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# 일정경사 수심단면에서 평균수위의 상승/저하 효과를 고려한 해빈류의 예측

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# Prediction of Longshore Current with Set-up/down Effect on a Plane Beach

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## ABSTRACT

The numerical model for prediction of longshore current with set-up/down effect on a plane beach is developed using the longshore component of the depth-integrated momentum balance equation. To predict the longshore current, the wave height model should first be formulated because the longshore current depends on the wave height directly. Two wave model, regular wave model and random wave model, are developed based on the energy flux balance equation. Also, the numerical model estimating the set-up inside the shoreline is developed using both the on-offshore momentum equation and the moving boundary technique. The numerical models are verified by the analytical solution, and compared with laboratory data. It is found from the comparison that developed models may be predicted accurately the longshore current with set-up/down effect on a plane beach.

## 1. INTRODUCTION

When the oblique incident wave is approaching to the shoreline, the wave will be transformed by shoaling and breaking. This phenomena are closely associated with the water depth, energy dissipation and momentum exchange. Also, the related factors are interacted each other such that the longshore current may be changed as the wave height is transformed. It is very important for coastal engineers to know the characteristics of these wave transformation at the interested area to plan and design coastal structures. The required informations such as the wave height, wave set-up/down and longshore current induced by the oblique incident waves should be quantified by a methodology.

In particular, the magnitude of set-up/down and longshore current should be predicted to analyze the sediment transport processes near the shoreline. The longshore current model is required as a sub-model of the sediment

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transport model because the sediment transportation depends directly on the intensity of longshore current. Also, the magnitude of set-up/down is necessary to define the onshore limitation where the sediment is moving. Until now, many researchers have studied to predict the wave height and longshore current near the surf-zone on a beach.

As noted the previous paragraph, the wave height should be first estimated to predict the longshore current correctly. There are several model for estimating the wave height near the surf-zone. All of wave model are based on the conservation of wave energy. As the method considering the energy dissipation near the surf-zone, wave models may be classified into the regular wave model and the random wave model. Now, we explain two model among many wave models. Dally and Dean(1985) intuitively presented the regular wave model through the concept of stable wave height to describe the energy dissipation inside the surf-zone. Also, Thornton and Guza(1983) suggested the random wave model using the probability density function of breaking wave and periodic bore formula.

The longshore current is induced by the breaking of the wave incoming obliquely. The longshore current is caused by the excess flux of momentum due to the presence of waves(Longuet-Higgins and Stewart, 1964). Using the radiation stress, Bowen(1969) suggested the longshore current model, and derived the exact solution on a plane beach with the several assumptions. Also, Longuet-Higgins(1970) formulated the another longshore current model using the concept of lateral mixing. The characteristics of longshore current on a plane beach were investigated using the analytical solution. Although we can understand the basic characteristics associated with the longshore current from these simple models, it is impossible to apply these models to field because many assumptions are included into them. Several researches associated with the longshore current have carried out by Ebersole and Dalrymple(1980), Symonds and Huntly(1980), and Baum and Basco(1986). The developed numerical model for predicting the longshore current on a arbitrary beach are based on the very simple wave height model of Dally and Dean(1985)'s regular model. Recently, Larson and Kraus(1991) suggested another numerical model included the wind effect based on Baum and Basco(1986)'s model. By this model, the effect of wind can be neglected. In addition, Simth, et al.(1993) evaluated the applicability of the numerical model considering the effect of the turbulent kinetic energy and the Reynolds stress by comparison with the many kinds of field measurements. However, all of these numerical model are used the regular wave model to obtain the wave height on a beach. It may be implied that these numerical model can not be applied to the random wave. Also, the effect of set-up/down at the shoreline may not be considered correctly.

As pointed out in the literature review, it is necessary to predict the longshore current with set-up/down effect based on the random wave model. The objective of this study is to develop the numerical model for predicting the longshore current with the effects of the set-up/down. To do this goal, the following procedures are studied.

As the first step for the reliable prediction of longshore current, two wave height models are researched. One is the regular wave height model, the other the random wave height model. Both models can be formulated by energy flux balance equation. However, how to consider the energy dissipation near the surf-zone is different from each other. To verify the numerical model developed in this study, we derived the analytical solution of each model on a plane beach, and compared the numerical results with the analytical solutions.

Using the wave height data calculated from the wave height model, the numerical model for predicting the magnitude of the set-up/down is also developed using the momentum equation in the direction of on-offshore. By the same procedure as that of wave height model, the developed numerical model is verified by comparison of the numerical results with the analytical solution. In particular, the moving boundary techniques are introduced to evaluate the magnitude of set-up/down inside the shoreline on a plane beach.

Finally, the numerical model for longshore current is developed by the longshore component of the depth-integrated momentum balance equation. The implicit finite difference method, Crank-Nicolson scheme, is used to develop the numerical model. Also, the analytical solution is derived as a function of mixing parameter under the simple conditions. The developed numerical model is verified using the analytical solution and the laboratory measurements on a plane beach.

## 2. Wave Model

The governing equation of wave model can be easily derived by the energy flux balance. When the oblique incident wave is approaching to the shoreline, the wave will be transformed by the shoaling, and is eventually breaking at any point where does not satisfy the stable condition. These processes can be described by the following Eq. (1).

$$\frac{dF}{dx} = \langle \varepsilon \rangle \tag{1}$$

in which $F$ is the energy flux and $\langle \varepsilon \rangle$ is the averaged wave energy dissipation per unit area.

