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A Practical Method of Prediction of Resistance for Displacement Vessels

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Abstract

The prediction of the total resistance of a ship is generally based on considering it to be a simple sum of the viscous resistance and the wave resistance. An experimental approach for predicting full-size ship resistance on this basis is practical but obviously has the deficiency that a model has to be built for each prototype of interest and the resulting tank tests are time consuming.

On the other hand, purely theoretical calculations of the wave resistance, using, for example, the Michell theory, require relatively little computer time and give an excellent portrayal of the overall variation of the vessel resistance as a function of forward speed. Unfortunately, there are sufficient differences between this theory and the measured results to make this method impractical for design purposes.

The proposal examined here is to use a data bank of experimental resistance results to modify the theoretical predictions. It is demonstrated that the technique will produce remarkably accurate resistance predictions and can take into account the effects of the water depth, any restriction of canal or river width, as well as the prismatic coefficient, and other geometric parameters.

1 Introduction

1.1 Background

The wave resistance of a ship is defined as the drag associated with generating the wave pattern behind the vessel. In addition to this component of drag, one must add the viscous resistance, which can be estimated by one of the flat-plate skin-friction formulas, to be found in Lewis, 1988. These two components together constitute an approximation of the total drag on the ship.

The subject of wave resistance is one which has been studied for over a century now. The work of Michell, 1898 was the first which resulted in a usable formula for the wave resistance of a ship traveling at a constant speed in deep water. The assumptions in his theory were that the effects of viscosity and surface tension could be ignored. Additionally, the ship was considered to be *thin*.

In the current work, the theory of Michell, as extended by Lunde, 1951 for a river or canal with a rectangular cross section, has been used. That is, the effects of finite water depth and lateral restriction on the width of the waterway are included.

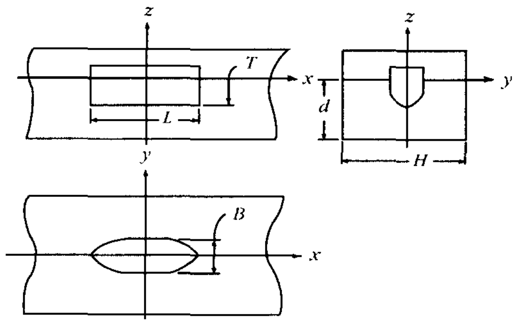


Figure 1. The Towing Tank

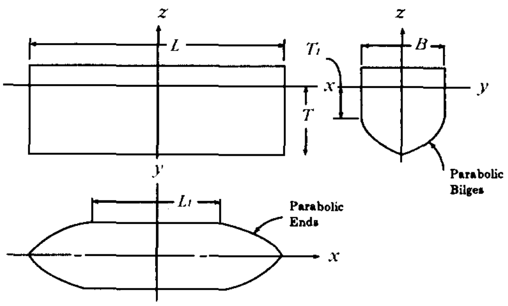


Figure 2. The Wigley Hull

1.2 Current Work

The work to be described has its origins in a series of collaborative papers by Doctors, Renilson, Parker, and Hornsby, 1991, Doctors and Renilson, 1992,1993 and Doctors, 1994,1995.

There, both catamarans and a monohull were tested in a towing tank filled with water to various depths. Attempts to correlate the experimental results for the resistance with the linearized theory were made. It was found that the theory could be used quite accurately to predict the effects of /it changes in the water depth, the spacing between the demihulls of a catamaran, as well as the effect of sloping river banks.

The purpose of this paper is to describe a more detailed series of numerical and experimental investigations in which the influence of the prismatic coefficients, defined as

$$C_p = \nabla / A_M L,$$

is to be studied. Here, ∇ is the immersed volume of the vessel, A_M is the maximum-section area, and L is the length.

2 Analytic Work

2.1 Linearized Theory

The theory of Lunde [3] has been used for this research. That is, the effects of finite water depth d and lateral width H of the towing tank or the waterway are included.

