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A Comparative Study of Mathematics Curriculum Between Korea and the United States

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국문 초록(Abstract in Korean): 본 연구는 학교 교육과정 가운데 특히 수학과 교육과정에 초점을 맞추고, 미국과 한국을 중심으로 수학과 교육과정의 변화를 초래한 근본적인 원인을 분석하고, 두 나라의 중학교 수학과 교육과정의 체제와 내용을 비교해 보고자 시도하였다. 이러한 비교는 교육과정의 공통성과 차이성을 찾아서 한국 교육과정의 사회적 및 개인적 적합성을 평가하고, 이후 한국의 교육과정 개선을 위한 방안들을 모색하기 위한 것이다.

이미 미국의 경우 1980 년대 들어 서면서 정보화 사회에 적응할 수 있는 수학과 교육과정의 개발 작업에 노력해 왔으며, 한국도 1980 년대 후반부터 제 6 차 교육과정의 개발을 위한 연구를 시작하였다. 그 결과, 미국은 NCTM (미국수학교사협회)을 중심으로 새로운 수학교육의 표준을 설정하고, 향후 수학교육이 지향할 방향과 전략을 설정한 바 있다. 또한 한국은 제 6 차 교육과정 개정 작업을 통하여 1992 년에 새로운 교육과정을 고시하였다.

물론 양국의 수학과 교육과정을 비교、분석하기 위해서는 그 범위와 대상을 폭 넓게 정할 수도 있겠지만, 본 연구에서는 분석의 대상을 최근 미국의 수학과 교육과정의 근간을 이루고 있는 NCTM 의 일련의 교육 표준화 관련 연구들과 한국의 제 6 차 교육과정에 나타난 수학과 교육과정으로 제한하였다.

본 연구에서는 양국의 수학교육을 이해하기 위하여

1) 양국의 수학과 교육과정에 나타난 수학교육의 일반적 성격, 기본 방향 교육 목표를 비교、분석하였고,
2) 양국의 중학교 수학 교육과정에 나타난 교육 내용을 비교해 보았다. 이를 위해서, 본 연구는 NCTM 의 교육과정 안에 명시된 중학교 과정의 수학과 교육 목표 및 내용을 준거로 하여 한국 교육과정의 관련 내용을 분석하고 비교학적으로 해석하는 방식을 취하였다.

물론 한 국가의 교육과정 체제를 목표 및 내용 요소의 비교만으로 파악할 수 없다고 본다. 향후 미국과 한국의 교육과정을 이해하기 위한 연구들은 내용의 조직, 방법, 평가, 그리고 운영계획 등에 관한 분석으로 확대되어 시도되어야 할 것으로 본다.

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I. Background of the Problem

Schools have been asked to respond to a number of the social and political issues that are commanding the nation’s attention. The call for reform is a way of asking that institutions be rededicated to national ideals to which people are deeply committed. The act of reform, in contrast with mere change, is an act of social commitment. The enlightened public awareness of the close connection between a nation's social progress — including economic well-being, political development and cultural maturity — and the state of education systems means that the structure of a nation’s economy is very much related to the success of its educational system. Peoples all over the world believe in the value of education and want its benefits for their children, who must adapt to a rapidly changing society.

In the context of these developments, many countries have changed or are in the process of changing their national curricula. Several countries, including Australia, Japan, Korea, The Netherlands, and the United States began to institute changes in mathematics education in the 1980s. That is not the coincidence it appears to be, but an active effort to cope with rapid social transformation. Most visible, of course, has been the development of computers and the explosive growth of computer applications in response to the world’s proliferating information needs and capabilities. As national economies adapt to information-age needs, workers in every sector of society must learn to interpret intelligent, computer-controlled processes. Most of these applications have required the development of new mathematics in areas in which this was not feasible before the advent of computers. As Kirsner noted:

> The present curriculum has not kept pace with the explosion of mathematical knowledge that this century has witnessed · · · Nor has it adjusted to the realities of technology or the demands of the workplace (Kirsner 1990, pp. 1–2).

In addressing the many forces at work during any change in education, it is important to consider three factors in the context of the larger society that must be considered in restructuring the mathematics curriculum. These are:

1. The rapid changes society is undergoing at present.
2. The changes occurring in the field of mathematics.
3. The evolution of new theories for the learning and teaching of mathematics.

The changes taking place in society seem to involve not only unprecedented momentum and diversity, but they are transforming the culture technologically. The

Information Age has arrived, and most societal institutions are experiencing profound changes as a result. Various areas of human life have been or are being transformed by the presence of new electronic technologies. Computerized electronic technology makes possible not only the wide and rapid distribution of information, but its manipulation, analysis, and recombination as well.

If these powerful technologies of the Information Age were as widely available in schools as they are in other societal organizations, what changes in curriculum content and in the perspectives of instruction would be indicated? A number of changes could occur: the addition of new knowledge to the existing curriculum, without changing pedagogy to incorporate the new information technologies; changes in the content of the traditional curriculum; and reorganizing the curriculum and the styles of classroom teaching (White 1987, p. 2).

