Analysis of Lifetime Data using Proportional Hazards Model

- 비례위험모형을 이용한 수명데이터의 분석 -

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요 지

본 연구에서는 의학분야의 생존분석에서 적용되어 왔던 Cox의 비례위험모형을 신뢰성예측에 적용할 때의 분석절차 및 그에 따른 소프트웨어를 다룬다. 이 비례 위험모형은 신뢰성공학분야에 적용될 경우 많은 잠재력을 가지고 있으나, 그 분야에 적용된 경우가 많지 않고, 이미 적용된 사례들도 잘못 적용되어 왔다는 지적이 많은 실정이다. 본 연구에서는 시스템 , 서브시스템, 부품수준에서의 각 라이프사이클을 거치며 얻어진 수명데이타를 분석하여 신뢰도를 예측할 수 있는 모형을 설정하고, 그에 따른 소프트웨어를 다루며, 이 방법의 개관, 장단점, 주의점등을 고찰한다.

1. Introduction

Life cycle Life test refers to the process of testing a product over an extended period of time or usage cycles in order to observe the failure event and hopefully to study the failure process. Points of interst are the failure mode, failure mechanism, and the time or usage experienced before failure. Reliability measures deals with sustained, failure-free product-process performance that meets or exceeds customer's needs and expectations. Those product and process performance varations is resulted from configuration applications and materials environments and operating methods. Even under well defined and controlled applications and laboratory environments, product life tends to vary considerably.

Hence, estimates of product life from small samples may not be extremely reliable. Product variation, application variation, and environmental variation tend to act together to produce a great deal of variation in field performance over time. [9]

Reliability prediction are generally concerned with projecting the type of experience a customer may have with the product relative to performance, over time, for a given application, and within a given envrionment in light of such uncertainty. Obtained reliability

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prediction estimates are useful for:

1) identifying potential reliability problems.
2) reliability design trade-off study in order to select best one from competing designs.
3) determining warranty costs, availability, operational effectiveness.
4) planning maintenance and logistics support strategies.

Accurate reliability prediction are very important since they can considerbly effect reliability estimates. Otherwise, it can mislead to the conclusions of life cycles and costs. Explanatory variables upon which the time to failure depends are usually not accounted fully for reliability analysis.

Proportional hazards modelling identifies the effects of various explanatory variables or factors which may be associated with variations in the length of life of eqipment, factors such as temperature, pressure, speed, material, use condition, operating history, design changes, etc., may be considered. Repairable as well as non-repairable systems may be studied. Data may be censored or uncensored. [5,10,14] Since the PH model is exceptionally flexible, it is possible to construct more accurate reliability prediction model that can explain the performance variation effects in terms of explainatory variables.

Statistician have done much recent work on the theory and application of proportional hazards(PH) models, particulary in medical research. The PH models have much potential for application in reliability engineering, because they permit consideration of explanatory variables, also called covariates, in reliability models. Two PH models have outstanding potential for wide application: the parametric Weibull PH model and the nonparametric "Cox" PH model (named after D.R. Cox, who derived it). The Weibull model is especially useful, because:

1) It include the exponential distribution as special case.
2) It is resonably familiar to scientists and engieers, because of the relationship to the exponential distribution and because of widely-used graphical methods (Weibull plotting).
3) It coincides with and accomodates the accelerated testing models (including the classical models such as the Arrhenuis and inverse power law).

In this paper, overview of such the proportional hazards modeling approach in the reliability engineering field is described along with software tools and its implementation to lifetime data analysis for the field acquired reliability database. Also, we discuss potential advantages and disadvantages that will be arised out of engineering applications. This paper will be illustrative and described in a tutorial manner the basic undestnading and techniques for proper application of PH modelling in reliability prediction.

2. Basic Proportional Hazards Models

Suppose that T denotes the random variable that is the lifetime of system, subsystem, part or component. Let f(t) be the probability density function of such failure times. Corresponding reliability function will be R(t).The hazard (or failure rate) function which is indication of the failure characteristics of system, subsystem, part, component or equipment is:

$$h(t) = \frac{f(t)}{R(t)} \tag{1}$$

where h(t)dt is the conditional probability of failure in (t, t+dt) given survival to time t. Usual analysis on h(t) is done with the notion of function of failure time with fixed conditions and variables. No restrictions on h(t) are imposed with such as physics of failure, aging characteristics and other explanatory variables which might be important factor that effects the reliability of such item under consideration.

2.1 Basic Model

Under PH models [1,2,5,6,11], the hazard rate of an item is not only a function of failure time (age), but is also affected by explanatory or covariates as below:

$$h(t) = h(t, z; \beta) \tag{2}$$

where z = row vector of covariates

β = colurm vector of regression coefficients.