The experimental setup is shown in Figure 1. The general formula for the wave resistance is

$$R_W = \frac{2\rho g}{H} \sum_{i=0}^{\infty} \epsilon_i \frac{w^2 k (U^2 + V^2)}{2k - k_0 \tanh(kd) - k k_0 d / \cosh^2(kd)} \tag{1}$$

where

$$\epsilon_i = \begin{cases} \frac{1}{2} & for \quad i = 0 \\ 1 & for \quad i \geq 1 \end{cases} \tag{2}$$

and ρ is the water density and g is the acceleration due to gravity.

The longitudinal and transverse wavenumbers in Equation (1) are

$$w = \sqrt{k^2 - u^2} \tag{3}$$

$$u = 2\pi i / H \tag{4}$$

while the circular wavenumber k is given by the solution of the implicit dispersion relationship:

$$\begin{aligned} f &= k^2 - kk_0 \tanh(kd) - u^2 \\ &= 0 \end{aligned} \qquad (5)$$

$$df/dk = 2k - k_0 \tanh(kd) - kk_0 d/\cosh^2(kd) \qquad (6)$$

Finally, the fundamental wavenumber is

$$k_0 = g/U^2 \qquad (7)$$

where U is the speed of the ship.

The index i of the summation in Equation (1) has been dropped from all the symbols except for the sake of brevity.

We next consider the two finite-depth wave functions in Equation(1), which are

$$U = [P^+ + exp(-2kd)P^-]/[1 + exp(-2kd)], \qquad (8)$$

$$V = [Q^+ + exp(-2kd)Q^-]/[1 + exp(-2kd)], \qquad (9)$$

in which the Michell deep-water wave functions $P^\pm$ and $Q^\pm$ are defined by

$$P^\pm + iQ^\pm = \int_{S_0} \mathcal{B}(x,z) exp(iwx \pm kz) dx dz \qquad (10)$$

Here, x and z are respectively the longitudinal and vertical coordinates, $\mathcal{B}$ is the local beam, and the integration is to be performed over the centreplane area S_0.

A modified version of the hull defined by Wigley, 1934 was used here. The hull has parabolic section bilges and parabolic waterplane ends, as shown in Figure 2. The hull has a length L, a draft T, and a beam B. The vessel has a variable amount L_1 of parallel middle body, which allows one to vary the prismatic coefficient, a parameter which is known to have a strong influence on the wave resistance. The vessel can also have a wall-sided region of the draft T_1 near the waterplane, thus allowing one to investigate the influence of the vertical prismatic coefficient, but this feature was not considered in the current study.

Because the dependence on x and z can be separated, this hull is called a "simple" ship and the first wave function in Equation (10) can be expressed as

$$P^\pm = P^{(x)} P^{(\pm,z)}, \qquad (11)$$

while the second wave function $Q^\pm$ is zero because of fore-and-aft symmetry. The x-dependent factor in Equation (11) can be obtained by analytic integration of the demihull form Equation(10) to give

$$P^{(x)} = -\frac{4B}{w(A_2 - A_1)} \left\{ \cos(A_2) - \frac{\sin(A_2) - \sin(A_1)}{A_2 - A_1} \right\} \qquad (12)$$

where

$$A_1 = \frac{1}{2} w L_1, \qquad (13)$$

$$A_2 = \frac{1}{2} w L \qquad (14)$$

In a similar manner, the z-dependent factor is given by

$$P^{(\pm,z)} = \pm\frac{1}{k}\left\{1 + 2\frac{exp(\mp C_2) - exp(\mp C_1)}{(C_2 - C_1)^2} \pm 2\frac{exp(\mp C_2)}{C_2 - C_1}\right\} \tag{15}$$

in which

$$C_1 = kT_1 \tag{16}$$
$$C_2 = kT \tag{17}$$

2.2 Method of Applying Correction

Two approaches for correcting the resistance for the influence of water depth were tried. In the first method, the assumption was made that the influence was to alter the ratio of the wave resistance. That is,

$$R_W^{Pred.}(F, d, C_P) = \frac{R_W^{Theory}(F, d, C_P)}{R_W^{Theory}(F, d^*, C_P)} R_W^{Exp.}(F, d^*, C_P) \tag{18}$$

In Equation(18) the experiment is done with a base water depth d^* and the prediction for the resistance at a different depth d is computed for the same Froude number F and the same prismatic coefficient C_P.