Many researches, such as Horikawa and Kuo(1966) and Battjes and Jansson(1978), have been studied to express the energy dissipation mathematically. Among them, there are two models which have been generally accepted in the related studies. One is Dally and Dean(1985)'s model, the other Thornton and Guza (1983)'s model. The details of each models are followed.

### 2.1 Dally and Dean's Model

Based on the many kind of measured data, Dally(1980) have intuitively suggested the concept of wave height stabilization which a wave breaking dissipates energy continuously until some stable wave height is reached where breaking stops and wave reforming and shoaling begins again. It can be expressed as the following mathematical form (2).

$$\langle \varepsilon \rangle = -\frac{K}{h}(F - F_s) \tag{2}$$

in which $K$ is an empirical breaking wave decay coefficient, $h$ is the still water depth, and $F_s$ is the energy flux of a stable wave defined by Dally and Dean(1985). Therefore, the governing equation analyzing the wave height with respect to the distance on-offshore can be finally obtained by substitution of Eq. (2) into Eq. (1).

$$\frac{dF}{dx} = -\frac{K}{h}(F - F_s) \tag{3}$$

To define the energy flux of stable wave height, $F_s$, in Eq. (3), the following stable wave criterion suggested by Horikawa and Kuo(1966) based on the laboratory tests will be introduced.

$$H_s = \Gamma h \tag{4}$$

in which $H_s$ is the stable wave height and $\Gamma$ is an empirical constant. If the beach profile is plane and the water depth in the interested area is shallow, Eq. (3) and (4) are transformed as the following Eq. (5):

$$\frac{dH^2}{dx} + \left(K - \frac{s}{2}\right)\frac{H^2}{h} = K\Gamma^2 h \tag{5}$$

in which $s$ is the bottom slope. Eq. (5) is the first order nonlinear nonhomogeneous ordinary differential equation, so that it can be solved by the integrating factor with the determined integration constant by application of boundary condition that breaking wave height is proportional to the local water depth as:

$$H_b = \gamma h_b \qquad (6)$$

in which $\gamma$ is the breaker index as a constant and subscript $b$ denotes conditions at incipient breaking point. The analytical solution is given as a function of the local water depth only in the surf-zone.

$$H = h_b \left[ (\gamma^2 + \alpha)\left(\frac{h}{h_b}\right)^{(K/s-1/2)} - \alpha\left(\frac{h}{h_b}\right)^2 \right]^{1/2} \qquad (7a)$$

$$\alpha = \frac{K\Gamma^2}{s(5/2 - K/s)} \qquad (7b)$$

From Eq. (7), it is easily found that the wave height tends to increase for increasing the bottom slope and decreasing the breaker index.

### 2.2 Thornton and Guza's Model

In contrast to the monochromatic waves, it is hard to model the mathematical formulation for the random waves well. Because there are both broken and unbroken waves at each spatial point and the percentage of broken waves varies as a function of position, a probability density function(pdf) associated with the distribution of wave height should be introduced to characterize the transformation of wave height from offshore to shoreline. Thornton and Guza(1983) used the Rayleigh pdf to define the wave height distribution at the breaking point. Even though mathematical formulation of this approach is more difficult than that of monochromatic model, this random wave model can be overcome the problems of monochromatic model such that underestimate the wave height near the shoreline. Similar to Dally and Dean's monochromatic wave model, the governing equation of random wave model is also Eq. (1). By the requirement for separation of the causes of the energy dissipation into breaking and friction, Thornton and Guza (1983) has suggested the following Eq.

$$\langle \varepsilon \rangle = \langle \varepsilon_b \rangle + \langle \varepsilon_f \rangle \qquad (8)$$

where $\langle \varepsilon_b \rangle$ represents for the averaged energy dissipation caused by the breaking and $\langle \varepsilon_f \rangle$ is for the mean energy dissipation caused by the bottom friction.

First, the averaged energy dissipation is defined by the periodic bore and pdf of breaking wave height as:

$$\langle \varepsilon_b \rangle = \frac{B_b{}^3}{4} \rho g \frac{\langle f \rangle}{h} \int_0^\infty H^3 p_b(H) dH \qquad (9)$$

in which $B_b$ is a breaker coefficient and $\langle f \rangle$ is mean frequency. In addition, pdf of breaking wave height in Eq. (9), $p_b(H)$ is assumed by Rayleigh pdf specified by the significant wave height, $H_{rms}$ and the weighting function derived from the measured data.

$$p_b(H) = W(H) \frac{2H}{H_{rms}{}^2} e^{-(H/H_{rms})^2} \qquad (10)$$

in which $W(H) = (H_{rms}/\gamma h)^4 [1 - e^{-(H/\gamma h)^2}]$.

