation, we set the optical axis to get though the primary component of the binary system(i.e. $X_1 = 0.0$)

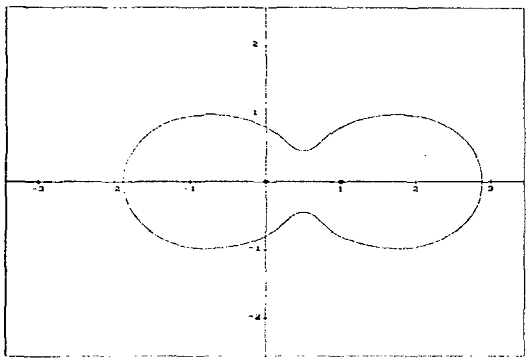
1-a) γ =0.0 2-a) γ =0.3

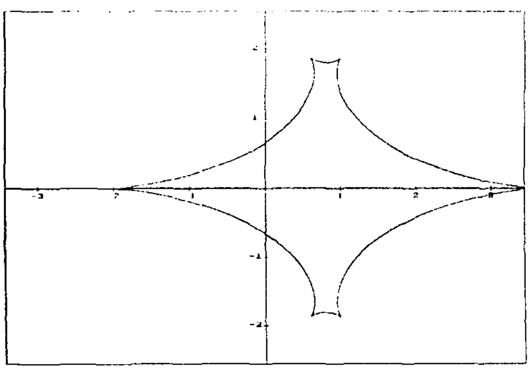


1-b) 2-b)

Fig. 1. The structures of the critical curves (a) and the corresponding caustics (b) for various γ -parameter (0.0, 0.3, 0.6, 1.3, 2.0, and 2.5). In Fig. (1-a) and (1-b), two ghost critical curves and ghost caustics are shown. When a gamma parameter is larger than 1 and 2, the critical curves are divided into two parts and four parts, respectively(see Fig. 5 and 6).

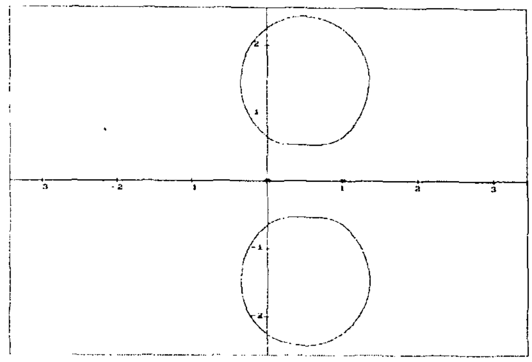
3-a) $\gamma=0.6$

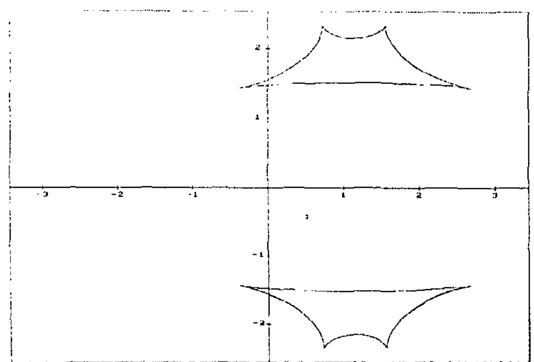




3-b)