The Froude number is defined in the usual way as

$$F = U/\sqrt{gL}$$

In order to be able to effect the prediction using Equation (18), one must first subtract the frictional resistance. The frictional drag on the model was computed on the basis of the 1957 International Towing Tank Committee(ITTC) formula, described by Lewis, 1988(Section 3.5).

In the second approach, the assumption was made that the influence of depth was to cause a *shift*, or difference, in the wave resistance. That is,

$$\begin{aligned} R_W^{Pred.}(F, d, C_P) &= R_W^{Theory}(F, d, C_P) - R_W^{Theory}(F, d^*, C_P) \\ &\quad + R_W^{Exp.}(F, d^*, C_P) \end{aligned} \tag{19}$$

It is interesting to note that using different formulations for the frictional drag will alter the result given by Equation (18). On the other hand, the result of Equation(19) is *unaffected* by the choice of method for the friction calculation. This is one reason for preferring the latter approach.

2.3 Procedure for Smoothing the Data

The straightforward application of the prediction procedure, as described above, suffers from the fact that errors in the data for the base case will affect the outcome. For this reason, it was decided to implement a smoothing of the data based on the "graduation" developed by Whittaker, 1923 and Henderson, 1924. The algorithm has been modified here to permit uneven spacing of the points.

We start by considering the function:

$$E = \sum_{i=0}^{N} (y'_i - y_i)^2 + \alpha \int_{x_1}^{x_N} \left(\frac{d^2y'}{dx^2}\right)^2 dx \tag{20}$$

where $P(x_i, y_i)$ is the original set of N points or knots and the smoothed set of points is $P'(x_i, y_i')$. The parameter α is arbitrary and can be selected to alter the relative importance of the two terms on the right-hand side.

One may now imagine the curve to be represented by a thin elastic beam, or spline, connected via elastic springs at its knots to the original unsmoothed points. Assuming that the spring-beam system is in equilibrium, it is clear that the strain energy in the springs is represented by the first (summation) term in the Equation(21) while the second(integral) term represents the strain energy in the beam.

We are only interested in the relative effect of the two terms, and this can be entirely handled by the smoothing parameter α. The case of a perfectly supple beam (no smoothing) is modeled by choosing α equal to zero and a rigid beam is modeled by a value of infinity.

The computer program effectively minimizes the energy in the system, and therefore solves the spring-supported-beam problem, which by nature, assumes the minimum-energy configuration. The above equation was discretized by replacing the second-derivative term by the appropriate finite-difference formula, thus giving

$$E = \sum_{i=0}^{N}(y'_i - y_i)^2 + \alpha \sum_{i=2}^{N-1} \left(\frac{y'_{i+1} - y'_i}{x_{i+1} - x_i} - \frac{y'_i - y'_{i-1}}{x_i - x_{i-1}} \right)^2 / \frac{1}{2}(x_{i+1} - x_{i-1}) \tag{21}$$

The next step is to construct the derivatives $\partial E / \partial y_i'$ and equate them to zero, in order to obtain the conditions for the minimum value of E. This provides a set of N equations for the N unknown values of y_i'. The resulting matrix solution is very rapid, since one can take advantage of the fact that the matrix is a pentadiagonal one.

2.4 Error in the Prediction

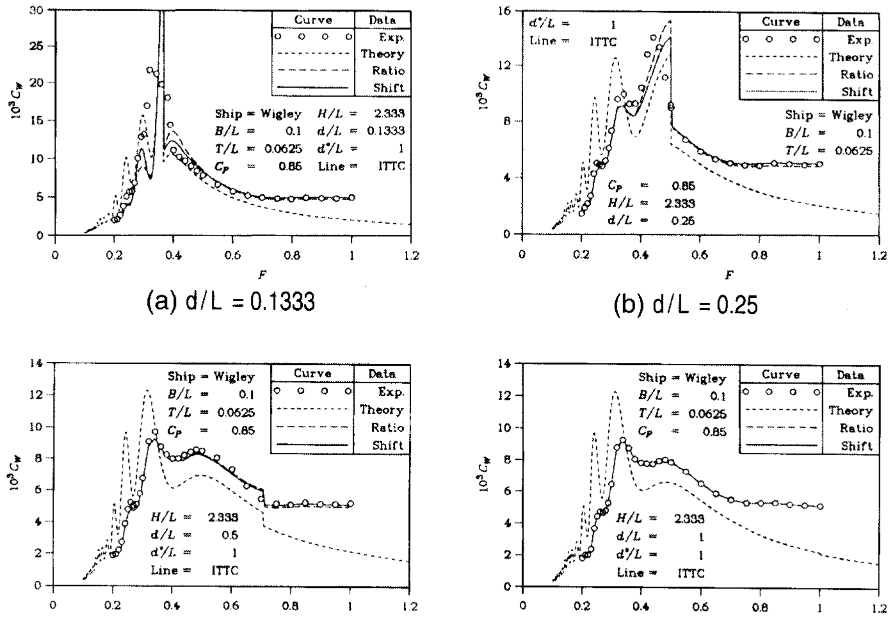
As well as preparing graphs comparing the predictions for the resistance with the experimental data, it was considered necessary to develop a precise definition of error for comparing the different numerical approaches.