Rapid change in technology has occasioned a reexamination of the mathematics curriculum including its goals, content, teaching activities, and ways of evaluation or assessment. Advanced technology has dramatically changed the nature of the discipline itself and its impact on human life. Changes in technology offer impressive opportunities to improve teaching, but promise to have an even more profound effect on the content of the mathematics curriculum. For example, availability and use of hand-held calculators has made it unnecessary for students to develop the ability to perform with pencil-and-paper such calculations as long division and addition of long columns of multi-digit numbers.

Given current technology, it seems more appropriate for students to understand the principles behind such arithmetic operations instead of spending long hours to execute the algorithms. In summary, we should provide our students an opportunity to learn technological and reflective knowledge, so that they will be able to live in society more intelligently.

Secondly, there are changes in mathematics itself. In the past quarter of a century, significant changes have occurred in the nature of mathematics and its uses. In part, it is because of the nature and rapidity of these changes that new perspectives in mathematics education, the so-called social constructivist philosophy of mathematics has emerged (see Romberg 1992b; Ernest 1991). Not only has much new mathematics been discovered, but also the types and variety of problems to which mathematics is applied have grown at an unprecedented rate.

New subjects are slowly and sometimes cautiously introduced into curricula — a prominent example is discrete mathematics, and there seems to be a revival of geometry. Again, some of these subjects are taking their place in the curriculum because new technologies have opened new possibilities. However, a phenomenon like the revival of geometry has offered us new insights into mathematics education, in large part because current research is exploring new understandings of childrens' learning processes. The central place of calculus, the emphasis on fractions and percentages, and the role of logarithms have been explored in the last decade with mixed results. But these changes too represent changes in mathematics that throw a new light on mathematics curriculum.

Thirdly, the content of mathematics itself is changing; we are also witnessing the evolution of new theories for the learning and teaching of mathematics. There is a growing consensus among cognitive scientists and mathematics educators that abstract mathematical concepts are constructed by learners through reflection and conceptual activity (Resnick 1987).

At the Freudenthal Institute in The Netherlands, the theory for "realistic mathematics education" evolved during 20 years of developmental research that in several important respects correlates with the constructivist approach. One of the key components of realistic mathematics education is that students are able to reconstruct or reinvent mathematical ideas and concepts through exposure to a variety of "real-world" problems and situations (de Lange 1987; Freudenthal 1991). Of course, there is a slight difference between social constructivism and a realistic approach. The former is, in the first place, a theory of learning in general, while the latter is a theory of learning theory that stresses students' potentiality to pose problems and solve those problems in their own ways.

In summary, the three changes mentioned above might provide a possibility to spur educators' and publics' sincere concerns on education. The world of mathematics, science and technology has changed dramatically in the past twenty years in line with social changes. In the future, those changes will be more accelerated.

To assure that our students are prepared for the future, we have to reconsider goals, content, teaching, and assessment perspectives in education. Such reexamination will justify new, strikingly different school curriculum. Further, there is reason to believe that these new curricula will appeal to students who find current educational programs very unattractive.

In America, there are several recent initiatives that address these changes positively and are serving to stimulate public interest in education. These have developed on many fronts and have taken many forms, including national goals, curriculum frameworks, various national testing activities, and national efforts to reform teacher education and selection, not to mention the seven goals adopted by the President and governors' conference in 1990 and incorporated in the America 2000 proposal (U.S. Department of Education 1991), which articulates a future vision of America that could well speed up the reform of American education, and function as a guideline of reform efforts in specific subjects.

There has been a widespread uneasiness that the United States is an underachieving nation in school mathematics and that the current school curriculum is only helping to create a nation of underachievers. These vague anxieties seem to have been vividly confirmed by a series of international comparison studies on student achievement in mathematics.

The results of the international comparison studies have awakened Americans to the importance of mathematics as a tool for stability in an era of rapid change (McKnight et al. 1987; Dossey et al. 1988). Since the late 1960s, the major barometer of the status of mathematical health of U.S. students has been the mathematics assessment of the National Assessment of Educational Progress (NAEP), a program of the United States Department of Education (Romberg et al. 1991a). (In 1988, the United States Congress, reacting to public concerns over the state of education, increased the emphasis to be given to mathematics in the NAEP program by requiring assessments in mathematics on a biennial basis beginning in 1990.) In more specific studies, detailed information has recently been obtained on American students' poor performance in mathematics and on the reasons for it (Romberg et al. 1992a; Kitchen et al. 1993).

In Korea, a new curriculum revision at each level of schooling (K–12) has been completed, which has been accepted among those concerned with education as an active and relevant response to diverse changes in society. Since the early 1980s, there has been a rapidly escalating pressure for educational democratization in Korea. Democratization in education has two interrelated meanings. On the one hand, it means educational independence or autonomy — in other words, assuring political neutrality so that educational systems are free from domination by a particular political ideology. This goes so far as to create a mechanism for removing curriculum contents reflecting the ruling party's ideology. On the other hand, it means the democratic administration of educational systems, and the establishment of a democratic decision-making process. We label this process procedural democracy.

