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Development of a Teaching/Learning Model for the Mathematical Enculturation of Elementary and Secondary School Students[1]

KIM, SOO HWAN
Chongju National University of Education, 135 Sugok-dong, Heungdok-gu, Chongju, Chungbuk 361-712, Korea; Email: soowhan@sugok.chongju-e.ac.kr

LEE, BU YOUNG
Department of Mathematics, Dong-A University 840 Hadan-dong, Saha-gu, Pusan, Korea; Email: bylee@daunet.donga.ac.kr

PARK, BAE HUN
Department of Mathematics, Education, Korea National University of Education, Cheongweon-gun, Cheongju, Chungbuk 363-791, Korea
Email: baehun@cc.knue.ac.kr

The purpose of this study is to develop a teaching/learning model for the mathematical enculturation of elementary and secondary school students. It is clear that the development of teaching and learning in the classroom is essential for the realization of global innovations in mathematics education. Research questions for this purpose are as follow:

(1) What can be learned from literatures reviews of the socio-cultural perspective on mathematics education, and of ethnomathematics as a mathematics intrinsic to cultural activities?
(2) What is the direction of teaching and learning from the perspective of mathematical enculturation?
(3) What is the teaching /learning model for mathematical enculturation?
(4) What is the instructional exemplification based on the developed model?

This study promotes the establishment of mathematics education theory from the review of literatures on the socio-cultural perspective, the development of a teaching/learning model, and the instructional exemplification based on the developed model.

[1] This is a paper presented by the first author at Subgroup 2 (Preparing Teachers to Teach Diversity) of WG 21 (The Teaching of Mathematics in Different Culture), ICME-8, Seville, Spain, July 14-21, 1996.

1. Introduction

The researcher intended to illuminate humanistic education of mathematics as a value-directed and intentional course from a cultural perspective. White (1959) divides the components of culture into four categories; ideological, sociological, sentimental, and technological. Moreover whilst showing that these four components are interrelated he argues strongly that "the technological factor which means manufacture and use of tools and implements is the basic one; all others are dependent upon it" (Bishop 1988). Not only machinery, but also mathematical symbolic technology which are included in the technological component of culture have played important roles in human cultural development.

Modern mathematics, as a symbolic technology, has been a generative power in the development of culture, developed in a variety of societies historically. Therefore, we can define mathematical culture as a mathematical component of the larger culture. There are values of mathematical culture such as 'rationalism' and 'objectivism' in the ideological category composed of beliefs, dependent on symbols, and philosophies; 'progress' and 'control' in the sentimental category composed of attitudes, feelings concerning people; and 'openness' and 'mystery' in the sociological category composed of customs, institutions, rules and patterns of interpersonal behavior (Bishop 1988).

Korea is a culture which has a 5,000-year-old history and a language. Mathematics as a world-wide common language and a cultural phenomenon that is clearly supra societal in nature (Bishop 1988). Everybody recognizes the importance of mathematics to current and future societies. But, in spite of the value of 'openness' of mathematics in the sociological category, a lot of people still feel very mystified about just what the mathematics is. That is to say, although mathematics is the most extensively taught subject in the world, a lot of students are still worried by it. Furthermore some students abhor mathematics. This study is grounded on the point that the micro-innovation of teaching/learning in classroom practice is more important than the macro-innovation of the world-wide mathematics curriculum.

2. Mathematical Enculturation

Mathematical culture is part of the process of living, and therefore is not to be possessed exclusively by a few people, as we know from the value of 'openness' in it. ME (Mathematical Enculturation) is a process of interaction between teacher and student in formal and institutional circumstances designed for the performance and accomplishment of an intentional mathematics curriculum which is the objective expression of the object and contents of mathematics education. That is to say it is performed within the framework of knowledge, but is the process of interpersonal interaction being performed for the purpose of recreating and redefining the framework.

The object of ME is to enable one who takes school mathematics lessons to demonstrate mathematical power in his socio-cultural living. This does not mean that the culture of mathematicians should be transmitted to all students, but that mathematical culture as a valuable contributor to the development of human culture should be shaped by them. For the realization of ME as a cultural phenomenon which includes ethnomathematics as a mathematics intrinsic in cultural activities. We should consider both curriculum and teaching/learning of mathematics.