The β vector is estimated by the method of maximum likelihood, using a Newton-Raphson algorithm. The general multiplicative hazard function is:

$$h(t, z; \beta) = h_0(t) g(z; \beta) \tag{3}$$

where $h_0(t)$ is the baseline hazard function and $g(z; \beta)$ is a function relating the covariates. The $g(\cdot)$ can take various forms, but must assure that the $h(\cdot)$ function is non-negative for all values of the time to failure random variable T. The function which has been found most useful is

$$g(z; \beta) = \exp(z\beta). \tag{4}$$

In Equation (4), the first covariate in the z vector is generally defined as $z_1 = 1$ such that when all other covariates are set to zero, the following case arises:

$$g(z; \beta) = \exp(\beta_1) \tag{5}$$

Following this convention $h_0(t) = \exp(\beta_1)$ and Equation (2) for the hazard functiong becomes simply

$$h(t, z; \beta) = \exp(z\beta) \tag{6}$$

2.2 Proportional Hazards

The multiplicative family of hazard functions includes the proportional hazards class. These functions have the property that different individuals in a population have hazard functions retaining constant proportions over time. Specifically, for two individuals X and

Ys having hazard functions $h(t; z_x;\beta)$ and $h(t; z_y;\beta)$, the ratio

$$\frac{h(t; z_x;\beta)}{h(t; z_y;\beta)} \tag{7}$$

is constant for all t > 0. Plots of

$$\ln(t) \text{ versus } \ln[-\ln R(t; z_x;\beta)] \tag{8}$$

and

$$\ln(t) \text{ versus } \ln[-\ln R(t; z_y;\beta)] \tag{9}$$

are both linear. Inspection of log-tranformed Weibull distribution reveals that the linear relationship of log-transformed data points underlies graphical estimation of the Weibull parameter, using Weibull paper. In the event of Weibull proportional hazards, the plots of (8) and (9) are linear and parallel. The linear and parallel properties have to be observed in the life data at different levels of variables in order to apply PH model. These graphical properties are very valuable in descriptive analysis for

(a) Preliminary identification of a parametric life distribution(e.g., the Weibull);

(b) Association of covariates. When the data are stratified (based upon different values of the covariate) the plots, Equations (8) and (9) should be linear and parallel.

2.3 Weibull Proportional Hazard Model

The Weibull hazard function is

$$h(t;\theta;\delta) = (\delta/\theta)\,(t/\theta)^{\delta-1} \tag{10}$$

where θ is scale parameter

δ is shape parameter.

Therefores the ratio of the hazard functions, for two individuals, X and Y, is:

$$\frac{h(t; \theta_x; \delta_x)}{h(t; \theta_y; \delta_y)} = \frac{(\delta_x/\theta_x)(t/\theta_x)^{\delta_x-1}}{(\delta_y/\theta_y)(t/\theta_y)^{\delta_y-1}} \tag{11}$$

Recall that the slope of the Weibull plot provides the estimate of the shape pararneter δ. Clearly, for proportional hazards, the individuals, X and Y, must have the same slopes in their Weibull plots and

$$\delta_x = \delta_y = \delta. \tag{12}$$

Then Equation (11) becomes

$$h(t; \theta_x;\delta)/h(t; \theta_y;\delta) = (\theta_y/\theta_x)^{\delta} \tag{13}$$

and the ratio of the hazard functions is seen to be independent of time.

Stated another way, the individuals, X and Y, have hazard functions retaining constant proportions over time--proportional hazards.

2.3.1 Exponential Case

The exponential distribution of time to failure is a special case of the Weibull distribution, where $\delta = 1$ and the hazard function is

$$h(t; \theta) = (1/\theta), \qquad (14)$$

a constant with respect to time. The mean of the distribution is θ and is referred to as the Mean lime to Failure (MTTF). It is also the characteristic life. The exponential proportional hazard model is obtained by substitution of Equation (6):

$$h(t; z; \beta) = 1/\theta(z, \beta) = \exp(-z\beta). \qquad (15)$$

2.3.2 Weibull Case

Generalizing from the exponential case to the Weibull, the covariates are included in the Weibull distribution through the characteristic life. Thus, the Weibull reliability function is written:

$$\begin{aligned} R(t; \theta(z, \beta); \delta) &= \exp[-[\ (t/\theta(z, \beta))^{\delta}]] \\ &= \exp[-\ (t\ e^{-z\beta})^{\delta}] \end{aligned} \qquad (16)$$

2.3.3 Inverse Power Law

The Weibull proportional hazard model coincides with the accelerated testing model (where some stress variable acts multiplicatively on the time measure to accelerate failure). In fact, the Weibull PH easily accommodates the classical acceleration models, such as the inverse power law. For examples, Nelson[12] described the Weibull characteristic life, θ, as an inverse power function of applied voltage(V):

$$\theta(V) = \frac{1}{c\ V^{p}} \qquad (17)$$

where v = stress (in kV)

c,p = positive parameters characteristic of the material(insulation) and test method.