Therefore, substitution of Eq. (10) into Eq. (9) and integration give the averaged energy dissipation as a function of significant wave height.

$$\langle \varepsilon_b \rangle = \frac{3\sqrt{\pi}}{16} \rho g \frac{B_b^3 \langle f \rangle}{\gamma^4 h^5} H_{rms}{}^7 \qquad (11)$$

Second, the averaged energy dissipation induced by the bottom friction can generally be defined using the bottom shear stress and pdf of breaking wave height as:

$$\langle \varepsilon_f \rangle = \rho c_f \frac{1}{16\sqrt{\pi}} \left( \frac{2\pi \langle f \rangle H_{rms}}{\sinh kh} \right)^3 \qquad (12)$$

in which $c_f$ is the bottom friction coefficient and $k$ is the wave number. Eq. (12) is also a function of the significant wave height with respect to the distance on-offshore. Therefore, the governing equation for prediction of random wave height transformation at each position on the plane beach profile is obtained by substitution Eq. (11) and (12) into (1).

$$\frac{d}{dx}(H_{rms}{}^2 h^{1/2}) = \frac{3}{2}\left(\frac{\pi}{g}\right)^{1/2} \frac{\langle f \rangle B_b{}^3}{\gamma^4 h^5} H_{rms}{}^7 + \frac{4\pi^{5/2} C_f}{g^{3/2}} \left(\frac{\langle f \rangle}{\sinh kh}\right)^3 H_{rms}{}^3 \quad (13)$$

Following the similar procedure as Dally and Dean's model, we can derive the analytical solution of Eq. (13) under the negligence of friction term. The details of derivation is omitted and final equation is presented as:

$$H_{rms} = a^{1/5} h^{9/10} \left[1 - h^{23/4}\left(\frac{1}{h_o{}^{23/4}} - \frac{a}{y_d{}^{5/2}}\right)\right]^{-1/5} \quad (14a)$$

$$0 \leq h \leq h_o$$

$$a = \frac{23}{15}\left(\frac{s}{\pi}\right)^{1/2} \frac{\gamma^4 s}{B_b{}^3 \langle f \rangle} \quad (14b)$$

$$y_d = H_o{}^2 h_o{}^{1/2} \quad (14c)$$

in which $h_o$ is the wave depth at boundary offshore, and $H_o$ is the wave height at $h = h_o$. These results implies that the wave height depends on directly the energy dissipation near the surf-zone. Therefore, we can quantify the reasonable range of break coefficient, $B_b$ using the analytical solution.

## 3. Wave Set-up/down Model

To analyze the set-up/down, the momentum balance rather than energy balance should be applied to the interested area. As you known, the momentum balance implies that the sum of external forces is equal to the internal momentum exchanges. Therefore, we can derive two momentum equation with respect to the acted direction of each force. Among them, the set-up/down model is derived by the on-offshore component of the equation of motion.

To derive the governing equation of set-up/down model, it is assumed that the bottom friction in the on-offshore direction can be neglected, and waves and bottom topography is uniform in the longshore direction. Then, we can easily derive the on-offshore momentum balance equation.

$$\rho g(\langle \eta \rangle + h)\frac{d\langle \eta \rangle}{dx} + \frac{dS_{xx}}{dx} = 0 \quad (15)$$

in which $\langle \eta \rangle$ represents the variation of mean water level. The radiation stress of the on-offshore component $S_{xx}$ is given by

$$S_{xx} = E\left[n(\cos^2\theta + 1) - \frac{1}{2}\right] \quad (16)$$

in which $n = 1/2 + kh/\sinh 2kh$. The wave set-up/down derived by momentum balance in direction of on-offshore is caused by changing of the radiation stresses.

To derive the analytical solution, it is assumed that the variation of mean sea level, $\langle \eta \rangle$ is much smaller than water depth, $h$ in the region before breaking is occurred. When considering that the incident wave is approaching to the shoreline normally, Eq. (15) can be integrated with respect to distance on-offshore. Therefore, the following analytical solution in the region of seawards of the breaker line is given by

$$\langle \eta \rangle = -\frac{H^2 k}{8 \sinh 2kh} \quad (17)$$

In derivation of Eq. (17), the integrating constant is treated as a zero which means there is no any variation of mean sea level at far offshore.

Meanwhile, the Eq. (17) is never applied to the region of shorewards of breaker line because of the included assumption, $\langle \eta \rangle \ll h$. Therefore, the other analytical solution should be derived which is applicable in the region between the breaker line and set-up limit line. Then, the assumption is no longer available because the order of magnitude of sea level variation is approximately same as that of water depth. It also can be considered that the wave height depends on the shallow water depth and sea level. Therefore, Eq. (15) can easily be obtained by integration as:

$$\langle \eta \rangle = -\frac{1}{1+\frac{8}{3\gamma^2}} h + C \tag{18}$$

in which $C$ is a integration constant that should be determined by Eq. (17)

## 4. Longshore Current Model

The mathematical model for longshore current can be formulated by the longshore component of the depth-integrated momentum balance equation under the same assumptions as those of set-up/down model. The longshore current may be caused by the combination of several forces such as the bottom friction, the gradient of radiation stress, and lateral turbulent mixing.

The governing equation for longshore current analysis is given by the consideration of balance of all forces longshore.

$$\frac{d}{dx}\left[M\frac{dV}{dx}\right] - \langle B_y \rangle = -\frac{1}{\rho}\frac{dS_{xy}}{dx} - \rho_a C_D |W| W \sin\phi \tag{19}$$

in which, $M$ represents the lateral turbulent mixing stress, $V$ is the mean longshore current, $\langle B_y \rangle$ is the time-averaged bottom friction in direction of longshore. $\rho_a$ is the density of air, $W$ is the wind speed, and $C_D$ is the drag coefficient.