In Korea, the recently completed Sixth Revised Curriculum is reputed to be most democratic: yet, while the task forces that participated in the curriculum revision may have had an opportunity to exercise their expertise autonomously, they have had no chance, lamentably, to compare the result with the curricula of other countries.

Within the context of Korea's latest curriculum revision, the present study is certainly timely, since the explicit purpose of this paper is to analyze the present structure of mathematics curriculum of the United States and Korea and identify some of the common features and differences between them in order to understand certain characteristics of mathematics curricula internationally. In this paper, recent principles for the reform of mathematics education in the U.S., especially a series of new visions disseminated by the National Council of Teachers of Mathematics (NCTM) since 1989 — evolving out of NCTM's An Agenda for Action (1980) — will be described and compared with Korean mathematics curriculum in terms of some fundamental considerations of formal curriculum.

Major differences in national philosophy and structure make simple comparisons of educational programs across countries difficult. Because there are major variations in educational systems among countries, we have to acknowledge the difficulty in international comparison. Consequently, in this paper my goal will be restricted to an examination of how much curriculum and its expectations vary from America to Korea. In other words, this study limits the primary task to a comparison of goals and content structures that are explicitly documented in mathematics curriculum in Korea and the United States.

The justification for this approach is embedded in the assumption that the efficacy of a particular curriculum can not be evaluated in isolation, but must be judged authentically by comparing it with other curricula, as if we become able to understand our behaviors more objectively through reflecting on others in daily life. In spite of the difficulties involved, it might be very helpful to compare different curricula from various perspectives.

II. Overview of Curriculum Staucture

1. The United States: NCTM's Curriculum and Evaluation Standards for School Mathematics

NCTM's Standards (1989) are a product of recent reform activity in mathematics education that we mentioned in the previous section. It is a framework that set out the curriculum content and early criteria of evaluation. The Standards constitute a set of guidelines that suggest criteria for a quality mathematical sciences curriculum for K-12. It strongly endorses the first recommendation of An Agenda for Action of NCTM in 1980: "Problem solving must be the focus of school mathematics". The strategy NCTM used in developing the Standards evolved out of two conferences[1] held in 1983 in response to the perceived crisis in education.

The vision of school mathematics expressed in the Standards is captured in the following statements: All students need to learn more, and often different, mathematics, and that instruction in mathematics must be significantly revised (NCTM 1989, p. 1).

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[1] National Commission on Excellence in Education (1983). A Nation At Risk: The Imperative for Educational Reform. Washington, D.C.: U.S. Government Printing Office; and National Science Board Commission on Precollege Education in Mathematics, Science, and Technology (1983). Educating Americans for the Twenty-First Century: A Plan of Action for Improving Mathematics, Science, and Technology Education for all American Elementary and Secondary Students so that Their Achievement Is the Best in the World by 1995. Washington, D.C.: U.S. Government Printing Office.

In this statement, five ideas on education are implied:

A. Teaching mathematics to "all students" emphasizes that anyone who is to be a productive citizen in the 21st century must be mathematically literate — including not only "talented white males" but all under represented groups.
B. "More mathematics" implies that all students need to learn more than how to manipulate arithmetic routines. At present, nearly half of American students never study any mathematics beyond arithmetic.
C. "Often different mathematics" indicates that all students need to learn concepts from algebra, geometry, trigonometry, statistics, probability, discrete mathematics, and even calculus.
D. "To learn" means more than to memorize and repeat. Learning involves investigating, formulating, representing, reasoning, and using appropriate strategies to solve problems, and then reflecting on how mathematics is being used.
E. "Revised" instruction implies that teachers and students need to envision mathematics classrooms as discourse communities where conjectures are made, arguments presented, strategies discussed, and so forth (Romberg 1993, p. 37).

Further, the Curriculum and Evaluation Standards for School Mathematics, NCTM lists four societal goals:

The educational system of the industrial age does not meet the economic needs of today. New societal goals for education include

1) mathematically literate workers
2) lifelong learning
3) opportunity for all and
4) an informed electorate. Implicit in these goals is a school system organized to serve as an important resource for all citizens throughout their lives (NCTM 1989, p. 3).

These ideas are the essential qualities underlying NCTM's reform expectations and suggested by a series of documents on mathematic education. In other words, NCTM's Standards advocate 4 basic standards to be applied to the 14 curriculum content standards: mathematics as problem solving, mathematics as reasoning, mathematics as communication, and mathematics as connections that make a linkage within mathematics and between mathematics and the real world (NCTM 1989, pp. 5–6). NCTM's Standards are recognized by many American educators as the exemplar of what is needed in all curricular areas which will be reformed continuously during the next decade. They represent explicitly the new visions or ultimate goals mathematicians and mathematics educators are striving to accomplish.

2. KOREA: The Sixth Revised Curriculum

Education in Korea is the responsibility of central government, which is now in the process of distributing educational control to local educational authorities and individual schools. Under the supervision of the Ministry of Education, several professional organizations for educational research — for example, the Korean Education Development Institute (KEDI) — take part in developing curricula for K-12 students. The new revisions or reforms represent a radical change from current school practices with respect to what should be taught and how it is to be taught in Korean schools. The revised national curriculum of Korea (the Sixth Curriculum No. 1992-11, endorsed by Ministry of Education) represents clearly an increasing tendency to move from centralized governance in education to a decentralized educational system.