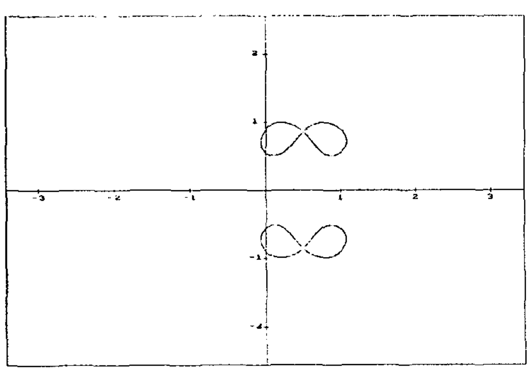
4-a) $\gamma=1.3$

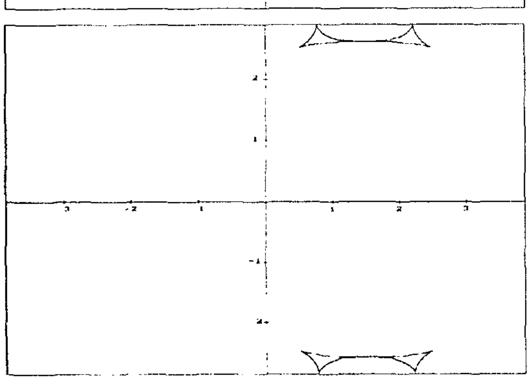




4-b)

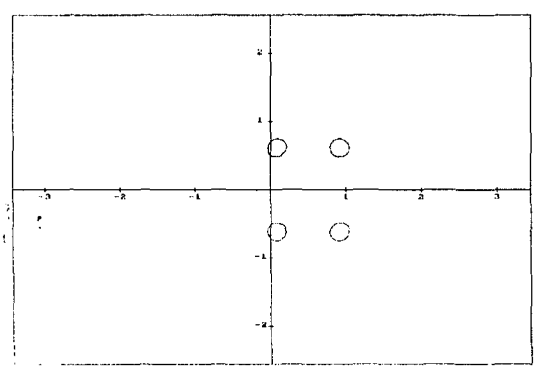
5-a) $\gamma=2.0$

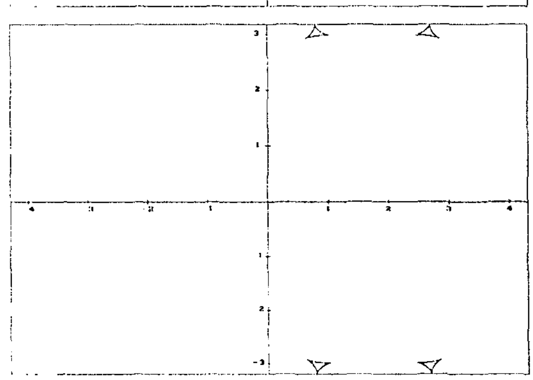




5-b)

6-a) $\gamma=2.5$





6-b)