The chosen error estimate was defined to be :

$$e = \sqrt{\frac{1}{W}\sum_{i=0}^{N} w_i \delta_i^2} / \frac{1}{W}\sum_{i=1}^{N} w_i y_i, \tag{22}$$

$$W = \sum_{i=1}^{N} w_i \tag{23}$$

Here δ_i is the error at each point(the difference between the predicted and the measured value of the resistance), w_i is a weight applied to each point, y_i is the measured resistance, and W is the sum of the weights. It can be seen that the numerator of Equation(22) is the weighted root-mean square error. The weights were selected to be the spacing between the resistance values. Thus, the result would not be sensitive to different choices of spacing of the points - such as concentrating readings near turning points of the curve. Next, the denominator is effectively the average value of the resistance over the range of speeds for which the calculation is made. Consequently, the measure of error has been normalized and it therefore represents the relative error.



(a) d/L = 0.1333

(b) d/L = 0.25

(c) d/L = 0.5

(d) d/L = 1.0

Figure 3. Basic prediction method

Other choices of definition of error are clearly possible. For example, the denominator could be chosen to be the peak resistance in the curve, instead of the average resistance. Additionally, one can apply the method to either the resistance itself or the resistance coefficient. The latter was done for the purpose of this paper.

3 Results and Discussion

The Wigley models tested had a length of 1.5 m. They had the standard beam-to-length ratio B/L of 0.1 and the standard draft-to-length ratio T/L of 0.0625. A series of four water depths will be discussed here. Five models were tested in the towing tanks. The prismatic coefficients C_P were 0.6667, 0.75, 0.80, 0.85, and 0.9, respectively. As noted above, the wall-sided region(the value of T_1) was kept at zero.

The results for the wave resistance of the fourth model, with C_P= 0.85, are shown in Figures 3(a) through (d), respectively. The ordinate is the wave-resistance coefficient, defined in the usual way, as

$$C_W = R_W / \frac{1}{2} \rho U^2 S$$

where S is the wetted-surface area.

The dimensionless base depth d^*/L for use in Equation(18) and Equation(19) was chosen to be unity. Thus, there is perfect agreement for both types of correction in Figure 3(d), When the depth $d = d^*$.