Recall that the values of the parameters (c and p) were estimated emperically, by the least squares method. Taking the natural log of both sides in Equation (17):

$$\ln[e(V)] = -\ln(c) - p\ln(V) \qquad (18)$$

then exponentiating both sides:

$$\begin{aligned} \exp[\ln[\theta(V)]] &= \exp[-\ln(c) - p\ln(V)] \\ \theta(V) &= \exp[-\ln(c) - pln(V)]. \qquad (19) \\ \theta(z; \beta) &= \exp(z\beta) \qquad (20) \end{aligned}$$

where $z_1 = 1,\ z_2 = \ln(V),\ \beta_1 = -\ln(c),\ \beta_2 = -p.$

2.3.4 Maximum Likelihood Estimates

SAS (Statistical Analysis System) procedures PHREG which was added in starting SAS version 6.8 along with existing LIFEREG and LIFETEST provide maximum likelihood estimates of the Weibull proportional hazards model. Also, NCSS (SOLO) microcomputer packages can be used to obtain the maximum likelihood estimates (MLE) of non-parametric proportional hazards model. Although complete Fortan code can be found in the Kalbfleisch and Prentice book for obtaining estimates of PH model, their code contain few bugs. It is necessary to take a carefull examinination of solution to verify with solution which is obtained from the other packages.

2.3.5 Multiple Covariates

The weibull PH model readily accommodates additional covariates. they are simply added to the vector z. For example, if two continuous covariate stresses (say voltage z_2 and temperature z_3) are applied simultaneously in a life test, the Weibull characteristic life might be modeled as

$$\theta(z, \beta) = \exp(\beta_1 + z_2 \beta_2 + z_3 \beta_3) \tag{21}$$

In data analysis, it is often helpful to make transformations on the units of measures such as scaling or logarithmic transformations.

A discrete covariate generally has two levels, which essentially stratify the population. If the shape parameters for the j levels, or strata are equals, i.e.,

$$\delta_1 = \delta_2 = \ldots = \delta_j = \delta \tag{22}$$

and the proportional hazards model applies, then the MLE's may be obtained by pooling the data from the strata, to achieve better parameter estimates in terms of reduced variance. Discrete covariates are included by forming indicator variables (0 or 1) for each level or strata. Suppose, for examples that test articles of one item are produced by three different manufacturers (A, B and C). Then the characteristic life, for one discrete covariates with three levels, is modeled as follows:

$$\theta(z, \beta) = \beta_A + \beta_B z_B + \beta_C z_C \tag{23}$$

where $z_A = 1, 1,\ z_B = 0, 1,\ z_C = 0, 1.$

The characteristic lives for each strata are calculated, based on the MLE's of the β vectors as follow:

$$\theta_A(z, \beta) = \beta_A \tag{24}$$

$$\theta_B(z, \beta) = \beta_A + (1)\ \beta_B + (0)\ \beta_C \tag{25}$$

$$= \beta_A + \beta_B$$

$$\theta_C(z, \beta) = \beta_A + (0)\ \beta_B + (1)\ \beta_C \tag{26}$$

$$= \beta_A + \beta_C$$

In this illustration, the A stratum is seen to define the "baseline" hazard function.

Discrete and continuous covariates may be included in the same model. Interaction between two covariates may be modeled by forming a third covariate from the product of the two covariates under study. Suppose there are two stresses in a life test and these stresses are continuous covariates (say voltage z_2 and temperature z_3) for which there is an interaction $z_4 = z_2 z_3$. Then the Weibull characteristic life is

$$\theta(z, \beta) = \beta_1 + z_2 \beta_2 + z_3 \beta_3 + z_4 \beta_4 \qquad (27)$$

where $z_4 = z_2 z_3$

Selection of the units of measure for the primary covariates can serve to increase or decrease the measured interaction. There are nonparametric methods available for use in analysis of life data.

2.4 Cox Proportional Hazards Model

Cox Proportional Hazards Model which was proposed by D. R. Cox is distribution free [4]. The method is based on Equation (3):

$$h(t; z; \beta) = h_0(t) \exp(z\beta) \qquad (28)$$

where important properties of $h(t; z; \beta)$ are independent of $h_0(t)$ for complete testing and minimally dependent upon the baseline hazard for several types of censoring. Software is becoming available for this methods. Various software such as SAS and NCSS(SOLO) includes distribution free Cox PH procedure.