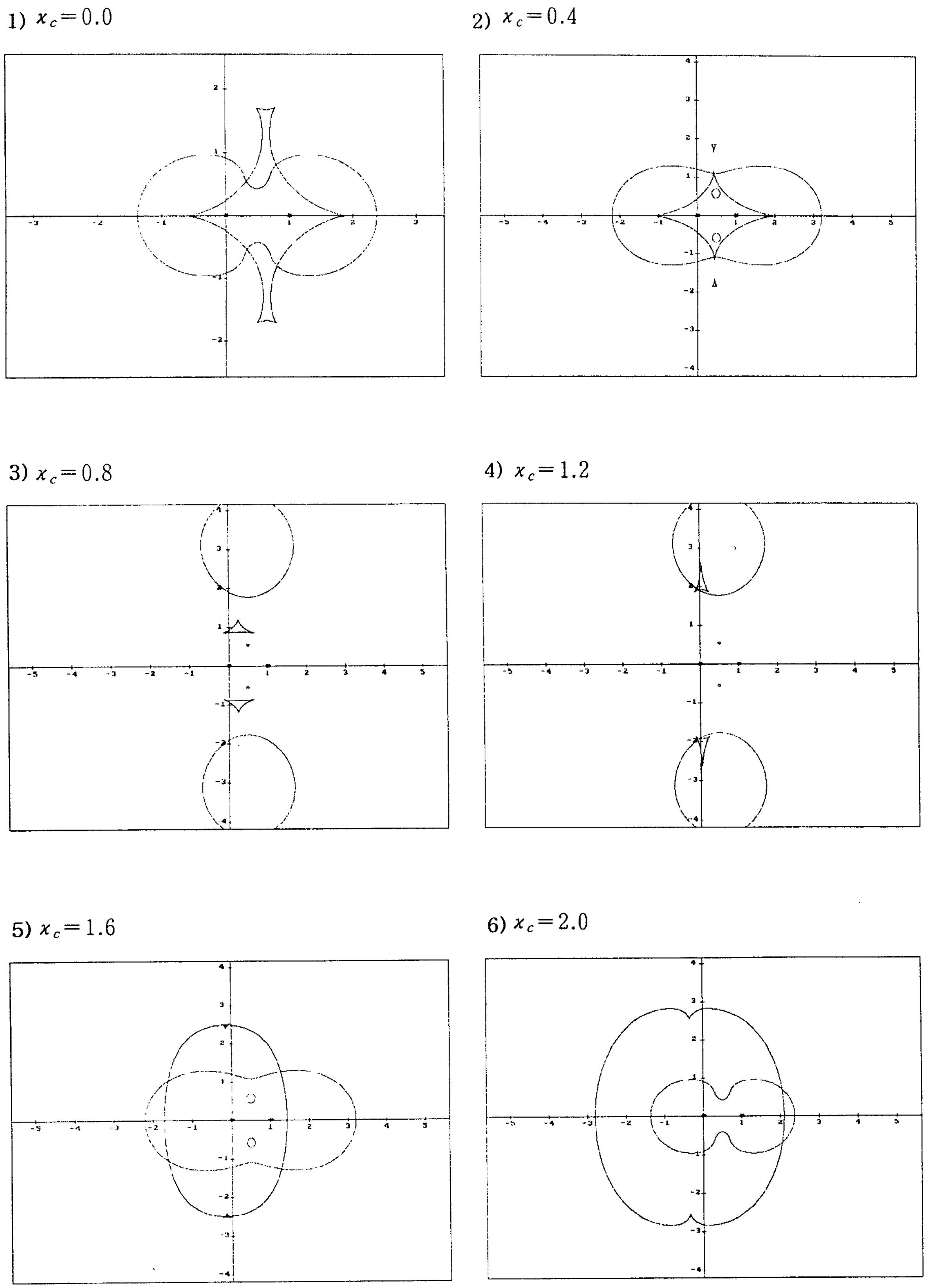


Fig. 2. The morphological variation of the critical curves and caustics for κ_c values (with fixed values of γ=0.3, d(separation)=1.0. Fig. 2 shows 2 ghost critical curves. Fig. 5 and 6 show the overfocusing cases. !Note: In these figures, the critical curves in the lens plane and the caustics in the observer plane are overlapped. Bold line represents the critical curves, and thin line represents the caustics.