The lateral turbulent mixing resulting from the Reynolds stress and the time-averaged bottom friction resulting from the bottom shear stress has been generally expressed, respectively.

$$M = \varepsilon(\langle \eta \rangle + h) \tag{20a}$$

$$\varepsilon = \Lambda u_m H \tag{20b}$$

$$\langle B_y \rangle = \frac{2}{\pi} c_f u_m (1+\sin^2\theta) V \tag{21a}$$

$$u_m = \frac{gHk}{2\sigma}\frac{1}{\cosh[k(\langle \eta \rangle + h)]} \tag{21b}$$

in which $\varepsilon$ is the eddy viscosity coefficient, $u_m$ is the amplitude of the horizontal component of the wave orbital velocity at the bottom, and $\Lambda$ is a empirical coefficient representing the lateral turbulent mixing. Also, the longshore component of radiation stress is given as Eq. (22)

$$S_{xy} = \frac{1}{2} nE \sin(2\theta) \tag{22}$$

To derive the analytical solution, it is assumed that the relative water depth, $kh$ is shallow and the angle of incidence wave is small. Also, the effect of wind and set-up/down can be negligible. Then, Eq. (19) can be written as:

$$\Lambda g^{1/2}\frac{d}{dx}\left[x(gh)^{1/2}(\langle \eta \rangle + h)\frac{dV}{dx}\right] - \frac{\gamma}{\pi} c_f (gh)^{1/2} V = -\frac{5}{16}\gamma^2 g^{3/2} h^{3/2}\frac{\sin\theta_b}{(gh_b)^{1/2}}\frac{dh}{dx} \tag{23}$$

Upon introducing the planar beach profile, $h = sx$ and rearranging each terms, Eq. (23) is simplified as the following Eq. (24).

$$p\frac{d}{dx}\left(x^{5/2}\frac{dV}{dx}\right) - q x^{1/2} V = \begin{cases} -r x^{3/2} & 0 < x < x_b \\ 0 & x_b < x < \infty \end{cases} \tag{24a}$$

$$p = \Lambda g^{1/2} s^{3/2}$$
$$q = \frac{\gamma}{\pi} c_f g^{1/2} s^{1/2} \tag{24b}$$
$$r = \frac{5}{16}\gamma^2 g^{3/2} s^{5/2}\frac{\sin\theta_b}{(g h_b)^{1/2}}$$

Also, Eq. (24a) can be normalized by the characteristics at the breaker line, $x_b$ and $V_b$ as:

$$X' = \frac{x}{x_b}, \quad V' = \frac{V}{V_b} \tag{25}$$

in which $V_b$ is longshore current when neglected the lateral turbulent mixing in the breaker line. Thus, Eq. (24a) is transformed Eq. (26) as the nondimensional form.

$$P\frac{\partial}{\partial X'}\left(X'^{5/2}\frac{\partial V'}{\partial X'}\right)-X'^{1/2}V'=\begin{cases}-X'^{3/2} & 0<X'<1\\ 0 & 1<X'<\infty\end{cases} \quad (26)$$

where $P$ is a nondimensional parameter representing the relative importance of the lateral turbulent mixing.

$$P=\frac{\pi s\Lambda}{\gamma c_f} \quad (27)$$

If the mixing parameter, $P$, is equal to zero, we obtain the simple solution, that is:

$$V'=\begin{cases}X' & 0<X'<1\\ 0 & 1<X'<\infty\end{cases} \quad (28)$$

From the Eq. (28), it is easily found that longshore current increases linearly from the shoreline to the breaker line, and at the breaker line longshore current is discontinuous. However, a analytical technique is needed when $P\neq 0$ because Eq. (26) becomes the nonhomogeneous ordinary differential equation. The first step is to find the particular solution can be defined as:

$$V'=AX' \qquad 0<X<1 \quad (29a)$$

$$A=\frac{1}{\left(1-\frac{5}{2}P\right)} \qquad P\neq\frac{2}{5} \quad (29b)$$

We can see that Eq. (26) is Cauchy equation to find the homogeneous solution. Then, the general solution of Eq. (26) can easily defined as:

$$V'=\begin{cases}B_1X'^{p_1}+AX' & 0<X'<1\\ B_2X'^{P_2} & 1<X'<\infty\end{cases} \quad (30)$$

To determine the integration constants $B_1$ and $B_2$, the boundary condition is applied, which $V'$ and $dV'/dX'$ should be continuous at the breaker line, $X'=1$. Then, we can obtain $B_1$ and $B_2$ as a relation of the mixing parameter, that is:

$$B_1=[P(1-p_1)(p_1-p_2)]^{-1} \quad (31a)$$

$$B_2=[P(1-p_2)(p_1-p_2)]^{-1} \quad (31b)$$

in which $p_1+p_2=-3/2$ and $p_1p_2=-1/P$. We have derived the analytical solution for the longshore current on a plane beach. Therefore, we analyze the characteristics of longshore current using Eq. (30), with Eq. (27) and Eq. (31) together.

The current profiles calculated by Eq. (30) are presented in Fig. 1 for various values of the horizontal mixing parameter $P$.

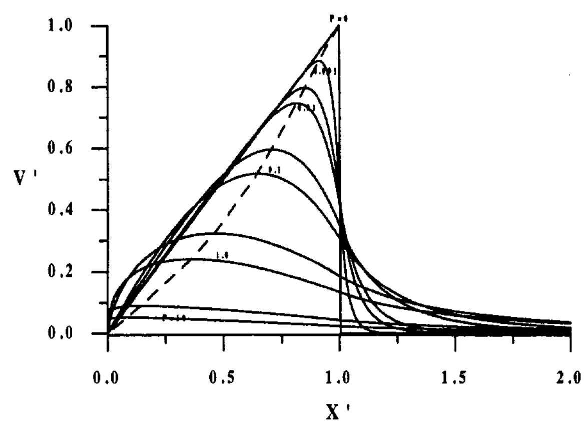


Fig. 1 Current profiles for a sequence of values of the mixing parameter $P$.