Firstly, in terms of the curriculum we need an activity-oriented curriculum for mathematical enculturation. ME curriculum consists of three components, namely a concept-based symbolic component, a project-based societal component, and an investigation-based cultural component (Bishop 1988). Modern school mathematics which is said to be Western mathematics is directed to practical, cultivating, cultural, and social purposes.

There are two kinds of knowledge in mathematics. One is conceptual knowledge, the other is procedural knowledge (Hiebert & Lefevre 1986). Conceptual knowledge (‘knowing that’) can be considered a system of propositions which are mathematically true. Procedural knowledge (‘knowing how’) can be considered a well-ordered system of language and symbolism for socio-cultural communication. We make good use of the mathematics as a world-wide common language in several fields of physics, astronomy, archaeology, arts, sports, etc. as well as in daily life. This is the reason for teaching and learning modern mathematics. The development of mathematics which is not only a generative power to develop a culture but also helps to integrate a culture begins with an ‘environmental request’ or a ‘demand within mathematics’. Therefore ME curriculum should satisfy these requests.

Secondly, in terms of teaching and learning, we need to place more focus on the process of mathematical enculturation than on the transmission of given knowledge. This is the process of shaping a new mathematical culture underlying the intention of the ME curriculum, and also is almost the same as the constructivist approach to teaching and learning, whenever the situation of construction peculiar to the culture can be set up. The contents of world-wide school mathematics are almost uniform, but we should take an interest in the ethnomathematics as a mathematics intrinsic to cultural activities and the practical mathematics as a tool in a daily life for efficient teaching and learning under the ME curriculum consisting of the activities based on concepts, investigation, and project.

That is to say, we should make the best use of the peculiar ways of thinking not only of our own ethnomathematics but also of other cultures’ ethnomathematics. Also, there is a difference between knowledge as a tool in daily life and knowledge as an object in a mathematics classroom. But there is evidence that one who has a concept as a tool can easily get that concept as an object. The understanding of several situations related to the core concept, and the representation of these situations by natural language, can be helpful to transfer from concept-as-tool to concept-as-object.

3. Interaction and communication for ME

The constructivist approach and meaning theory are very suggestive for teaching and learning mathematics. Kant put knowledge in the phenomenological domain of 'constructed reality' and laid the foundation for constructivism by integrating rationalism as an ideology underlying the subject and empiricism as an ideology underlying the object (Konold & Johnson 1991). Social constructivists argue that the objective knowledge of mathematics exist in human social activities, interactions, and rules through which personal subjective knowledge, language, and social life are supported, and that these need to be regenerated.

That is to say, subjective knowledge of mathematics in the individual domain is publicized, and becomes new objective knowledge as it passes with passing through the process of public criticism and reformation in the social domain. On the other hand, objective knowledge of mathematics in the social domain is transformed in to new subjective knowledge by passing through the process of personal reformulation and representation. ME can be considered as a circular process of public creation from the subjective knowledge, and personal reformation from the objective knowledge in the socio-cultural domain (Ernest 1991). We need to lay great emphasis on the socio-cultural negotiation between public creation and personal reformation in the process of ME. When the situations involving mathematical concepts as a tool in a daily life and the ethnomathematics intrinsic in cultural activities are transformed into the objects of learning in the classroom, these can be the sources of dynamic communication.

The difficulty of students learning mathematics is not so much the result of the deficiency of their concepts as it is the lack of communication (Mellin-Olsen 1991; Sinclair 1983). Mathematics is a human creative activity, and socio-cultural interaction and communication in the classroom plays an important role in teaching and learning mathematics. From this perspective, we can set the direction of teaching/learning as follows:

Firstly, if we consider teaching/learning mathematics as the process of students ME, we need to initiate the interaction between teacher and students as socio-cultural negotiation. To accomplish this, relativism as a view of mathematical knowledge should be regarded as of greater importance than absolutism by emphasizing 'rationalism' over 'objectivism', 'progress' over 'control', and 'openness' over 'mystery'. We can anticipate the activation of open interaction through lively discussion and investigation by regarding mathematics as a dynamic subject including the study of patterns. For example, in small group activity we should organize instruction to achieve interactions such as the exchange of mutual opinion and requests for the other person's explanation.