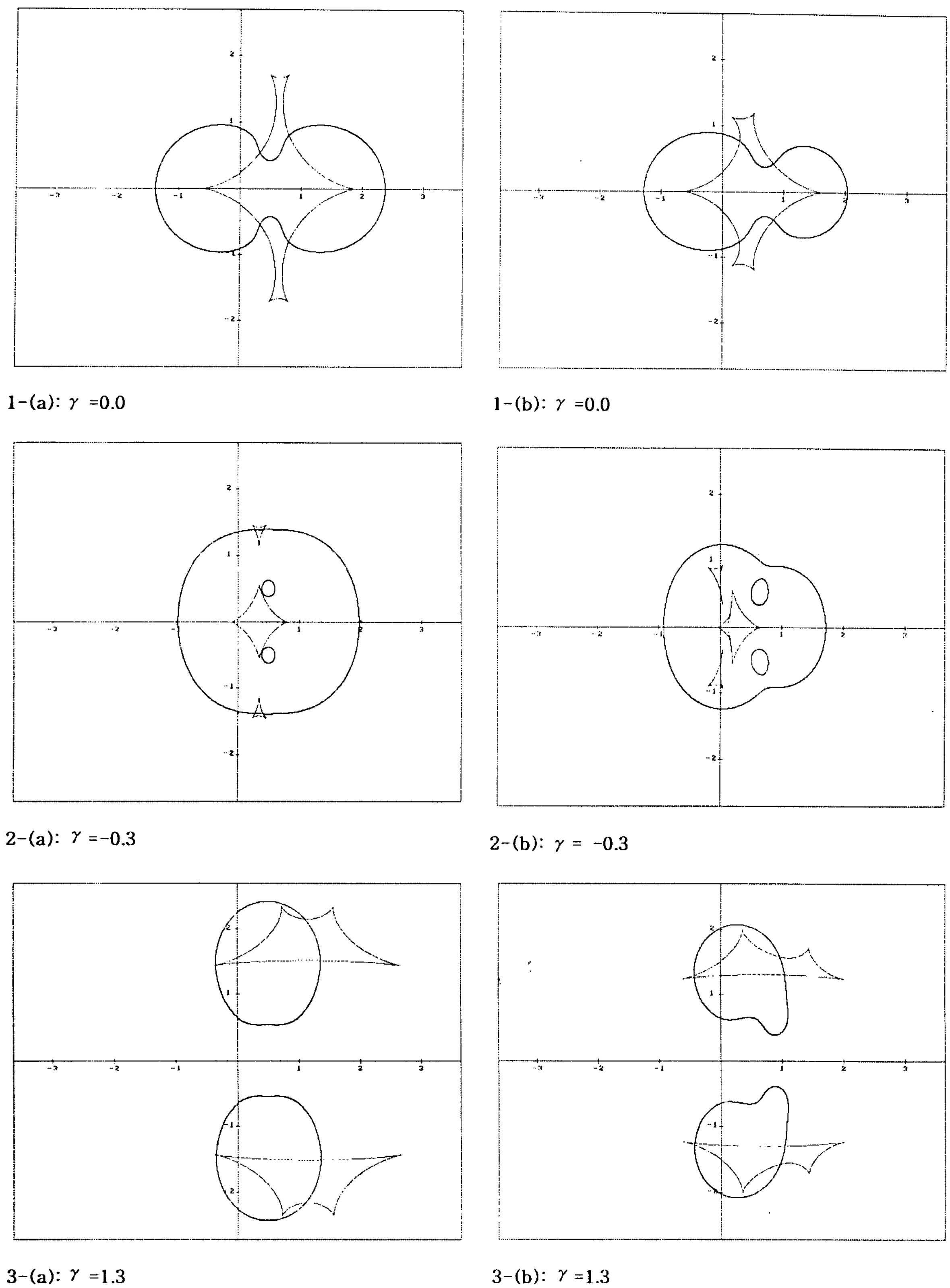


Fig. 3. The effect of mass ratio to the critical curve structures. Fig. 1,2,3-(a) show the case for an identical mass ratio(1:1), and Fig. 1,2,3-(b) show a different mass ratio(1:0.5). Two mass components are located at the points(0.0,0.0) and (1.0,0.0) on the normalized coordinates, respectively

IV. PROBABILITY DISTRIBUTION OF THE K-VALUE

The flux factor-K is the most important parameter describing the strength of an amplification event by microlensing, so statistical study of the K-value for a given caustic curve may help us to get an insight for each lens parameter. The concept of the factor K was firstly introduced by Chang(1979,1984), which was simplified later more adequately by some authors (Kayser and Witt 1989). Since the Jacobian of a lens equation vanishes on the critical line within the approximation of geometrical optics, this factor is of importance to analyze the lightcurve induced by the microlensing effects. The flux factor K proposed by Kayser and Witt(1989) is given by,

$$K = \sqrt{\frac{2}{|T_z|}}, \tag{8}$$

which is independent of any cosmological parameter, due to the absence of the terms including distance relation. where the tangential vector of a caustic curve, T_z,

$$T_z = J\begin{pmatrix} \frac{-\partial|J|}{\partial y} \\ \frac{\partial|J|}{\partial x} \end{pmatrix}, \tag{9}$$

