Trellis Detection of Tamed FM with the DLMS and Convergence

Min-Goo Kang*, Yang-Won Lee*, Hyung-Rae Cho**, Sung-Chul Kang***

Abstract

The Maximum Likelihood Sequence Estimation scheme is modified to improve the error perfomance of the correlative coding in the Tamed FM. To remove intersymbol interference, the Decision Feedback Equalization scheme with the delayed LMS alogrithm and the Viterbi algorithm(10-symbol delay) in the delayed adaptive equalization are proposed for the performance of decision-directed adaptive equalization under the High Frequency channels, and the condition of convergence is analyzed.

Ⅰ. TFM encoding

In the TFM, the phase shifts of the modulated carrier over a one bit period are restricted to $\pm\pi/4$ or $\pm\pi/2$ [rad] determined by three consecutive input binary data bits in accordance with the encoding rule as shown below,

$$\Delta\Phi(mT)=\Phi(mT+T)-\Phi(mT) = \pi/2(b_{m-1}/4+b_m/2+b_{m+1}/4) \quad \cdots\cdots (1)$$

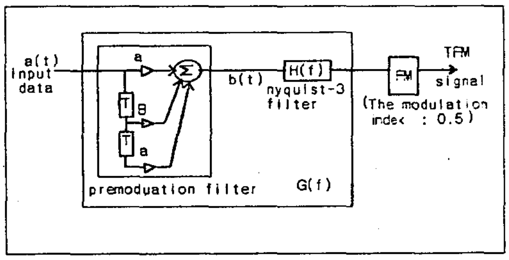

Fig.1. Basic scheme for the generation of a TFM signal

In equation (1), b_{m-1}, b_m and b_{m+1} represent the binary data at the time t=(m-1)T, mT and (m+1) T respectively. The input data are 1 or -1. Using the delay operator D, that equation is described by,

$$x\Phi(D)= \pi/2(D^{-1}/4+1/2+D^{+1}/4)\cdot B(D)= \pi/8D^{-1}(1+D)^2\cdot B(D) \quad \cdots\cdots (2)$$

If the TFM encoding polynomial is S(D) and its frequency spectrum is S(f), S(D) and S(f) are defined by,

$$S(D)=\frac{\Phi(D)}{B(D)}=\pi/8\,D^{-1}(1+D)^2$$

$$S(f)=S(D)\mid_{D=\exp(j2\pi fT)}$$

$$= \pi/8\exp(-j2\pi fT) * (1+2\exp(j2\pi fT)+\exp(j2\pi fT))$$

$$= \pi/8(1+0.5(\exp(-j2\pi fT)+\exp(j2\pi fT)))$$

$$= \pi/8(1+\cos(2\pi fT))$$

$$= \pi/2\cos2(\pi fT) \quad \cdots\cdots (4)$$

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The phase shifts of the modulated carrier of TFM can be extended to include a wide range of values by modifying the premodulation filter charcteristics as follows. Let the premodulation filter G(f) be given by,

$$G(f) = S(f) \times H(f) \qquad (5)$$

H(f) is a low-pass filter satisfying the Nyquist-3 criterion. This filter. i.e., junction with the frequency modulator, serves to ensure the phase $\Phi(t)$ after a smooth transition settles at one of prescribed values at the end of a bit period. In general, H(f) can be written as $H(f)=w(f)(\pi fT/\sin(\pi fT))$. where H(f) is a low-pass filter satisfying the Nyquist-1 criterion. In this thesis, w(f) is considered to have a raised-cosine characteristic, such as,

$$w(f)=\begin{cases} 1 & ,0\le|f|((1-\alpha)/2T) \\ 1/2(1-\sin((f|-1/2)/\pi\alpha) & ,((1-\alpha)/2T\le((1+\alpha)/2T) \\ 0 & ,elsewhere \end{cases} \qquad (6)$$

where α is the rolloff factor in the range from 0 to 1. The chosen value of α has a significant influence on the noise floor level of the signal spectrum as well as on the realizable BER performance with the proposed noncoherent detection scheme.