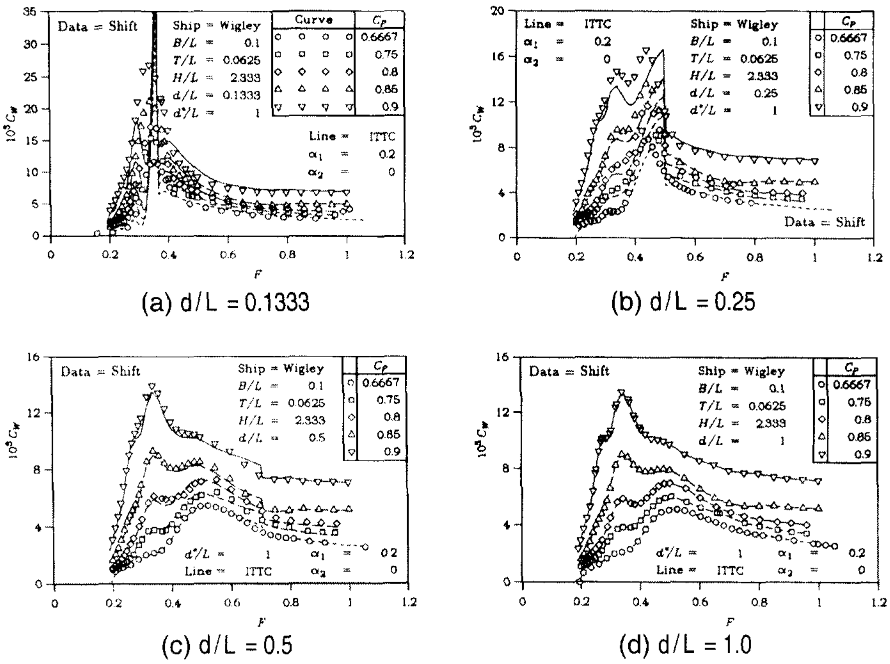


Figure 4. Smoothing experimental data

In examining the four parts of Figure 3, we can see that the use of the theory alone greatly underpredicts the wave resistance at high Froude numbers. Additionally, it suggests unrealistically high values of the resistance when the depth Froude number, given by

$$F_d = U/\sqrt{gd},$$

equals unity.

Next, we observe that substantial improvement in agreement with the experiments is obtained by using either of the two corrective approaches described above. Indeed, the agreement is within a few percent at high Froude numbers. At the lower Froude numbers, the interesting jump in the resistance curve at a depth Froude number of unity (which corresponds to a Froude number that depends on the depth) is also predicted well – particularly for the depth-to-length ratios d/L of 0.25 and 0.5 in Figure 3 (b) and (c).

In very shallow water, such as in Figure 3(a), the prediction of the correction technique has deteriorated. Also, the corrective method predicts sharp jumps in the wave resistance in Figures 3(b) and (c), which are not observed in practice. These two points will be addressed again later.

Results for the three models are plotted together for the four different depths in Figure 4. Smoothing has now been applied here to the raw experimental data (indicated by the parameter $\alpha = 0.2$.) The improvement in the curves is evident, since the unwanted kinds in the predictions – particularly evident in the curves in Figure 3(c) – have now been eliminated.

In the next demonstration, smoothing(given by $\alpha = 5000$)is also applied to the theoretical curves in the four parts of Figure 5. It is clear that there is considerable advantage in using this idea to eliminate the unnatural sharp jumps and, in particular, the very high values of the resistance