which is valid on the all caustic lines, except for the cusps. On the relation of maximum amplification given in eq. (10), the K value, the form factor and R_s (the source size in normalized unit) represent the flux variation by appearing of two additional images during the HAEs (Chang, 1981). The form factor, f, is determined by a source brightness profile, for a uniform surface brightness of a source, f=1.39 and for the case of a limb darkening, f=1.47 (Kayser & Witt, 1986). Therefore, the total magnitude variation during a HAE should satisfy the relation given by,

$$\Delta m_{HAE} \leq 2.5log(1 + \frac{Kf}{\sqrt{R_s}}), \tag{10}$$

which describes the maximum bitghtness variation in magnitude that can be occurred when a HAE happen with the crossing of a source whoes size is much smaller than one Einstein radius on the caustic line. If a certain HAE is detected with photometric observations, we may use an useful confining relation to determine the radius of a source. Using above relation, we have an equation confining the source size;

$$R_s \leq (\frac{Kf}{10^{(0.4m_{HAE})} - 1})^2, \tag{11}$$

If we know the K-value of a given lensing event and its maximum magnitude as well, we will be able to determine the maximum source size by eq. (11) with observed light curves. We investigated the probability distribution of K values on the caustics with a Monte Carlo simulation. To generate uniform deviated random numbers through 0 to 2π, we use the RAN1 routine (Press et al. 1992) that has sufficiently long period($\approx 2^{31}$). With the parametric equation of critical curves given in eq. (7), we obtained the probability distribution of normalized K values on a caustic curve. We compute the probability distribution of the K-value with a Monte Carlo calculation.

$$\frac{dP}{d\Delta K} = \frac{1}{N}\frac{\delta n_i}{\delta\Delta K}, \tag{12}$$

where P is the probability of each event, n_i is the number of each event and ΔK is the bin size of the K values($\equiv 0.2K$). We used total 40,000 points on each caustic line, and the statistical error is set to the 90% confidence level (i.e. 1.647σ)

V. CONCLUSION

We have investigated structures of the critical curve and the amplificaton properties due to binary microlens systems. For the case of the system consisting of equal masses without external shear, the critical curves of the