These current profiles have the following properties.

First, as $P\to 0$, the profile tends to the triangular form Eq. (28) appropriate to zero mixing. There is a single maximum current $V'_{max}\to 1$ just to the left of the breaker line. To the right of the breaker line we have $V'\to 0$. However, the longshore current profile is discontinuous at the breaker line when $P=0$. On using the values of $p_1$ and $p_2$ we find that in the limit, as $P\to 0$, $V'_b\to 0.5$. In the other words, the current at the breaker line is the mean of the limiting currents on either side. Now as $P$ increases from zero to infinity, $V'_b$ decreases monotonically from 0.5 and 0.

Second, the current profile generally has a single maximum value $V'_{\max}$ lying within the surf-zone ($0 < X' < 1$). It is possible to find the position $X'_m$ of this maximum, we differentiating Eq. (30).

## 5. DEVELOPMENT OF NUMERICAL MODELS

So far, the basic characteristics associated with the wave transformation have sufficiently been analyzed and understood through the analytical solution derived using each governing equations. Then, the considered mathematical formulation will be applied to develop the numerical model. In this section, four numerical models will be developed to predict the longshore current with set-up/down effect on a plane beach. The processes of calculation is proceeded from the seaward end of the numerical grid in the finite difference method.

Energy conservation is first applied to develop two numerical wave height models for the incoming waves obliquely. To calculate the wave height, several initial conditions(water depth, wave height, mean frequency, and incident angle) should be given from the observations as a input data. Then, we will obtain the wave number and wave phase celerity by the dispersion relationship, and the wave angle of incidence will be obtain by Snell's law to consider the refraction of waves. Here, it should be noted that the still water level is not agreement with the mean water level because it is added the displacement quantity due to wave set-up/down. Therefore, the water depth should be re-calculated with the magnitude of set-up/down. Finally, the longshore current is calculated by the wave height and set-up/down data.

### 5.1 Regular Wave Model

The differential equation of Eq. (3) derived from the energy flux balance equation can be

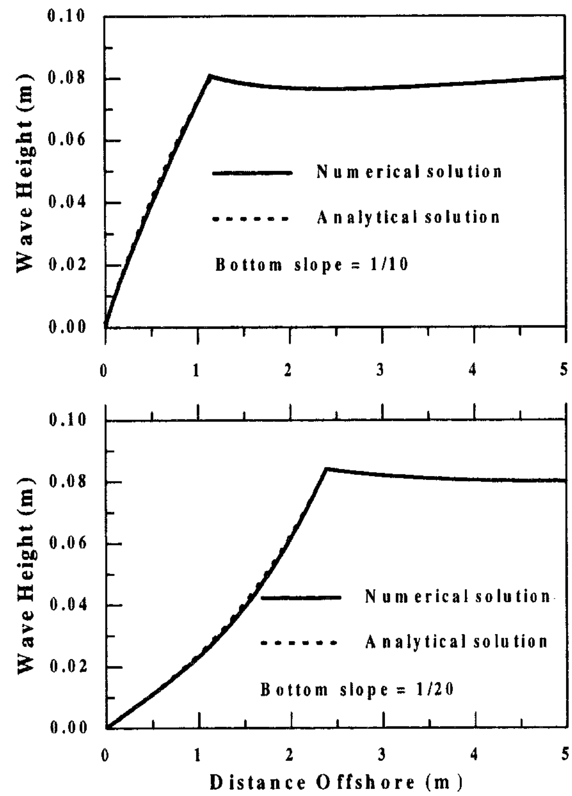


Fig. 2 Comparison of the analytical solution with the numerical solution on two bottom slopes. ($H_0 = 0.08m$, $T = 1\sec$, $K = 0.24$, $\Gamma = 0.3$, $\gamma = 0.7$)

written again as the following difference form.

$$(H^2 C_g \cos\theta)_{i+1} = (H^2 C_g \cos\theta)_i - \frac{K\Delta x}{(\langle\eta\rangle + h)_i}\left[H^2 C_g - \Gamma^2(\langle\eta\rangle + h)^2 C_g\right]_i \quad (32)$$

in which $i$ is a spatial index. If the wave energy flux is stable at point, $K$ is equal to zero because there are no dissipation. Therefore, the simplest forward scheme is given as:

$$H_{i+1} = \left[\frac{(H^2 C_g \cos\theta)_i}{(C_g \cos\theta)_{i+1}}\right]^{1/2} \quad (33)$$

If breaking is initiated, energy dissipation takes place, and $K$ is not zero. Therefore, wave heights can be calculated sequently until the shoreline.

$$H_{i+1}=\left\{\frac{(C_g)_i}{(C_g\cos\theta)_{i+1}}\left[H^2\cos\theta-\frac{\Delta xK}{h}\left(H^2-\Gamma^2(\langle\eta\rangle+h)^2\right)\right]_i\right\}^{1/2} \quad (34)$$

The analytical solution for the characteristics of regular wave transformation have been researched previous section. Fig. 2 shows the comparison of the analytical solution with the numerical solution. Two results are very well agreement with each other. Therefore, it is seen that the numerical model for prediction of regular wave is developed correctly.