S(f) is a 3-tap transeversal filter that extends the influence of input data over three bit periods for the phase function $\Phi(t)$. i.e., therefore, S(f) introduces the correlative encoding property into the premodulation filter. The ampilitude response of S(f) is given by,

$$|S(f)| = B(1+2a/B \cos(2\pi fT)) \qquad (7)$$

where a and B are the tap coefficients of S(f). With this modified premodulation filter, the phase shifts of the modulatlated carrier during the m-th bit can be expressed as,

$$\Delta\Psi(mT)= \pi/2(a\times b_{m-1}/4+a\times b_{m+1}/4) \qquad (8)$$

where a and B satisfy the condition (2a + B) = 1, so that the maximum change in $\Phi(t)$ during a one bit period is restricted to $\pi/2$ rad. Consequently, the modulation index of the resulting TFM is always equal to 0.5, the same to the Minimum Shift Keying(MSK) signal. This suggests that the basic coherent demodulator for the MSK signal is also applicable to TFM signals.[1,2]

Ⅱ. Trellis detection algorithm.

A whole class of signals, each with a given set of allowable phase shifts in accordance to eq.(8), can be realized by varying the tap coefficients a and B of the transversal filter S(f) and the rolloff factor α of the Nyquist-3 filter. We call this technique of signal generalized TFM. Because the detected value is dependent to the output sequence b_n at the receiver.

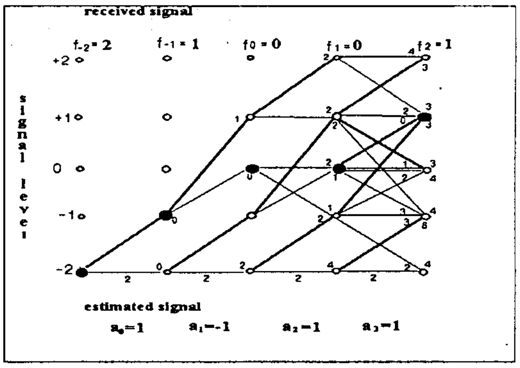


Fig. 2. Trellis diagram for initialization

Trellis detection algorithm is a modified Viterbi decoding algorithm made by Viterbi for convolutional decoding. Trellis detection algorithm is based on the MLSE and decreases the number of calculation using the trellis characteristics of TFM based on the rule for encoding and level transition probability of (D^2+2D+1) in the table 1,2.

Table 1. Rule for encoding and phase shifting of (D^2+2D+1)

a_{k-1}	a_k	a_{k+1}	b_k	$\Delta\phi$
-1	-1	-1	-2	-π/2
-1	-1	1	-1	-π/4
-1	1	-1	0	0
-1	1	1	1	π/4
1	-1	-1	-1	-π/4
1	-1	1	0	0
1	1	-1	1	π/4
1	1	1	2	π/2

Table 2. Rule for level transition probability of (D^2+2D+1)

R1\R2	0	1	-1	2	-2
0	1/8	1/16	1/16	0	0
1	1/16	1/16	1/16	1/16	0
-1	0	1/16	1/16	0	1/16
2	0	1/16	0	1/16	0
-2	0	0	1/16	0	1/16

The Fig. 2 describes the initialization of Trellis detection algorithm. At starting point, the initial states is initialized. This function is named Trellis diagram initialzation stage. The Fig. 3 presents the result of the Trellis diagram initialzation. The real line branch is for a_0 = -1.

The bit-by-bit detection is the symbol detection at t=kT times, but the symbol of the trellis algorithm is detected from the trellis diagram using the encoding rule and the level transition probabilty at a_k =+1,-1.

The number of branch $r_2=\pm2$ is two, That of branch $r_2=\pm1$ and 0 is four individually, and so total number of branch is 16. The flow chart of TDA is shown at Fig. 3. The first step in the trellis diagram initialization like Fig. 2. And the second is the path expansion at this stage, the CPM (Cummulative Path Metric) is calculated, the number of received branch at ±2 in 2 , and at ±1 , and 0 are 4 , so that, the all of branches is 16.

And one path from two branchs is removed by calculating the CPM after k^{th} path . Therefore, 8 survival paths are found by camparison of distance from 16 expansion paths at this time, if the CPM is the same, CPM is campared until $t=r_{t-2}$.