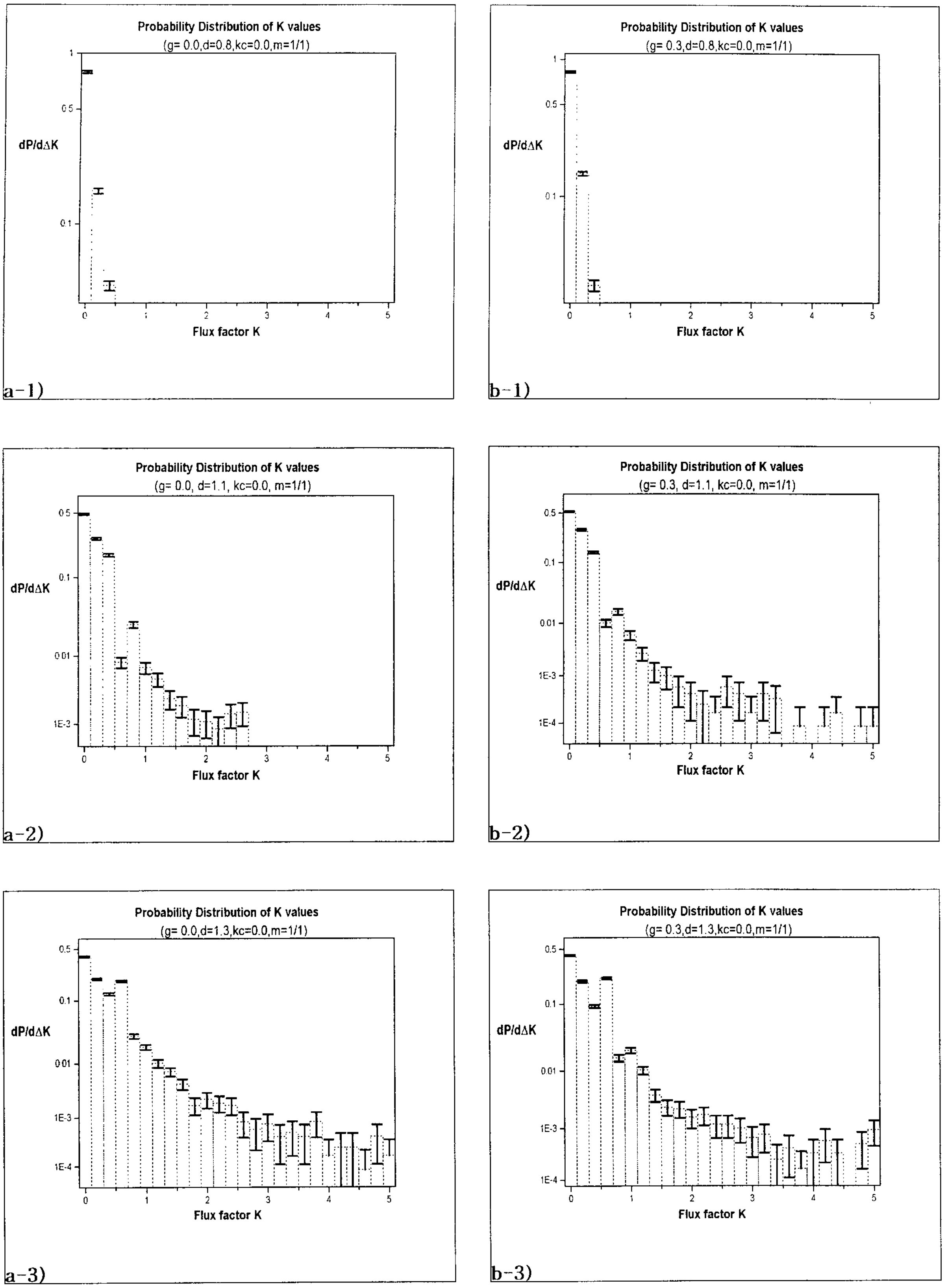


Fig. 4. Probability distributions of the K value for various γ parameters.

Fig. a-(1,2,3) and Fig. b-(1,2,3) show for the case of γ=0.0, and γ=0.3 respectively with a separation parameters(d=0.8, 1.1 and 1.3).