### 5.2 Random Wave Model

The analytical solution have been derived by neglecting the bottom friction. But, it can not be neglected whenever the energy dissipation due to the bottom friction is dominated. In this reason, the bottom friction term is added to the more accurate description in this present model. Then, the difference equation of governing equation (1) and (8) can be written as a following form.

$$(EC_g\cos\theta)_{i+1}=(EC_g\cos\theta)_i+\langle\varepsilon_b\rangle_i\Delta x+\langle\varepsilon_f\rangle_i\Delta x \quad (35)$$

The transformation of random wave near the surf-zone on a plane beach can be calculated by Eq. (35). The numerical model results are good agreement with the analytical solution as shown in Fig. 3.

### 5.3 Set-up/down Model

The wave height calculated by each wave model is used to calculate wave set-up/down. Therefore, $\langle\eta\rangle$ can be obtained by the difference form which is formulated from Eq. (15).

$$\langle\eta\rangle_{i+1}=\langle\eta\rangle_i-\frac{[(S_{xx})_{i+1}-(S_{xx})_i]}{\rho g(\langle\eta\rangle+h)_i} \quad (36)$$

An iterative procedure is followed that first

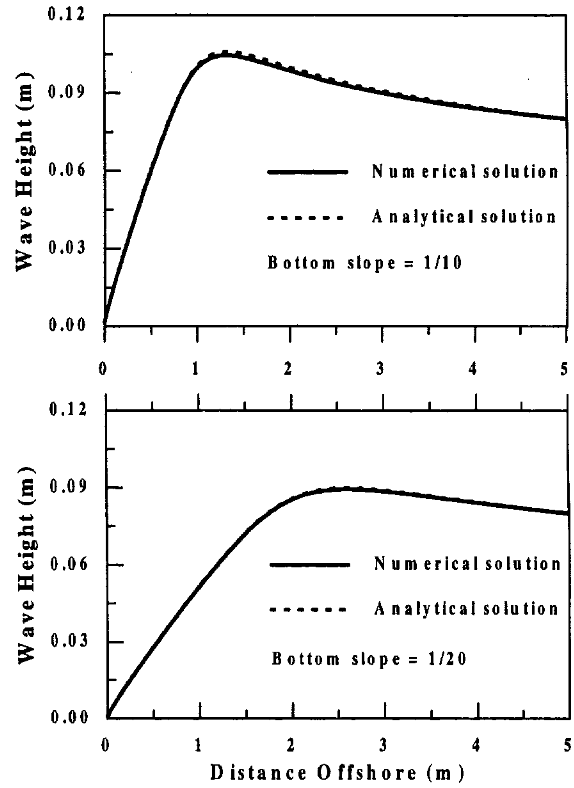


Fig. 3 Comparison of the analytical solution and the numerical solution on two bottom slopes. ( $H_0=0.08m$, $T=5\sec$, $B_b=0.7$, $\gamma=0.6$)

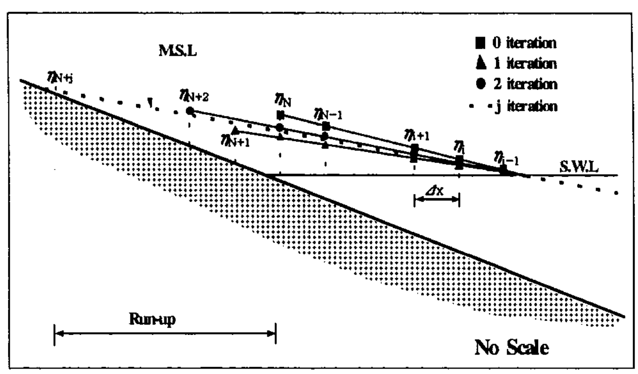


Fig. 4 Schematization for the determination of wave set-up limit line by the moving boundary technique.

wave height based on the still water depth have been employed to estimate $\langle\eta\rangle_1 \sim \langle\eta\rangle_N$. The gradient of between

$\langle \eta \rangle_{N-1}$ and $\langle \eta \rangle_N$ is used to obtain the unknown $\langle \eta \rangle_{N+1}$. This progress are iterated until the $\langle \eta \rangle$ equal to zero following a zigzag course with the mean water level as the central figure. Therefore, the total spatial index number becomes $N+j$ by the addition of $j$. Fig. 4 shows the convergency of $\langle \eta \rangle$.

### 5.4 Longshore Current Model

If the effect of wind is neglected, the governing equation (19) can be rewritten as following form using Crank-Nicolson scheme.

$$\frac{M_i}{\Delta x^2} V_{i-1} - \left( \frac{M_{i+1} + M_i}{\Delta x^2} + B_i \right) V_i + \left( \frac{A_{i+1}}{\Delta x^2} \right) V_{i+1} = -\frac{(S_{xy})_{i+1} - (S_{xy})_i}{\Delta x} \tag{37}$$

in which $B = \langle B_y \rangle / V$. Eq. (37) is simplified as:

$$-a_i V_{i-1} + b_i V_i - c_i V_{i+1} = r_i \tag{38a}$$

$$a_i = -\frac{M_i}{(\Delta x)^2} \tag{38b}$$

$$b_i = -B_i - \frac{(M_{i+1} + M_i)}{(\Delta x)^2} \tag{38c}$$

$$c_i = -\frac{M_{i+1}}{(\Delta x)^2} \tag{38d}$$

$$r_i = -\frac{(S_{xy})_{i+1} - (S_{xy})_i}{\Delta x} \tag{38e}$$

Because, the systems of equation is tri-diagonal matrix, it can be solved by Thomas algorithm. Fig. 5 shows the comparison of the numerical solution with the analytical solution. The difference of the right hand side of maximum velocity is caused by assumptions of the analytical solution.