If there is no symbol edtection during N-symbols because of many symbol error, the survival path is the shortest CPM. If a signal $\hat{a}_{k-d}$ is detected from 1 branch, then remove the other. In the case of bit-by-bit detection, the symbol error is the bit error, but in TDA, the bit error is corrected by viterbi Algorithm.[3,4]

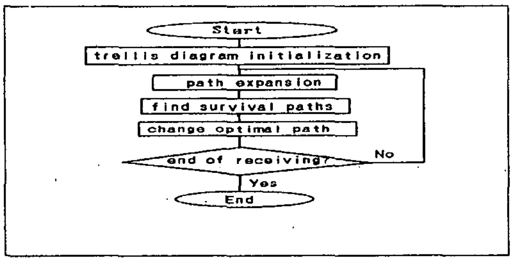


Fig. 3. Flowchart of the Trellis detection algorithm

Trellis detetion algorithm finds the similarity between the received sequence and trellis paths, gets the optimum path using the real path for each level of the trellis diagram. In Fig. 3, the simiarity is correalated to the distance. Since the bad path is not used on each level of the trellis diagram by a metric parameter, the optimum path is obtained. The optimum path is called survival path. There are defintions for the trellis algorithm as belowing,

1) Branch Metric is the level distance of branch between t=(k-1)T and t=kT.
2) Cummulative Path Metric is the metric summation.
3) The maximum path length is confined 10 symbols against error propagation.

In Fig. 3, the path metric is calculated at the path expansion. If the path after k^{th} path is the same, one path from two branchs is removed. At this time, the survival path can be obtained from the Comparision between Distances. Then 8 survival paths are survived from 16 expansion paths.[3-5]

If the cummulative path metric is the same, cummulative path metrics are compared until $t=r_{k-2}$. At the control path length:, if any symbol can't be detected during N-symbols due to many symbol errors, the survival path is the shortest cummulative path metric. And the new path a_{k-d} is detected from one branch and the other branch is removed.

The Trellis Detection Algorithm and linear equalizer (Model-A) is shown at Fig. 4. The output signal of the frequency discriminator passes the linear equalizer. The output of linear equalizer is feedback through the hard-decision circuit of typical FM receiver with fiequency discriminator.[6]

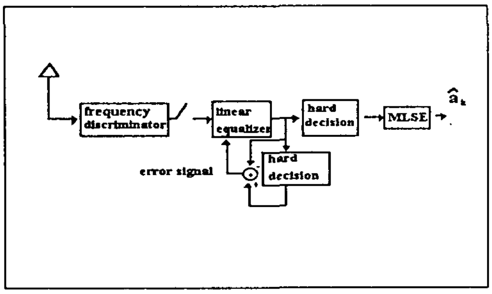


Fig. 4. Detection of the MLSE and the linear equalizer (Model_A)

But, the proposed model has linear equalization and MLSE, that is, Trellis Detection Algorithm using correlative redundancy. The model-B is the Trellis Detection Algorithm with delayed LMS and MLSE. The error is from the summation the output signal of linear equalization and MLSE output. Therefore, The error signal from the output signal of Model-B is more correct than that of Model-A. The output of frequency discriminator is the summation of 3 inter-symbols. Fig. 5 shows a model for a DLMS.

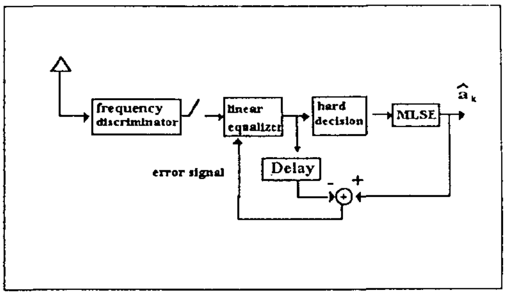


Fig. 5. Detection of the MLSE and the Delayed LMS (Model_B)

Ⅲ. Channel Modeling

The discrete-time characteristics of telephone channel modeling is described by eq.(9), and the frequency characteristics is shown at Fig. 6. The discrete-time telephone channel is described by,

$$F(z) = 0.04 - 0.05z^{-1} + 0.07z^{-2} - 0.21z^{-3} 0.5z^{-4} + 0.72z^{-5+0.36z^{-6}} + 0.21z^{-8} + 0.03z^{-9} + 0.07z^{-10} \quad (9)$$