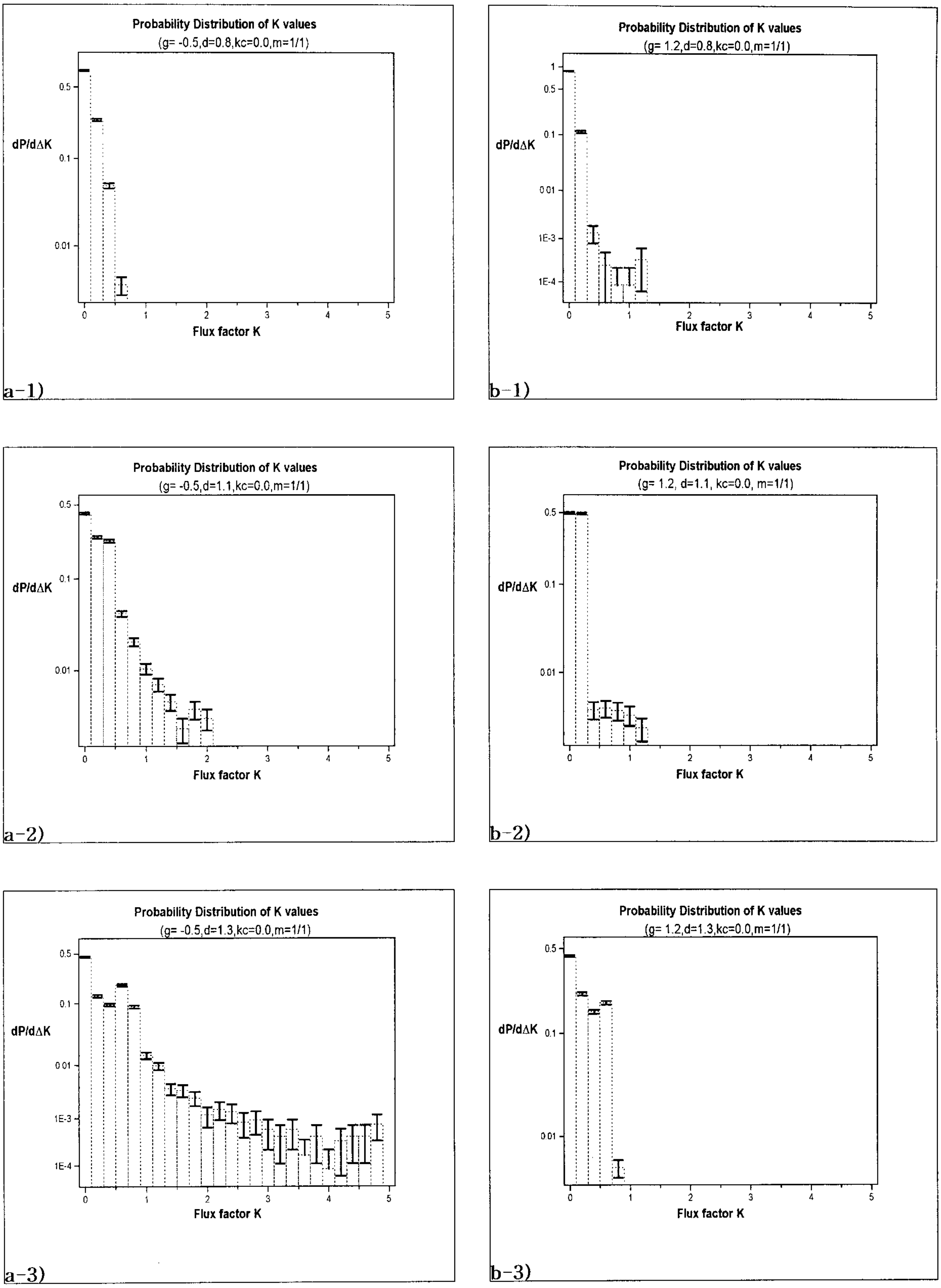


Fig. 5. Probability distributions of the K value for γ parameters.
Fig. a-(1,2,3) and b-(1,2,3) show for the case of γ=-0.5 and γ=1.2 respectively with a separation parameters(d=0.8, 1.1 and 1.3).

binary systems of which separation is smaller than one Einstein radius are no longer Cassini oval(see Fig.1(1-a)). Ghost critical curves are then formed. With a positive external shear, the separation parameter should be less than that of no shear cases in order to form the ghost critical curves. The external shears have made much more complexities for the caustic curves such as self-intersecting points disscussed by Schneider et al.(1992). Even though the critical curve is a single closed shape, its corresponding caustics can have very complicated structures (i.e. butterfly and swallowtail structures). We found that, especially for the cases of $\gamma < 0.0$, the arbitrary mass ratios are strongly responsible for the complexities of the caustics(see Fig. 3, 2-(a,b)). Whereas $\gamma = 0$, there is usually no self-intersection points (see Fig. 3,1-(a,b)), except for the cases of overfocusing and a different mass ratio with a very closed separation(< 1.0).