All numerical models have been formulated to predict the longshore current near the surf-zone on a plane beach. Whole procedure for numerical calculation is summarized in Fig. 6.

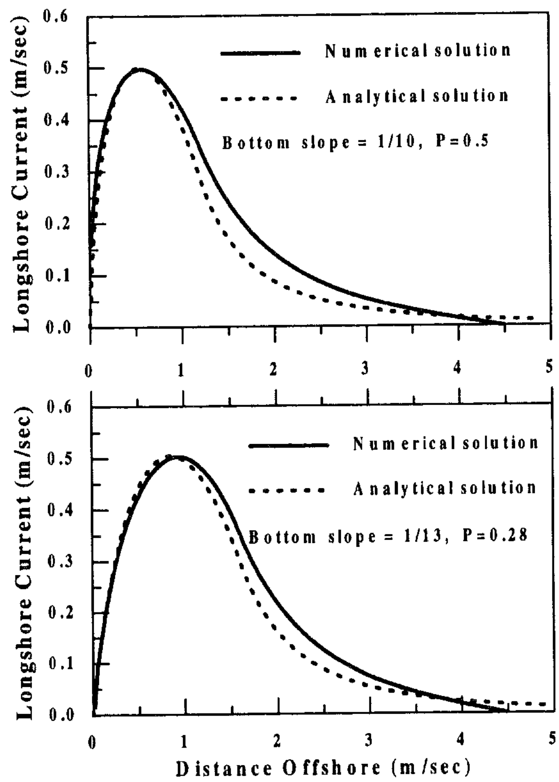


Fig. 5 Comparison of the analytical solution with the numerical solution on two bottom slopes. ( $c_f = 0.01$, $\Lambda = 0.3$)

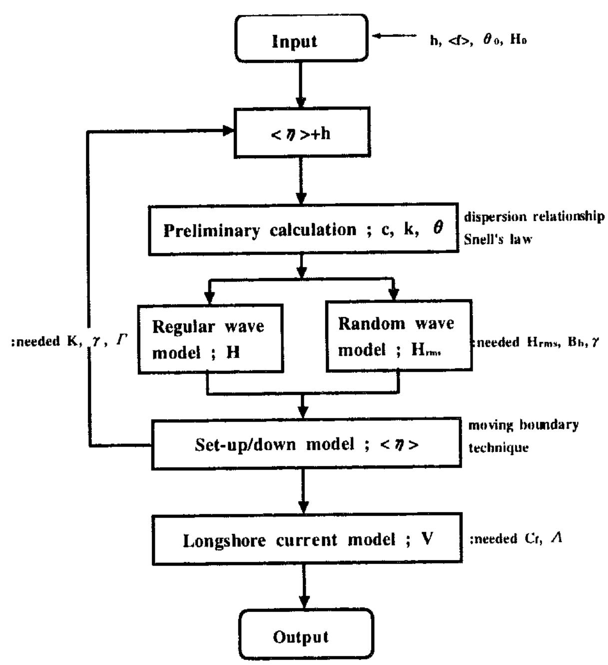


Fig. 6 Schematization of solution procedure.

# 6. VERIFICATION OF NUMERICAL MODELS

In this section, we are going to verify the applicability of developed numerical models. The developed numerical models are compared with two laboratory measurements.

## 6.1 Comparison with the Laboratory Data

To verify numerical models for wave height and set-up/down, the laboratory test have been performed with two-dimensional wave basin. It have been made of iron, having a length, width, and height of 25.0m, 0.8m, and 1.0m respectively. Wave heights are measured by the wave height amplifier and recorder of six-channel. Also, set-up have been measured. Fig. 7 shows the formulation of wave height calculated by the regular wave model and the random wave model, set-up/down are also shown with the calculated results of each wave models together. Model parameters for the regular wave model are used $K=0.2$, $\gamma=0.78$, $\Gamma=0.35$, and for the random wave model are $B_b=0.6$, $\gamma=0.8$, $C_f=0.01$. In the seaward region of break point, the profile of random wave model is smaller than that of the regular wave model. But, the random wave model is larger than the regular wave model inside the surf-zone. The calculated wave heights by the regular wave model are almost agree to the break point. The set-up/down model is also a good agreement with the measured data.

On the hand, the random wave model also agreed with the measurements in spite of this laboratory test is for the regular wave. Also, the set-up/down profile calculated from the random wave model results shows a good tendency with the measurements more than the regular model results.

## 6.2 Comparison with the Laboratory Data of Visser(1982)

Visser(1982) conducted seven measurement runs in a large wave basin, taking care to reduce side wall effects on the circulation. This