The development and control of the curriculum, which is in the process of being distributed to every level of the schools, however, is still under the jurisdiction of the Ministry of Education.

The national curriculum on mathematics education for middle school students contains a general overview presenting principles of school mathematics, and is them divided into four parts: goal statements, content specifications, teaching strategies, and assessment. In the curriculum document, those experiences considered necessary for the future adult life are prescribed and form the “core and essential curriculum” for all students at the 7–9 grade level.

In that part of the content to which all students are exposed, there are five strands of mathematical knowledge. These strands are intended to emphasize the major kinds and sources of mathematical activity rather than to suggest how the content should be delivered to students. The strands are number and formula, equations and inequalities, functions, statistics, and geometrical figures, which are prescribed as essential if students are to become intellectually informed citizens in the future. The new revised mathematics curriculum was intended to incorporate the following educational principles or general characteristics:

Mathematics can be characterized as a specific subject that enables students: to understand basic concepts, principles, laws or rules in mathematics; to develop an ability to observe and think about a wide variety of phenomena mathematically; and to develop am ability and an attitude to think of a variety of problems logically and solve them rationally · · · Mathematics as a subject empowers students to think mathematically, develops in them an attitude that honors social norms and orders, and a capacity for rational and creative thinking, which is essential to citizens in a democratic society (Ministry of Education 1992, pp. 44–45).

III. Comparison of Expectations: Goals and Content

In this chapter, curriculum expectations of two countries will be compared on two levels; that is, general goals or principles in mathematics, and objectives specified for specific levels of mathematical content.

This approach is explicitly manifested in NCTM's document. The NCTM document includes a set of standards relevant to each grade level. As indicated earlier, there are four standards — problem solving, communication, reasoning, and connections — that are to be accomplished regardless of student's grade level. The equivalent of general goals in the formal curriculum document, these are presented as the first four curriculum standards at each grade level, K-4, 5–8, and 9–12 in the Standards (1989). The expectation in the United States is that all other mathematical topics will be developed with these general standards in mind.

The first four curriculum standards for each grade level can perhaps be understood as the reflection of a consensus regarding mathematical education goals or expectations in America. Expectations expressed in the Standards are a kind of global umbrella covering all other mathematical topics.

Thus, this comparison of expectations begins with an examination of the intent in each country — in other words, of how much to emphasize explicitly problem solving, communication, reasoning, and connections in their mathematics curriculum. Following that, more specific expectations will be compared.

Before undertaking an analysis and comparison of both curricula, we need to define the term "curriculum", more clearly. Because there are so many definitions of this word, we, as researchers, are faced with the necessity of defining our usage.

Curriculum, generally speaking, is a product of decision-making that appears to be a maze of influence, power, and struggles won by some and lost by others. Any curriculum represents assumptions concerning both what are considered ends desirable state for students to achieve and the appropriate content they are to know.

Professional organizations usually have their own definitions for curriculum. At this point, we define the curriculum as "a set of intentions, a set of intended learning outcomes"(Posner & Rudnisky 1986, p. 7–8). Thus, the curriculum indicates what is to be learned; the goals indicates why this content is to be learned, and the instructional plan indicates how to facilitate learning. NCTM defines the mathematics curriculum as follows:

> A curriculum is an operational plan for instruction that details what mathematics students need to know, how students are to achieve the identified curricular goals, what teachers are to do to help students develop their mathematical knowledge, and the context in which learning and teaching occur (NCTM 1989, p. 1).

This definition is what we would call the "intended, formal, or explicit" curriculum of mathematics. In this paper, this viewpoint will be adopted as a starting point.

In general, a curriculum is thought of as comprising various components: aims and objectives, content, teaching methods, and evaluation and assessment. In the present analysis, however, the primary focus is on the comparison of curriculum goals and content of both countries, because curricula may vary greatly in what they say about teaching methods or strategies. Frequently, too, they leave the assessment of the pupils in the hands of the teacher; in other words, this is seen and accepted primarily as a teacher's role.

The major source used in comparing Korea's mathematics curriculum with the NCTM Standards was a mathematics section of the national curriculum for middle school students (Grade 7–Grade 9), which had become effective in March of 1995. In July 30, 1992, the Korean Ministry of Education ratified a new national curriculum for middle school from Grade 7 to Grade 9.

The reason for adopting the middle school curriculum as an object of analysis and comparison is to reduce the discrepancies associated with differences in curriculum systems between the U.S. and Korea. For example, in Korea, there are three levels of school grades, 1–6, 7–9, 10–12 (kindergarten is not included as part of the compulsory education enterprise); on the other hand, grade levels divided in the NCTM Standards are K-4, 5–8, 9–12.

1. Comparison of General Characteristics or Goals

We begin with an examination of statements that express general expectations on mathematics education. According to Graue & Smith (1992), there are three dimensions for instructional assessment in mathematics. These dimensions, which refer to mathematical processes, content, and dispositions in goal-directed activity, might be useful to reflect the complex nature of learning, and should be understood not as separate, independent aspects, but instead as interrelated and mutually constitutive (p. 11). The three dimensions express three aspects of student learning, so we think of them as forming three strands of mathematical knowledge.