Secondly, we need to develop communication in the process of teaching/learning mathematics for ME. Communication in teaching/learning can be summarized as hearing, speaking, reading, and writing. Within the educational context of a mathematics classroom there are two main reasons for students hearing and speaking, namely talking for themsemves (Pimm 1987) or exploratory talk (Barnes 1976) or else to organize their own thoughts for others (Pimm 1987) or explanatory talk (Barnes 1976). The strategies of mataphor and analogy in oral communication in the mathematics classroom create the mathematics register as a mathematical 'common language'. Also with written language, at least as much as with speech, we can distinguish between writing for oneself and for others.

Classifying the range of recording styles which may be observed in students' written mathematics, there are three styles, verbal, mixed, symbolic. The symbolic provides a high-status, written recording style towards which most mathematics teachers are aiming. Unfortunately, many attempt to move students along the recording continu-um toward the symbolic end much too rapidly (Pimm 1987). With apologies to Kant, while semantics without syntax is blind, syntax without semantics is empty. Mathematics is not the manipulation of symbols according to prescribed rules: mathematical activity can be both purposeful and meaningful to human beings (Pimm 1987)

It is urgent to activate students' interaction and communication skills through activities located in their environments. It is necessary to review mathematics as a dynamic subject as well as a language different from natural language. By using the techniques of analogy and metaphor in explaining mathematical ideas, I believe we will be able to initiate students' interaction and communication using diverse socio-cultural activities.

4. A teaching and learning model for ME

Mathematical enculturation is performed within the framework of mathematical knowledge, but it is a process of socio-cultural interaction for the purpose of recreating and redefining the framework. The basic philosophy of a teaching/learning model for ME (Mathematical Enculturation) is to increase the number of people at the formal level of mathematical culture. We can think of three levels of the mathematical culture (Davies

1973; Bishop 1988):

I (informal) level including ad hoc measurements to account for particular situations: ‘it is not very far to town’;

F (formal) level including concepts of number, symmetry and logic built into everyday life; and

T (technical) level including the symbols and arguments needed by mathematicians in research.

The ME general model for realizing ME is as following <fig. 1>. It can be divided into the ME fundamental model, the ME extensional model, and the ME complete model. If the mathematics curriculum as a activity-oriented cultural phenomenon were fully implemented, the ME fundamental model could be successful in students’ ME. But on the premise that the current mathematics curriculum does not guide a varieties of investigation and project activities, the ME extensional model and the ME complete model are presented as alternatives for initiation these activities.

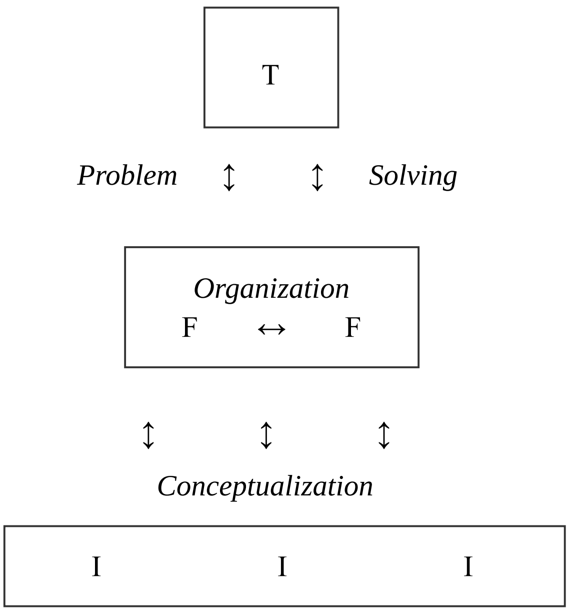

Figure 1. ME general model

The ME fundamental model initiates interactions and communications between teach-er and children for teaching and learning the concept-based symbolic components.

The first conceptualization stage activates the interactions between informal and formal levels of mathematical culture. We put regards on children’s mathe-matical activities and communication of children in this stage. We emphasize sociological 'openness' more than 'mystery' in the values of mathematical culture.