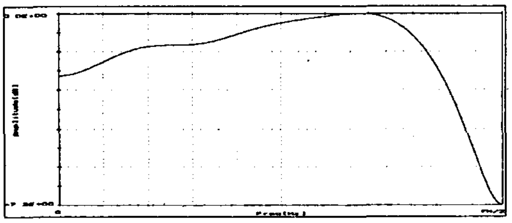


Fig. 6. Frequency characteristics of Discrete-time Telephone Channel

Those of HF (high frequency) channel is represented at eq.(10) and Fig. 7

$$F(z) = 0.408 + 0.816z^{-1} + 0.408z^{-2}$$

(Eigen value = ∞) .... (10)

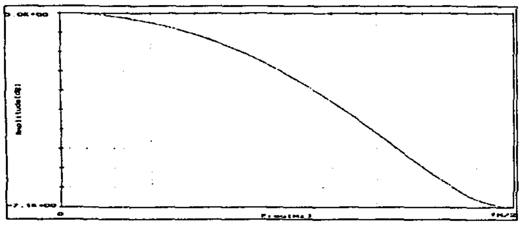


Fig. 7. Frequency characteristics of High Frequncy Discrete-time Channel

Ⅳ. Simmulation and Result

For the more spectral efficiency and high SNR of the multilevel TFM, the trellis detection with DLMS is proposed for the BER enhancement at telephone and HF channels.

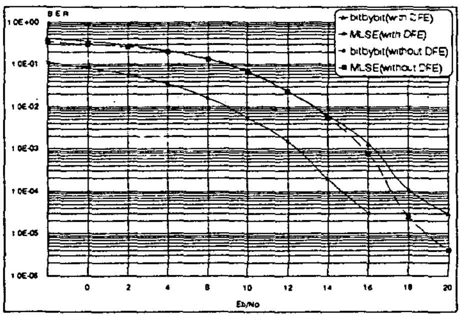
At the BER curve for Telephone channel, the DFE is 1~2 dB better than linear equalizer and the TDA with DLMS is improved about 4dB, compared with bit-by-bit detection with linear equalizer

The TDA with DLMS is improved some 4dB TDA with Delayed LMS is the best. As a result, the trellis detection with DFE is 4dB better. The trellis detection with DLMS is 3dB better

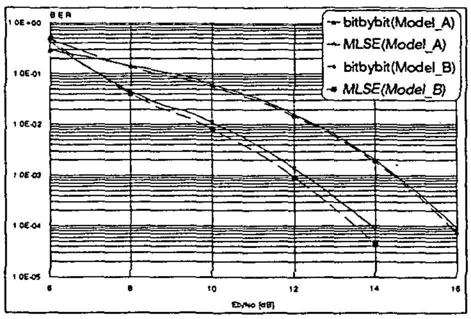
Ⅴ. Conclusion

The MLSE detection used with the DFE obtains a 4dB performance improvement when compared to the bit-by-bit detection at the BER level under the telephone channel. In Fig. 8 and Fig. 9, the comparison of BER between bit-by-bit detection and trellis detection with the decision feedback equalization is shown

The MLSE detection with delayed LMS obtains better performance improvement than that the

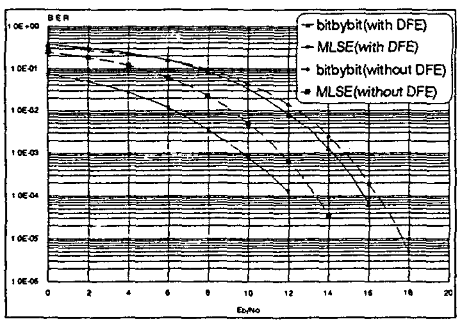


(a) bit-by-bit & MLSE with DFE scheme

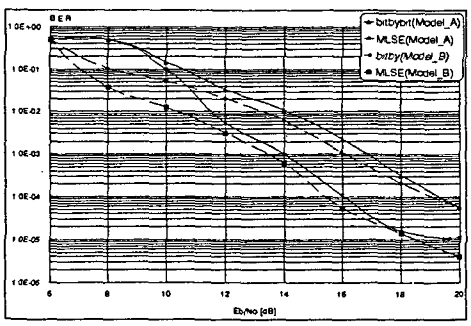


(b) bit-by-bit & MLSE with DLMS

Fig. 8. BER curves of telephone line



(a) bit-by-bit & MLSE with DFE scheme



(b) bit-by-bit & MLSE with DLMS

Fig. 9. BER curves of HF channel.