3. Software Tools

Although software tools is becoming widely available to support analysis of PH data arising from tests as well as field use, their application to reliability data is not

straightforward, particularly for repairable systems. Examples of available software include Statistical Analysis System (SAS) [3], Biomedical (BMDP), NCSS (SOLO) [13], GLIM, SURVREG, SYSTAT, SURVIVAL and a variaty of microcomputer packages with limited capabilities. The available packages include:

1) Graphical aids (parametric and nonparametric)
2) Estimation of reliability distribution properties (point and interval estimates),
3) Formal statistical tests of significance
4) Prediction models

Even though a number of programs and packages are now available to implement the

proportional hazards model, the most widely accessible computer programs although not necessarily easiest to implement nor complete, are PHREG procedure along with LIFETEST LIFEREG procedures within the SAS packages:

1) LIFETEST provides product limit estimates for censored failure data. It also provides formal statistical tests useful in determining whether stratas defined by values of a covariates are significantly different. Tests of association for other covariates are also

provided.

2) LIFEREG provides maximum likelihood estimates of Weibull parameters (as well as parametric estimates for other optional distributions) and of regression coefficients for covariates. Estimates of the variances of parameters and of the maximized log likelihood are useful in testing hypotheses concerning parameters and in forming interval estimates (confidence intervals) on the parmeters.

3) PHREG procedure performs regression analysis of survival data based on the Cox proportional hazards model. It performs a stratified analysis to adjust for subpopulation differences. The PHREG procedure allows us to: a) test linear hypothesis about the regression parameters b) perform conditional logistic regression analysis for matched case-control studies c) create a SAS data set containing survivor function estimates and residuals d) create a SAS data set containing estimates of the survivor function at all event times for a given realization of explanatory variables.

Those who uses Macintosh OS can obtain SURVIVAL 6.04 version from info-mac site that have capability of analysing upto 30 covariables and unlimited data point of Cox Proportion Hazards Models as long as the memory is permitted. It's code was written in Mac Think Pascal.

Fortran codes for the proportional hazards model can be found in the book by Kalbfeisch and Prentice [8], although these contain some errors. Currently, modified and extended versions of these programs are being worked with and developed in the several english institutions and orgranizations.

4. Analysis Procedure of Lifetime Data using Proportional Hazards Model

The reliability analyst should be familiar with the design and use of the items, for which failure data is collected. This knowledge should include:

(a) Failure modes,
(b) Environmental factors,
(c) Test procedures,
(d) Data collection procedures, and
(e) Maintenance procedures.

The analyst should also use a combination of statistical techniques in analyzing the data. The PH model and accelerated testing can be utilized as an exploratory approach and for improved predictions. Through a working knowledge of the equipment and procedures using a variety of analytical tools, engineers and scientists can gain valuable insights leading to better design and operational practices.

In order to apply PH model, the data collected have to be stratified into separate reliability estimates for levels of the covariates. The plots of $\ln(t)$ versus $\ln[-\ln[R(t)]]$ satisfy that they are resonably linear and parallel. Visual inspection provides a valuable first indication of the model, but can be deceiving, due to scaling effects. Therefore, it is necessary to perform statistical tests.

4.1 Tests of Homogeneity

The tests for homogeneity of the data set basically test whether the differences among the strata (Defined by a covariate) are statistically significant. The statistical hypothesis is

$$H_0: R_1(t) = R_2(t) = \ldots. = R_C(t) \tag{29}$$

for the c strata (c levels of the covariate). There are three non-parametric tests commonly used, and available in SAS LIFETEST:

1) Logranks
2) Wilcoxon and
3) Likelihood Ratio.

The same formula is used to calculate the logrank and Wilcoxon test statistics, with a difference in weighting factors. The test statistics have, as limiting (asymptotic, or large-sample) probability distributions, the chi-square.

The chi-square is widely tabulated in statistics texts. The logrank test places greater weight on larger survival times, while the Wilcoxon test places more weight on early survival times, Consequently, the logrank test is more inclined to detect a significant difference among strata at low values of time T. The Wilcoxon test is more useful in detecting differences among the reliability functions at the far right (high values of T). Ideally, both tests will provide results that are in agreement.

The likelihood ratio test assumes the exponential distribution for all strata. This amounts to assuming a Weibull distribution, with shape parameter $\delta = 1$ for all strata. This assumption is graphically interpreted as follows: The parametric Weibull plots would be parallel, with a slope of one (1). The slope of one on the graph applies only to the case in which both the horizontal and vertical axes are properly scaled.