In the first strand, four processes of mathe-matical learning are included, which are directly connected with the NCTM's Curriculum and Evaluation Standards. The essential mathematical processes focus not just on knowing mathematics but on doing mathematics.

The first four NCTM Standards — problem solving, communication, reasoning, and connections — can be interpreted as an ultimate goal to be considered in any effort to teach mathematics, as essentially the mathematical processes compatible with learning and teaching mathematical content. Mathematical processes means activity, in other words, that refers to "doing mathematics":

> "Knowing" mathematics is "doing" mathematics. A person gathers, discovers, or creates knowledge in the course of some activity having a purpose. This active process is different from mastering concepts and procedures. We do not assert that informational knowledge has no value, only that its value lies in the extent to which it is useful in the course of some purposeful activity ⋯ But instruction should persistently emphasize "doing" rather than "knowing that" (NCTM 1989, p. 7).

According to this argument, any specific mathematics content area, such as statistics or geometry, should embody these four basic standards and, furthermore, provide students an opportunity to experience them. For a student to be mathematically powerful, however the meaning of "mathematically powerful" is to be construed, the student must have the mathematical understanding and experience to undertake the routine tasks of everyday life. In other words,

> to be mathematically powerful in a mathematical and technological society, students should develop the capability to explore, conjecture, reason logically, and finally integrate a variety of mathematical processes effectively and efficiently to solve problems. In becoming mathematical problem solvers, students need to value mathematics, to reason and communicate mathematically, and to become confident in their power to use mathematics coherently to make sense of problematic situations in the world around them (Zarinnia & Romberg 1992, p. 263).

Now let us compare mathematical goals of both countries in terms of these general principles of mathematics education.

A. Problem Solving

For many years, the mathematics education community has vigorously acknowledged the importance of problem-solving activity for students. The NCTM Standards especially included learning to solve problems as one of the main goals of school mathematics teaching, along with learning to value mathematics, becoming confident in one's own mathematics abilities, learning to reason, and learning to communicate mathematically.

Problem solving is an act of doing, not a result. It is a "highly integrated process of addressing a non-routine task that may require both conceptual and procedural knowledge and may involve one or more specific content areas" (Romberg, Wilson, Smith, & Smith 1991). In sum, the curriculum must afford students opportunities to solve problems that require them to address relevant and appropriate mathematical ideas.

It is recommended that problem solving be the central focus of the mathematics curriculum, and problem solving activities be presented as problem situations that motivate students and serve as a context in which mathematical ideas are learned. Problem situations imply complex, messy, and culturally-based problems that are open to multiple strategies and solutions. Problems that are messy or ill-defined could provide more freedom on the learners part to pursue questions that reflect their own interests. One way to promote such interests is to provide problem situations that are relevant to students or pose real world applications (Lajoie 1991, p. 6).

In the Korean curriculum document, there are several statements that put stress on the importance of problem solving in mathematics education. The following are relevant to this standard:

- To develop an ability and an attitude to think of a variety of problems logically and solve them rationally.
- A capacity for and interest in problem solving rationally is essential to successful learning in most school subjects.
- To plan and implement a variety of rational and creative strategies or methods to solve the problems.
- The experience of inquiring mathematically about the various phenomena that confront us in the real world.

In this overview, we can recognize the similarity of goals or orientation in both countries' mathematics education. As you might see, both NCTM's Standards and Korea's curriculum emphasize the importance of problem solving in the real world. This similarity can be understood as reflecting a shared philosophical assumption on mathematical power. It means that the view of mathematics as knowledge for the sake of knowledge will not be able to dominate the community of mathematics any longer.

B. Communication

Communication is essential to mathematically powerful individuals. According to social constructivism, mathematics is a product of social construction — in other words, the basis of mathematical knowledge is linguistic knowledge (Ernest 1991, p. 42). Language includes a variety of signs and symbols that are used as tools to communicate each other among peoples. Mathematics is no exception. Mathematics also functions as a language for the simple representation of complex phenomena, and for communicating facts, ideas, and emotions.

In the process of communicating with others about the problems that they are engaged in, students develop the power to reflect on, evaluate, and clarify their own thinking, to model situations, to formulate definitions, and express ideas (Zarinnia & Romberg 1992, p. 264). Unless students explicitly comprehend relationships between mathematical concepts and symbols, they are liable to view mathematical algorithms as objects to be mechanically memorized.

This is a standard that seems to be narrowly interpreted in Korea's mathematics curriculum, even though there are a few references to it. Among the characteristics and goals of mathematics education, we can find statements on understanding fundamental mathematical concepts and symbols that are necessary and basic in mathematical thinking about the real world (e.g., to understand basic concepts, principles, laws or rules in mathematics; to understand and discriminate correctly among the terms and symbols that are required for a knowledge of mathematical concepts). However, communication as a form of mathematical power is not limited to understanding these communicative tools. It should be extended to any activity that involves transforming a variety of phenomena around us into mathematical language. Opportunities for communication about mathematics provides students with ways to reflect on and clarify their own thinking, as well stimulate students to take the perspective of another so that they develop common understandings of mathematical ideas.