MLSE detection with linear equalizer, when compared to the bit-by-bit detection at the BER level under the HF channel. In Fig. 8 and 9, the MLSE detection with DLMS gets 2dB BER improvement that of bit-by-bit detection in the HF channel.

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Appendix-A : Delayed LMS and it's Convergence[4]

The output of the equalizer is given by

$$y_k = c^{(k)} r^{(k)} \quad \text{......(A1)}$$

where $r^{(k)}$ is the colume vector of received samples on the delay line at time K and $c^{(k)}$ is the vector of M tap coefficients of the transversal filter. The steepest descent algorithm for adjusting the equalizer to minimize the mean square error takes a step in the direction opposite to the direction of the gradient of the error D time units ago, that is, at time K+1 the weight vector becomes

$$c^{(k+1)} = c^{(k)} - s_k g^{(k-D)} \quad \text{......(A2)}$$

where s_k is a parameter controlling the step size and $g^{(k)}$ is the gradient vector

$$g^{(k)} = E\left[\frac{\partial e_k^2}{\partial c^{(k)}}\right] \quad \text{......(A3)}$$

e_k is the error, that is, the difference between the desired and actual response,

$$e_k = d_k - y_k \quad \text{......(A4)}$$

The weight adjustment algorithm then becomes

$$c^{(k+1)} = c^{(k)} + \Delta_k E[e_k - \ D^{r^{(k-D)}}] \quad \text{......(A5)}$$

$$c^{(k+1)} = c^{(k)} - \Delta_k (Ac^{(k-D)} - a) \quad \text{......(A6)}$$

where Δ_k is the step size parameter (equal to $2s_k$), A is the correlation matrix of the input sequence (assuming $r^{(k)}$ and d_k are stationary sequences)

$$A = E[r^{(k)} r^{(k)T}] \quad \text{......(A7)}$$

and a is the cross-correlation vector

$$a = E[d_k r^{(k)}] \quad \text{......(A8)}$$

The optimum value of weight vector (that which minimizes the mean square error) is given by

$$c_{opt} = A^{-1} a \quad \text{......(A9)}$$

In order to decouple the weight adjustments, define the transformation

$$c' = Pc \quad \text{......(A10)}$$

where P is an orthonormal matrix which diagonalizes A

$$A = P^{-1} \Lambda P \quad \text{......(A11)}$$

and Λ is the diagonal matrix of the eigenvalues of λ_i of A. Then

$$c'^{(k+1)} = c'^{(k)} - \Delta_k(\Lambda c'^{(k-D)} - P\alpha) \quad \text{(A12)}$$

The optimum decoupled weight vector can be written as

$$c'_{opt} = \Lambda^{-1} P\alpha \quad \text{(A13)}$$

Let us define the tap weight error as

$$h^{(k)} = c'^{(k)} - c'_{opt} \quad \text{(A14)}$$

Then,

$$h^{(k+1)} = h^{(k)} - \Delta_k \Lambda h^{(k-D)} \quad \text{(A15)}$$

Furher assume Δ_k to be constant and equal to Δ. Then for each j, $j = 1, \cdots, M$, the z-transform of the jth tap coefficient error is

$$H_j(z) = \frac{z^{D+1} h_j^{(0)}}{z^{D+1} - z^D + \Delta\lambda_j} \quad \text{(A16)}$$