The second organizing stage elaborates the relations between formal levels of mathematical culture. We are concerned with the construction of the extended network of mathematical knowledge. We emphasize ideological 'rationalism' over 'objectivism' in the values of mathematical culture.

The third problem solving stage activates the interactions between formal and technical levels of mathematical culture. We have a high regard for stimulating creativity though the discovery of mathematical concepts or procedures. We emphasize sentimental 'progress' over 'control' in the values of mathematical culture.

The ME extensional model initiates interactions and communications among teachers, students, and parents for teaching and learning investigation-based cultural components. This model is complementary to the ME fundamental model. The investigation of cultural components including investigative materials existing both commonly in all cultures and uniquely in some cultural area should be regarded as of great importance.

The ME complete model activates the interactions and communications among teachers, students, and people in the society for teaching and learning project-based societal components. This model completes students' ME. Exemplification of a project of past, present, and future societies should be regarded as of great importance.

5. Project: The year of the Mouse and the 60th birthday

This paper includes some specific examples appropriate to these models: The investigator has developed instructional projects of 'The year of the Mouse and the 60th birthday' for fifth grades and 'Building a Su Weon castle' for eighth grades. The essence of 'The year of Mouse and 60th birthday' is as follows:

The Problem Situation

This year is the year of the Mouse, the 13th binary term of the sexagenary cycle, and therefore the baby who is born this year is called 'the zodiacal sign of Mouse'. However there is a lot more to 'the zodiacal sign of Mouse' than the babies who are born this year. What is the reason? For investigating the ethnomathematics intrinsic to this ethnic custom, the students already have examined into the age and the zodiacal sign of their family member. The zodiacal signs are Mouse, Cattle, Tiger, Cat, Dragon, Snake, Horse, Sheep, Monkey, Fowls, Dog, Pig.

<Project> The year of the Mouse and the 60th birthday
<Model> ME complete model
<Environment> Concept · Investigation · Project-based Environment
<Purpose> Mathematical Enculturation
- Investigating, Counting, Designing, Explaining
- Explaining by collection and arrangement of data (a bar graph, pie graph, frequency table)
- The conceptualization of multiple and least common multiple by investigation

<Method> The interaction and communication by small group activity

Problem 1 Explaining by collection and arrangement of data

Let each small group make a bar graph, pie graph, and frequency table of the number of the people who are in the year of Mouse, Cattle, Tiger, Cat, Dragon, Snake, Horse, Sheep, Monkey, Fowls, Dog, Pig.

Problem 2 The age distribution of people in the year of Mouse

Exmine the age distribution of people who are in each zodiacal sign of Mouse, Cattle, Tiger, Cat, Dragon, Snake, Horse, Sheep, Monkey, Fowls, Dog, Pig. Can you find some rules?

Problem 3 60th birthday anniversary

We give a banquet on our parents' 60th birthday. What is the meaning of the number ‘60’ in this case? There are the 12 Earth’s (horary) Branches and the 10 Celestial (calendar) Stems by the ying-yang, the cosmic dual forces in Oriental culture.

6. INVESTIGATION BY GAME: PRIMITIVE COUNTING STICKS

<*The Origin*> Once upon a time the primitive counting sticks were used to count. The game of primitive counting sticks seems simple, but careful observation and delicate hands are required for skillful performance.

<*Number of players*> 2–5 persons

<*Rules of the Game*>

1) Determine the order of playing (e. g., the person whose birthday is the earliest

goes first.).

2) The first player holds the primitive counting sticks in one hand and sets them up on the floor of a room or the desk, and then scatters them naturally by releasing his hold.
3) Picking up the sticks each player continues as long as he does not touch any other sticks, but loses turn if he does.
4) The picked up sticks can be used to pick up the other sticks.
5) When all sticks are picked up, each player count up the scores of his sticks according to the color of them, and the player with the highest score wins.

<*Scores*>

Yellow Stick: 2 points
Red Stick: 3 points
Blue Stick: 5 points
Green Stick: 10 points
Black Stick: 20 points

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