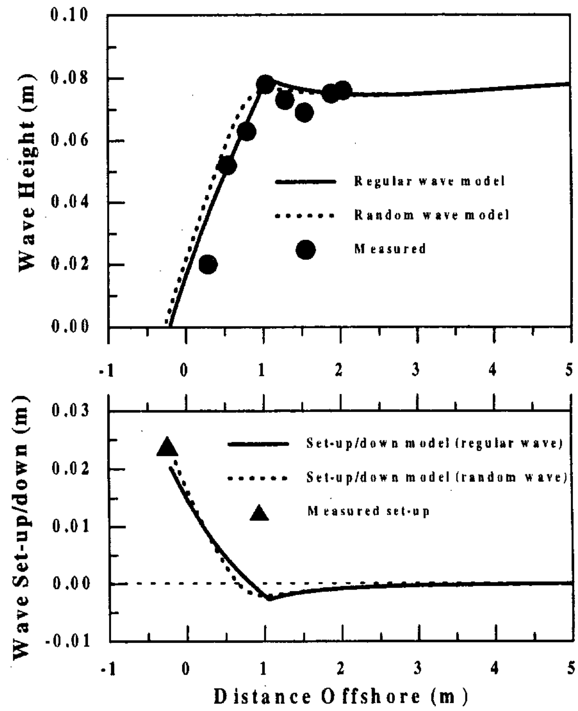


Fig. 7 Comparison of the numerical solution with the laboratory measurements. ( $H_0=0.78m$, $T=1\sec$ , $\theta=0°$ , slope=0.1)

laboratory test have been applied to the regular wave. The initial input data used as deep water conditions to calculate each model are $H_0=0.089m$ , $T=1.00\sec$ , $\theta=15.4°$ , and $s=0.101$.

The calculated longshore current profile shows a good agreement with the measurements in near the surf-zone as shown in Fig. 8. But, the seaward tail of the longshore current profile is over estimated. The calculated wave height by regular wave model and measurements agree up to the break point and for some distance into the surf-zone, but the measured wave height decay is steeper as closer to shoreline. Also, the calculated point of maximum set-down lies seaward of the measured point, and the more seaward start of set-up in the model causes the mean water level to be underestimated in the surf-zone.

The calculated random wave height is larger than the measurements in the surf-zone as shown in Fig. 9. In the shoaling region, wave height was underestimated. These results are

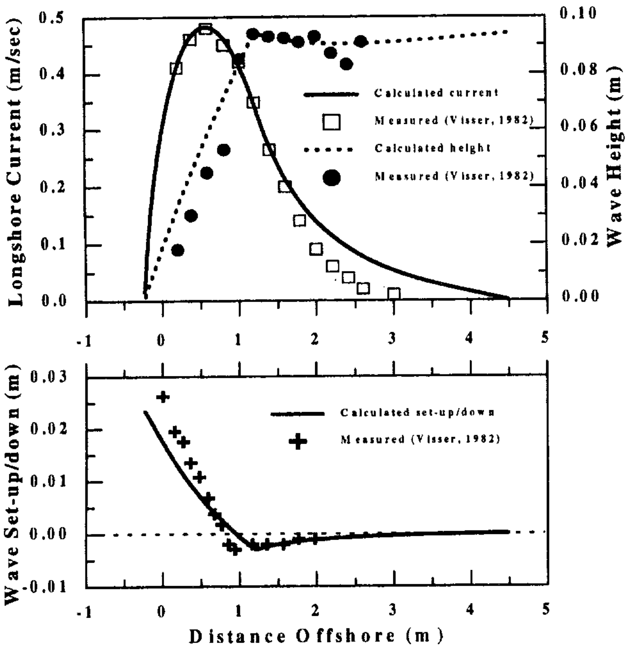


Fig. 8 Comparison of the numerical solution calculated by regular wave model with the laboratory measurements.

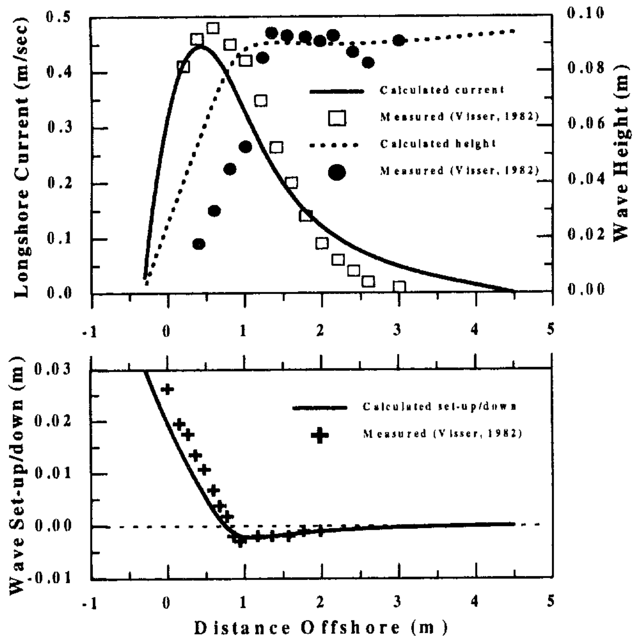


Fig. 9 Comparison of the numerical solution calculated by random wave model with the laboratory measurements.

due to characteristics of wave transformation as above mentioned. The set-up/down profile shows that the calculated set-down profile is seen to be a good agreement, but at the region of set-up, the profile is underestimated. These model results of wave height and set-up/down are not exact relative than that evaluated by the regular wave model. The longshore current profile by using the random wave model have been underestimated.

## 7. CONCLUSIONS

The conclusions may be summarized as follows; Numerical models have been developed to predict wave height, variation of mean water level, and the longshore current on a plane beach. The accuracy of numerical model have been confirmed by the comparison with the analytical solution and laboratory data. The set-up limit line may accurately be estimated by the moving boundary technique. The longshore current profiles give good agreement with measurements. The developed numerical model in this paper can be used as a sub-model of the sediment transport numerical model. The applicability of the developed numerical model for longshore current shall be more investigated by the filed data measured on a arbitrary bottom profile in the future.

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