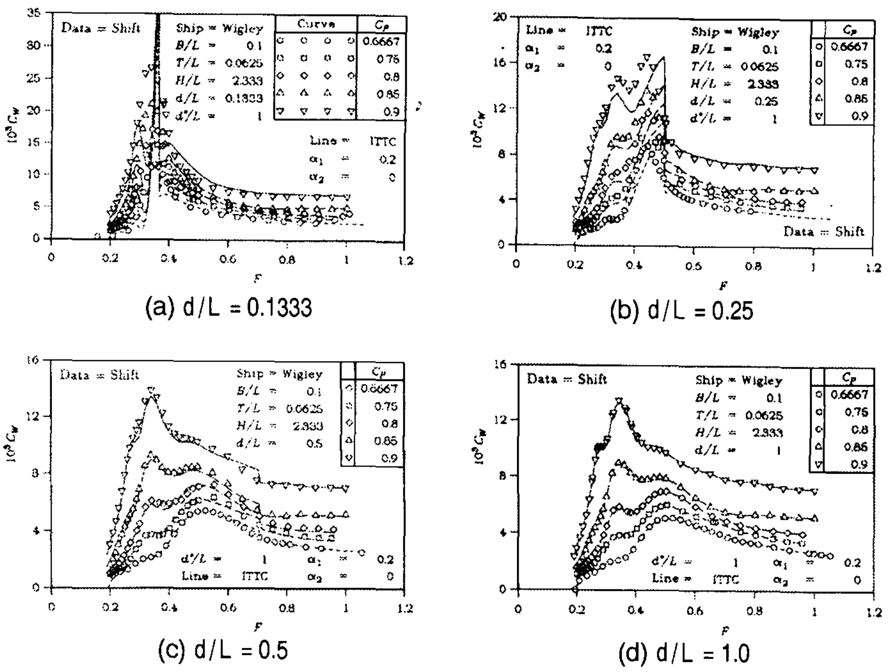


Figure 5. Smoothing all data

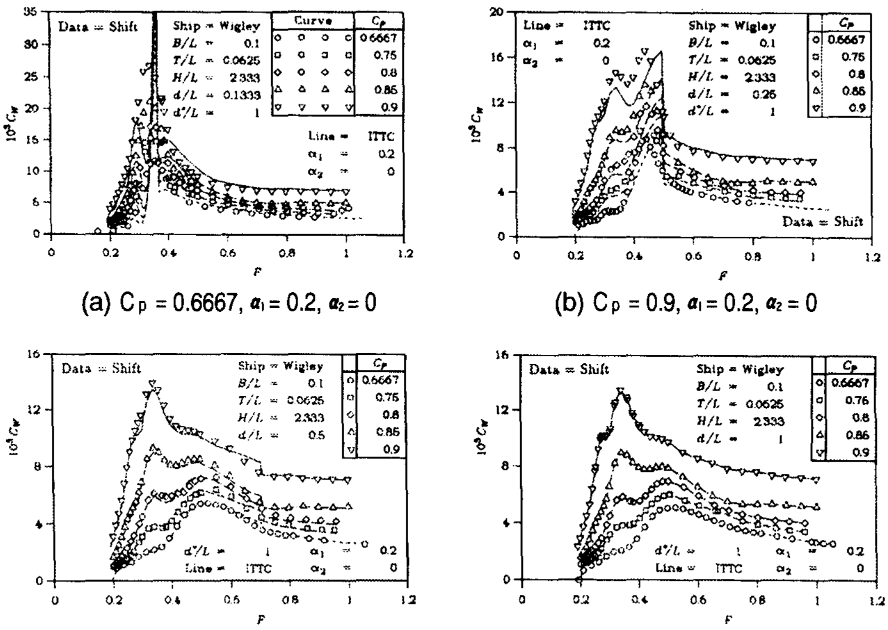
peaks in very shallow water, since the correlation between the predictions and the experimental data is now much improved.

Finally, we examine the question of the errors themselves, as defined by Equation(22). Samples of these computations are shown in Figure 6. Figures 6(a) and (b) show the errors for two extreme values of the prismatic coefficient, when smoothing is applied to the experimental data alone. In each case, three methods are tested: the pure theory, the ratio method and the shift method. It can be observed that the errors in the theory are worst for the greatest depth. This can be checked by reference to Figure 3, where there is a substantial percentage error at the high-speed end for the graph, for the case of $d/L = 0$. On the other hand, we demonstrate here an order-of-magnitude reduction in the error by employing the current techniques. Also, the shift method is seen to be reliably better than the ratio method.

Figures 6(c) and (d) show another pair of graphs, in which smoothing is applied to both the theoretical calculation and experimental data. As anticipated from Figure 5(in comparison to Figure 4), there is a further improvement in the accuracy of the prediction for resistance.

4 Conclusions

The research has demonstrated that the shift method is generally more reliable than the ratio method. This is because, firstly, the corrections tend to be more moderate and hence better behaved. Secondly, corrections across tank widths(not shown in this publications) are more continuous in the neighborhood of the critical speed. Thirdly, this method is insensitive to both the friction line used and the form factor used.



(a) $C_P = 0.6667$, $\alpha_1 = 0.2$, $\alpha_2 = 0$

(b) $C_P = 0.9$, $\alpha_1 = 0.2$, $\alpha_2 = 0$

(c) $C_P = 0.6667$, $\alpha_1 = 0.2$, $\alpha_2 = 5000$

(d) $C_P = 0.9$, $\alpha_1 = 0.2$, $\alpha_2 = 5000$

Figure 6. Prediction error

It has also been shown that a simple smoothing procedure is very effective for accounting for the scatter in the experimental data – as well as damping the unrealistic spikes and discontinuities in the linearized wave-resistance theory.

Future work is planned to extend the data experimental bank to include the influence of beam-to-length ratio and draft-to-length ratio.

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