Under this assumption, the experimental hypothesis is equality of the Weibull scale parameters across strata, i.e.,

$$H_0: \theta_1 = \theta_2 = \theta_C \tag{30}$$

for c strata. The test ststistic is calculated and treated as a chi-square random variables with c-1 degrees of freedom.

Flemings et.al [7], have proposed a generalized two-sample Smirnov test for the hypothesis.

$$H_0: R_1(t) = R_2(t) = R(t) \tag{31}$$

Note that this test applies to comparison of two reliability distributions (i.e., homogeneity across two strata). This test is powerful in detecting differences between two reliability functions, even when there is no significant difference at some time t [e.g., values of T where the product limit estimates for the two strata cross in ln-ln plot). Software is available through the SAS Users Group, to perform this test.[7]

4.2 Tests of Association

Typically, there are multiple covariates defined for an individual item. The general procedure is to:

(a) Stratify the data based on one covariate;
(b) Calculate the product limit estimates for each strata;

(c) Test the equality of reliability distribution (homogeneity across strata);
(d) Test for association of other covariates.

SAS calculates four tests of association between a covariate and the response (the time-to-failure random variable) . These tests are generalizations or the logrank and Wilcoxon tests for homogeneity. The test statistics are computed for each covariates and treated as chi-square variates with one degree of freedom. The tests are as follows:

(a) Univariate logranks
(b) Univariate Wilcoxons
(c) Stepwise logrank and
(d) Stepwise Wilcoxon.

In the stepwise tests a candidate covariate is selected by a method of steepest ascent, then a marginal test statistic is calculated, conditioned upon the covariates already setected.

4.3 Weibull Estimates

If we confine our interest to the Weibull PH model, The SAS procedure calculates MLE point estimates of the Weibull parameters and covariate regression coefficients. When a user specifies the Weibull distribution, SAS procedures provide the parameter and regression coefficient estimates of the extreme value distribution. These are chosen by maximizing log likelihood. Standard error for each estimate are calculated based on large-sample theory. Confidence intervals may also be constructed on the likelihood ratio statistic. The intervals are generally considered superior for small samples, but require a trial-and-error method.

4.4 Hypothesis Testing

The hypothesis tests on model parameters are based on three different approaches, all of which depend upon large-sample theory:

a) Asymptotic normality b) Likelihood ratio and c) Lagrange multipliers.

The hypothesis tests of most general interest are the following:

a) Equality of Weibull shape parameter for different strata defined by the value of a covariates; b) Equality of Weibull scale parameter for different strata, assuming equlity of shape parameter.

5. Conclusion

Although PH modeling approach in reliability engineering seems to be effective method, it should be used an exploratory tools with caution. If it is implemented without violating assumption of model, the advantages coming out of using the PH model are:

a) no assumption is needed about the lifetime distribution
b) it can be applied to non-repairable or repairable systems
c) censoring, tied values and zero values are easily handled
d) no assumption need to be made about the form of the base-line hazard function
e) the effects of explanatory variables are estimated
f) accelerated lifetest analysis is possible with time-dependent explanatory variables
g) non-stationary such as reliability growth can be incoporated within the model.

As with PH model, there are some disadvantage:

a) more failure data than usual analysis is needed, since arbitrary baseline functions and one or more regression coefficients must be estimated.
b) the explanatory factors for each part or systems must be known
c) requires the sophisticated software packages for analysis.

The primary objective of this paper is to facilitate use of the Weibull PH model as the basis for reliability analysis of components and equipment. This objective is accomplished through tutorial presentation of the concepts.

While PH model using covariates improve the reliability model by:

1) accounting for nonhomogeneity of the the test sample (e.g., test unitsrepresenting different design revisions or different manufacturers).
2) accounting for different treatments (e.g., different levels of environmental stresses applied to different test articles), but blindly applying this technique without checking assumption and condition are prohibitive in sense that the misuse of this model will cause serious effect on the result. It has many shortcoming and difficultties of applying it inthe field as well.

When it is implemented correctly, the PH modelling approach should benefit the design and development process through:

1) Identification of key explantory variables and extent of impact by those variables on the designs
2) Identification of useful accelerating stress variables allowing compression of test time
3) data and prediction models useful for performing trad-off studies among design alternatives
4) Find-and -fix, failure corrective action mutually beneficial to suppliers and users
5) Initial reliability estimates for preliminary system predictions and baselining of reliability efforts
6) Multiple phase reliability testing consistent with the needs of a multiple phase system life cycle and associated reliability models.

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