C. Reasoning

There will be no objection to the recognition that reasoning as a phenomenon unique to human beings represents the essence of mathematical thinking. Reasoning can be defined as a series of cognitive capabilities that function as heuristic tools for solving problems around us.

To reason mathematically encompasses conjecturing, gathering evidence, and approaching solutions. Because reasoning is not a product but a process, it cannot involve students in a predetermined way, furthermore, this is not possible. Its development can be facilitated in those environments that allow students to explore mathematical ideas without any external control, foster students' curiosity in mathematics, and encourage them to mathematize routine/non-routine problems in the world.

In Korea's curriculum document, we are able to identify many statements emphasizing this capability. To emphasize the importance of reasoning may be a logical component because it is necessary to reason rationally in order to solve problem effectively and efficiently.

The following are typical exemplars of mathematical reasoning in Korea's curriculum:

- To develop an ability to observe and think about a wide variety of phenomena

mathematically.
- To think about a variety of problems logically
- To increase students ability to think mathematically.

In adhering to these guidelines, students will be able to apply mathematical knowledge to solve problems that are encountered in the real world and become confident in their ability to address problem situations. As they reason through their problem situations, students develop the habit of conjecturing and of judging valid arguments. In addition, they use a deductive or inductive thinking strategy to solve problems. In conclusion, to get correct answers is no more important than to comprehend how a problem is solved.

D. Connections

This Standard covers two kinds of connections—connections within mathematics and connections between mathematics and other disciplines, such as art, psychology, business, or the natural sciences. The former focus on students' ability to integrate a broad range of mathematical strands — for example, number sense, algebra, statistics — rather than treating each strand in isolation.

The latter means that mathematics can be applied to other fields of knowledge. So, mathematics education should provide students with the opportunity to observe the transaction of mathematics with other school subjects.

These two ways of integrating mathematics are the main point of Connections Standard. The original intention of this standard is to encourage students to broaden their mathematical perspectives and to view mathematics as an integrated whole. It is also associated with mathematical communication capability both directly or indirectly. There are several sentences that are related to this standard in Korea's mathematics curriculum:

- Mathematics is a fundamental subject required for the learning of other subjects.
- Mathematics as a subject develops in students an attitude that honors social norms and orders that are essential to citizens in a democratic society
- Students need to know how to apply concepts and symbols of mathematics to the world in which we live.

In sum, we observe that there are some common characteristics in both countries' mathematics curriculum. But this is not say that are no differences. Though both curricula give priority to problem solving, NCTM's reformative ideas suggest more balanced emphasis on four standards around the problem solving process — in other words, on interrelationships of four standards as a particular capability — than Korea's.

In contrast, Korea’s mathematics curriculum does not explicitly use terminology relevant to those abilities, nor does it mention them with equal weight. Korea’s curriculum, however, has the special characteristic of stressing the affective learning outcomes that motivate students to learn mathematics to have an interest in mathematics learning, develop an attitude to think mathematically, and become rational citizens in democratic society. This characteristic has important implications for the possibility of curriculum integration, which means an integration of mathematics curriculum with other fields of school subjects.

2. Comparison of Specific Contents

In this section, we will try to compare more specific expectations or standards on mathematics education. The results of this comparison can be useful in understanding not only each country's specific instructional objectives but also the instructional content in mathematics, because, in general, these objectives serve as a prescription of what should be taught.

A. Standards 5-7: Number, Computation, and Estimation

The NCTM’s Standards emphasize developing number and compunction sense, creating algorithms, and using estimation. Students are expected to explore relationships between whole number, fractions, decimals, integers, and rational numbers. They should develop an understanding of ratio, proportion, and percent. In contrast, decreased emphasis is placed on repeated paper-pencil computation, memorizing rules and algorithms, finding exact answers, and rounding out of context.

In the case of Korea’s mathematics curriculum, even though it does not classify mathematical knowledge of number into several areas, instructional emphasis on number is very similar to NCTM’s Standards. This similarity can be interpreted as a reflection of the disciplinary structure of mathematics.

However, we can not but recognize that less emphasis is given to estimation than to number and number computation, though it is not certain whether estimation is implicitly implied in the goals. This fact is consistent with a less explicit emphasis on reasoning and connection in Korea’s curriculum.

In sum, both curricula emphasize the importance of number. This phenomenon might be the result of an interweaving between strands of knowledge in classical mathematics and knowledge that has a high application value in real world.

B. Standard 8: Patterns and Functions

The NCTM Standards emphasize identifying and using functional relationships.

Students should use tables, graphs, and rules to describe situations. They mention using different languages — verbal, graphic, and numerical — to represent mathematical situations.

In Korea, we cam find a list of more formal goals for function; it especially mentions a particular type of function that students are expected to solve. For example, the curriculum of Korea urges that students study linear functions and quadratic functions. But there is no mention of patterns as a goal of mathematics education.