Table. 1. Configurations of the critical curves and the caustics.

lens type / cases	Chang-Refsdal lens	Binary micro lens
Number of critical regions	1 ($\|\gamma\|<1$) 2 ($\|\gamma\|>1$)	1 or 3 (2 ghost) ($\|\gamma\|<1$) 2 or 4 ($\|\gamma\|>1$)
Number of caustic regions	for over focusing cases ($\|x_c \pm \gamma\| > 1.0$), from 1 to 5	*for special γ cases; from 1 up to 21 (including self-intersection regions)

For various values of gamma parameter, the evolution of the probability distribution of the K factor has been calculated, as varying the separation between two components from 0.8 to 1.3(in normalized units). The histograms of probability distribution of the flux factor K (hereafter P_k) are presented in Figs. 5 and 6. We have calculated them varying two parameters, the γ -parameter and the separation of the binary system. We have read the following tendency from the P_k. We have found that the greater the separation of the system, the higher the value of the flux factor with $|\gamma| < 1.0$.With $\gamma < 1.0$, the highest value of K could reach up to 5 or even heigher value, but their probability of $K > 3.0$, are very rare, which is in the order of 10^{-3} down to 10^{-4}. We thus conclude that such a high and rare values of K may not be responsible for the high amplification events (HAEs) due to the convolution effect with a source profile (Grieger et al, 1984). In the view of the relation of optical depth, our results are well agreed with that of Witt(1990).

In the cases of $|\gamma| > 1.0$, the all K values are of relatively smaller value(< 1.5) as compared with the case of $|\gamma| < 1.0$. Contrary to $|\gamma| < 1.0$, the separation of the binary system would not influence to the flux factor K. From these results of the present studies, we also are dare to predict that there will be no significant differences of HAEs in the N-body microlensing, whether external shearing effects are taken into account or not(see Witt, 1990). With $|\gamma| < 1.0$ the separation of microlens may take a great extent role to decrease or/and increase the value of the flux factor K for n-body microlensing. It is important to mention, however, that $|\gamma| > 1.0$, the optical depth of the n-body lensing system would not influence to the variation of the K values. Our results show that the most dominant factor for changing the K value in the microlensing seems to be the mean separation of the micro-lenses.

ACKNOWLEDGEMENTS

This work was supported in part by KOSEF under the grant 971-0203-012-2. One of the authors(Lee D.W.) would like to express many thanks to the staffs of the Kyunghee observatory for the allowance of using its facilities.

REFERENCES

Alard, C. 1995, Astrophysical Applications of Gravitational Lensing, Proc. IAU Sympo. 173, 215, eds. C. S. Kochanek J. N. Hewitt (Kluwer: Boston)
Alock, C., Akerlof, C. W.. Allsman, R. A. et al. 1993, Nature, 365, 621

Aubourg, E., Bareyre, P., Brehin, P. et al. 1993, Nature, 365, 623
Bourassa, R. R. & Kantowski, R. 1975, Ap.J, 195, 13
Chang, K. and Refsdal, S. 1979, Nature, 279, 561
Chang, K. Ph. D thesis, 1981, Hamburg Univ., Germany
Chang, K. 1984, A & A, 130, 157
Chang, K. and Refsdal, S. 1984, A & A, 132, 168
Grieger, B., Kayser, R. & Refsdal, S. 1988, A & A, 194, 54
Kayser, R. and Witt, H. J. 1989, A & A, 221, 1
Mao, S. and Paczynski, B. 1991, Ap.J, 374, L37
Paczynski, B. 1986, Ap.J, 301, 503
Paczynski, B. 1995, Astrophysical Applications of Gravitational Lensing, Proc. IAU Sympo. 173, 199 eds. C. S. Kochanek & J. N. Hewitt (Kluwer; Boston)
Press, W. H., Tehkolsky, S. A., Vettering, W. T. & Flannery, B. P. 1992, Numerical Recipes in C (Cambridge Univ. press)
Refsdal, S. 1964, MNRAS, 128, 295
Schneider, P. and Weiss, A. 1986, A & A, 164, 237
Schneider, P., Ehlers, J., & Falco, E. E. 1992, Gravitational Lenses, (Springer-Verlag; Berlin)
Udalski, A., Szymanski, M., Kaluzny, J, Kubiak, M, Mateo, M. 1992, Acta. A., 42, 253
Witt, H. J. 1990, A & A, 236, 311
Young, P. 1981, ApJ, 244, 756