The limit as $k \to \infty$ of $|h_j^{(k)}|$ is zero if and only if all of the poles of $H_j(z)$ are within the unit circle in the complex z-plane. If

$$\lim_{k \to \infty} |h_j^{(k)}| = 0 \quad \text{(A17)}$$

then $\lim_{k \to \infty} c'^{(k)} = c'_{opt}$

Stability also requires that all of the poles of $H_j(z)$ be within the unit circle. Thus, we conclude that the adjustment algorithm is stable and converges if and only if, for each j, all of the roots of the characteristic polynomial

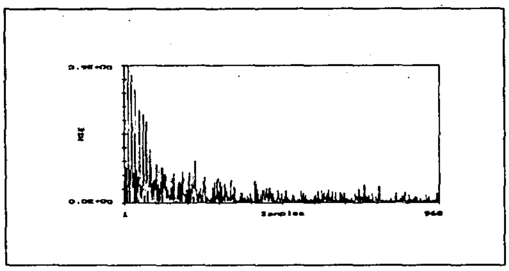

Fig. A1. convergence curve of the linear equalizer

$$F(z) = z^{D+1} - z^D + \Delta\lambda_j \quad \text{(A18)}$$

lie within the unit circle.

For the analysis of convergence curve, the linear equalizer has 31-forward taps and coefficient is 0.001, The linear equalizer converges very well like Fig. A1

Appendix-B : Stability and Convergence criterea

The particular form of $F(z)$ allows us to detemine the location of its roots. Define

$$\beta = \Delta\lambda_j \quad \text{(A19)}$$

The values of β for which $F(z)$ has roots on the unit circle can be determined by setting

$$z = e^{j\phi} \quad \text{(A20)}$$

Since β is real, form(A18),

$$\beta = \cos D\phi - \cos(D+1)\phi \quad \text{(A21)}$$

$$\sin D\phi - \sin(D+1)\phi = 0 \quad \text{(A22)}$$

The values of ϕ satisfying the latter equation define the points at which roots can lie on the unit circle, namely

$$\phi = \frac{l\pi}{2D+1} \quad \text{(A23)}$$

where l is odd and $|l| \leq 2D+1$ *or* $l = 0$.

The values of β corresponding to $F(z)$ having roots on the unit circle can be determined form (A21) and (A23),

$$\beta = 2\sin\frac{l\pi}{2(2D+1)} \quad \text{(A24)}$$

The overall behavior of the system can be ascertained by using root locus techniques. For $\beta = 0$, D roots lie at the origin and one root lies at z = 1. As β is increased, the root at z = 1 and one of the other roots move to toward each other on the real axis to meet at $z = D/(D+1)$.

The other roots move radially form the origin. For larger values of β, the two real axis roots split to form a complex conjugate pair which cross the unit circle at

$$\phi = \pm \frac{\pi}{2D+1} \quad \text{(A25)}$$

The values of β for which all roots lie inside the unit circle correspond to the interval $(0, \beta_{max})$ where β_{max} corresponds to $l=1$ in (A24). In terms of Δ and λ_j this interval is

$$0 < \Delta < \frac{2}{\lambda_j} \sin \frac{\pi}{2(2D+1)} \quad \text{(A26)}$$

The step size Δ must satisfy (A26) for each λ_j. Thus, the system is stable and converges if and only if

$$0 < \Delta < \frac{2}{\lambda_{max}} \sin \frac{\pi}{2(2D+1)} \quad \text{(A27)}$$

where λ_{max} is the largest eigenvalue of the correlation matrix A.

The system of DFE and DLMS has 6-forward taps, 5-backward taps, and coefficient is the same 0.001 like the linear equalizer, and it converges very well like Fig. A2.

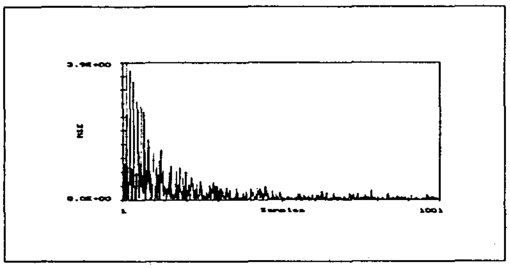

Fig. A2. convergence curve of DFE and DLMS

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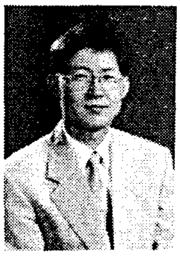

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