C. Standard 9: Algebra

NCTM places emphasis on student inquiry and investigation. Students should use a variety of methods to solve linear equations and investigate inequalities and nonlinear equations. On the other hand, less emphasis is placed on symbol manipulation and memorizing procedures.

Korea curriculum does not use the term “algebra” as a strand name; in its place, the document refers to “equation and inequality” directly. The difference in uses of terminology may not be important, because we may use a term in our own way in a particular context. Regardless of the differences in language, we observe a similarity of emphasis in both curricula.

D. Standards 10 and 11: Statistics and Probability

This standard is a typical exemplar that reveals a difference in professional interpretation of mathematical knowledge. In Korea, we are very accustomed to using a compound term for “statistics and probability”. But, in NCTM’s usage, these are separated into two independent strands. Thus, we will compare the characteristics of these standards without separation.

NCTM emphasizes statistical analysis of real-life situations, using statistical methods for making decisions. Students should create experimental and theoretical probability models and relate statistics and probability to real-life situations. Less emphasis is placed on memorizing formulas.

Korea’s curriculum also lists descriptive statistics as a goal. Basically, this means calculating, organizing, and representing data in a effective way. For example, it stresses students’ ability to represent gathered data into tables and graphs, to understand basic statistical concepts, and to calculate statistics. But there is only one goal statement for probability, which is expressed perhaps too vaguely.

E. Standard 12: Geometry

This standard has functioned as a catalyst for restructuring content change at the school level since the reform movement began in American mathematics education. Other countries including France, Japan, The Netherlands, and U.K. had emphasized geometry long ago. A major part of the mathematics curriculum in Australia, France, Germany, Japan involves quite formal geometry at middle school grade levels. Transformational and coordinate geometry are stressed in all do the countries (Howson 1991).

NCTM places emphasis on developing an understanding of geometric objects and relationships. Students should use geometry to solve problems. There is decreased emphasis on memorizing vocabulary, facts, and relationships in the NCTM Standards.

On the contrary, Korea's curriculum requires that students be exposed to more higher-order geometric knowledge. The curriculum explicitly lists those goals and prescribes the types of learning outcomes that will be produced as a result of geometric instruction. This can be regarded as a strong point in the comparison of the Korean standard to NCTM's.

F. Standard 13: Measurement

The NCTM emphasis is on estimation and using measurement to solve problems. There is decreased emphasis in the NCTM document on memorizing formulas and converting within and between measurement systems. However, there are no formal goals statements regarding measurement in Korea's curriculum. Of course, in one sense, measurement as an activity of problem solving may not exist independently; more explicitly, it may have no mathematical meaning without referring to particular strands. Though this may be the case, it would be well for us to state some goals on measurement more formally. At this point, this limitation has implications for future curriculum revision in Korea.

Through a series of comparisons such as those provided above, we can recognize that there are much many commonalities between NCTM's Standard for mathematics education and Korea's mathematics curriculum. At the same time, we can not but acknowledge that there are some differences. We can summarize those differences in terms of two aspects: one is a categorization of areas of mathematical contents, and the other is a relative difference of emphasis given to each area.

In general, NCTM distinguishes specific areas of mathematical contents into nine strands; on the other hand, Korea's curriculum specifies only five areas. The latter is less differentiated. It leaves us with insightful research questions about categorization of curriculum contents. How many areas of contents are legitimate and appropriate for mathematics education? Which categorization is authentic enough to represent structure of mathematical knowledge?

Next, number system and computation, functions, and statistics are emphasized in both countries. However, we note some variations in language usage and degree of emphasis on strands of mathematical knowledge. For example, NCTM's document uses the term algebra more prominently than the term equation and inequality, and stresses patterns, estimation, probability, and measurement as independent areas of mathematics content. But NCTM's level of coverage on geometry is very basic and elementary compared to Korea's curriculum.

These differences are partly due to an underlying philosophical assumption about mathematics education and past cultural practices in each country. It is, at the same time, partly due to practical considerations such as economy, politics, and traditions that have influenced the mathematics curriculum of both countries.

IV. Discussions and Suggestions for Next Steps

There were several problems that had to be faced in this analysis that make the comparison of expectations difficult. Above all, international comparison itself may present complex challenges, as mentioned earlier.

First of all, we have to acknowledge the importance of the fact that each country has a different school system. Also, as is widely known, there is no uniformity in educational enterprise, including the form of educational control. In America, there is no national curriculum developed at the federal level, even if standards are developed at the federal level, the government cannot mandate that schools comply with its recommendations. At this point, remarks of Ernest Boyer on the U.S. educational systems are relevant:

> Happily, we don't have in this nation a ministry of education. No education czar sets standards and measures compliance. Instead, we have 50 states and 16,000 school districts establishing policies and monitoring results (requoted from NCTM 1993, p.207).

On the other hand, Korea has an educational control agency at the national level, as well as a mandatory curriculum developed by the government. So, we should be very careful not to interpret the comparison of curricula in the two countries too literally. In other words, there are considerable variations in what is taught, in how should be taught, and in what is emphasized.

Secondly, throughout the world, it is well known that teachers do not slavishly follow their national curriculum, and what is "intended" by those who develop a national curriculum is never implemented uniformly in all classrooms. Moreover, what actually is learned by students may bear little relation to the mathematics curriculum.

Thus, the interpretations made in this paper may not correspond to actual student performance in the two countries. Nevertheless, the statements in a national curriculum carry a message about how school mathematics is generally perceived in that country — the system it employs, the aims and aspirations for students, and so forth. It is on this basis that this analysis has been carried out. If we had analyzed the actual teaching practices and the outcomes of teaching in both countries, the results of a comparison might have been produced very different results. Because there are no attempts to examine the variables of actual practices, we cannot conclude for certain from the results of this study that both countries' mathematics curriculum have similar expectations and the means for actualizing these expectations.

Thirdly, it is important to recognize that this study confined itself only to the comparison of curriculum at the middle grades. In order to understand the realities of the two curricular systems more fully and completely, however, we need to analyze and compare the whole mathematics curriculum. Because each curriculum uses different sequencing principles in its statement of goals and content, we cannot advocate any resolute position from the results of limited comparison.

In summary, we are not in a position to argue that curriculum in Korea is superior or inferior to American's, or vice versa, because each country has its own ways of life and every effort to improve education is derived from a different social context. Nevertheless, the evidence in this paper is sufficiently valid to identify the strengths and weakness of a particular curriculum compared to the other country's curriculum, and to illuminate each country's future direction of curriculum reform in mathematics.

Now, let us summarize the significance of this comparison and suggest some recommendations for successful curriculum implementation. In some senses, the goals of mathematics education in both counties seems to be radical — an idealistic or romantic vision of school mathematics. They certainly suggest a romantic view of goals and content when compared with most current practices in both countries, especially in the case of NCTM's Standards.

However, they are very positive and practical as reform initiatives for a rapidly changing society. Both curricula stress the need to respond to the information age. At the same time, they suggest that somewhat different mathematics needs to be taught to all students, that a shift from emphasizing mere conceptual understanding to applying procedural principles that are relevant to problem solving must occur, and that instruction based on students' real-world experience should be introduced in classrooms.

For any prospect of educational reform, there is certainly room for an optimistic view. Whether the new efforts will succeed or not depends on the cooperative atmosphere among teachers and other stake-holders in the new movement. Then, how are we to achieve success?

First of all, not only in-service teachers but also pre-service teachers must be taught in a manner similar to the way in which they are to teach — by reasoning, connecting, communicating, and problem solving. Teachers need both formal and informal opportunities for professional development in order to enhance their teaching capabilities.

Teachers are agents for transforming the intended curriculum into an operational curriculum. Insofar as the curriculum plays a role in the classroom, it does so under the direction of the teacher. Among all of those concerned with education, only the teacher knows the specifics of a particular learning situation and has the power to exercise her/his expertise.

Secondly, the education system as a whole should be responsive to social change. As mentioned earlier, the rapid escalation of technology is important not only with respect to the method of teaching but also with respect to the content of curriculum. Suppose it becomes possible for students to use a hand calculator routinely for solving problems in ordinary achievement testing on mathematics.

If this change should occur, we cannot but undergo a dramatic change in how to teach mathematics and in what to teach in mathematics education, at the same time. Arithmetical contents designed to develop students' computation skills will disappear from textbooks; the more complex problems that can be solved using calculators would substitute for the aforementioned skills in daily classroom activities. The teacher would no longer rely solely on traditional teaching activity using chalks and a blackboard, and a text. If we address these implications of technology, education reform will keep pace with the rapid changes in society.

Thirdly, we should maintain the alignment among curriculum components. Especially, alignment between goals, content, and evaluation or assessment procedures is one of the most important dimensions for successful implementation of new curriculum. Teachers, test developers, and educational administrators all must be concerned about the alignment of what they are teaching with evaluation. For evaluation to be properly aligned with curriculum content, the set of tasks on the evaluation instrument must reflect fully the goals and content specified in the curriculum.

To determine the alignment of evaluation with a curriculum, simply comparing the goals and content of an evaluation instrument with curriculum is insufficient. We should consider the degree to which individual test items measure the mathematical content that they purport to measure and choose wisely the format of evaluation, to assure that it is aligned with the curriculum.

Finally, we must not drop our guard against nominal education change. To appease demands for change, curriculum developers and educational administrators often try to change curriculum superficially.

For example, in retrospect in tracing mathematics curriculum revision in Korea, we can see that there is a massive gap between general abstract statements in the mathematics curriculum — which include goals, content, teaching strategies, and assessment — and assessment — and practical specifications, such as textbooks and teachers' guides.

In short, we did not adequately resist the status quo, which permitted topics in the curriculum to be selected not in terms of whether they were worthwhile to teach, but because they had always been part of the curriculum. We cannot afford to be in the position of teaching a Saber-Tooth curriculum (see Romberg 1993, p. 39) when our students face a radically charging world.

To avoid inertia and the uncritical use of outdated curriculum, it is essential that we make an effort to narrow the gap between changes in policy and practices in the classroom.

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