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A Study on the Dissipation Energy of Plate due to Cutting

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Abstract

This paper deals with the energy dissipation of ductile metal plate due to cutting. By using nondimensional analysis, we present that the dissipation energy of tearing behaviour can be formulated as a function of slenderness ratio expressed by cutting length, yield stress, plate thickness and elastic modulus. The validity of the proposed formula for Al-alloy, copper and mild steel is demonstrated by comparing the proposed formula with experimental results, which are shown in good agreements except for thick mild steel plate.

Nomenclature

C - variable coefficient depending on slenderness ratio
$C_{i,j}$ - coefficient depending on material and experimental condition
C_f - eduction factor accounting for the dynamic effect
F - total cutting force
F_C - cutting force component
σ_y - yield stress of the material
σ_o - static flow stress of the material
σ_{od} - dynamic flow stress of the material
t - plate thickness
t_{eq} - equivalent plate thickness
β - slenderness ratio $\left(\frac{l}{t}\sqrt{\frac{\sigma_y}{E}}\right)$
W - total dissipation energy
μ - coefficient of friction
θ - {half value of wedge angle
$\overline{\delta_t}$ - crack opening displacement
l - length of cut
V - Impact velocity
E - Young's modulus

1. Introduction

For the evaluation of structural safety of ships in collision and grounding, it is afundamental approach to investigate the failure mechanisms and its associated structural dissipation energy of ship hull platings [1]. As the structural tearing and crushing behaviour are quite complex one, most of the previous work has been done by combining experimentation with plasticity theory. In this paper,we report on the dissipation energy of Al-alloyed and mild steel plate due to cutting in a quasi-static fashion. A series of cutting tests of Al-alloyed plate by a rigid wedge,which is pushed into it, have been conducted first and then the dimensional analysis as well as the regression analysis have been applied to derive a formula for the evaluation of dissipation energy due to cutting process. For the regression analysis ,the previous cutting test data of Al-alloyed plates are used to determine the quotient of a formula, which contains a variable coefficient. On the contrary,all the previous formulae, which do not agree well with experimental observation, are presented with a fixed value of constant for a certain condition.The proposed formula in this paper with variable coefficient will be checked out for the validity by comparing with the experimental results of mild steel plate and some other formulae.

2. Summary of Previous Formulae

2.1 Experimental Formulae

Based on the fundamental tearing experiments of metal plates, the following simple empirical formulae of tearing energy were proposed by Vaughan [2], Woisin [3] & Lee [4].

Vaughan's formula for mild steel [2]

$$W = 5.5t^{1.5}l + 0.0044t^2l^2 \cdot \tan\theta \tag{1}$$

$$F = (5.5t^{1.5} + 0.0088t^2l \cdot \tan\theta)/\cos\alpha \tag{2}$$

Woisin's formula for mild steel [3]

$$W = 4.8tl^{1.7} \tag{3}$$

$$F = 8.16l^{0.7}/\cos\alpha \tag{4}$$

Lee's formula for Al.-alloyed plate [4]

$$W = 1250 \cdot l \cdot t^2 \tag{5}$$

where units of t, l, F and W are *mm, mm, KN* and *N· m*, respectively.

2.2 Empirical Formula by Lu & Calladine

Based on the quasi-static test results for unstiffened metal plates(mild steel, aluminium, brass, copper, dural), Lu & Calladine [6] proposed empirical formula by adopting dimensional analysis.

$$F = 1.3C_{1,3}\sigma_0 t^{1.7} l^{0.3}/\cos\alpha \tag{6}$$

2.3 Theoretical Formula by Wierzbicki & Thomas

An analytical model describing main features of the steady state process of cutting of a plate by a rigid wedge was developed by Wierzbicki & Thomas [7]. They assumed that the plate would curl up into two inclinded cylinders as the wedge advances into the plate. Based on this simplified model, they derived an approximate formula of the absorbed energy as functions of flow stress of the material, crack opening displacement, plate thickness and frictional coefficient.

$$W = 2.34\sigma_0 t^{1.6}\mu^{0.4}\overline{\sigma_t{}^{0.2}}l^{1.4} \quad for\ 10^\circ \le \theta \le 30^\circ, 0.1 \le \mu \le 0.4 \tag{7}$$

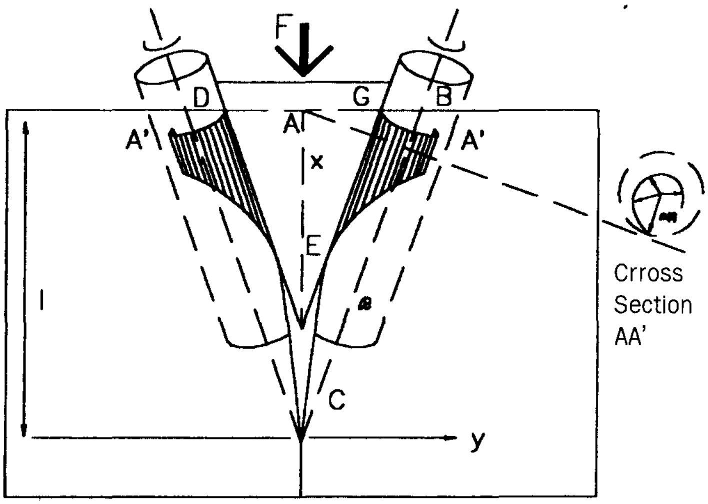

Figure 1. Thoretical tearing model of Wierzbicki & Thomas[7]

2.4 Empirical Formula by Paik

Based on quasi-static test for stiffened high-tensile steel plate, Paik [8] proposed empirical formula considering dynamic effect. Also, he formulated test constant with impact velocity.

$$W = C_f C_{1.5}\sigma_{od} t_{eq}^{1.5} l^{1.5} \tag{8}$$

$$F = 1.5 C_f C_{1.5}\sigma_0 t_{eq}^{1.5} l^{0.5} / \cos\alpha \tag{9}$$

$$C_{1.5} = 3.760\theta^2 - 1.156\theta + 1.112$$

$$C_f = 0.002V^2 - 0.071V + 1.0$$

$$C_f = 0.001V^2 - 0.042V + 1.0$$

3. Proposal of New Empirical Formula Based on the Dimensional Analysis

Based on the present test results of mild steel and experimental data for aluminium alloy plate by Lee [4], a new empirical formula relating to the absorbed energy and cutting length is derived. Following the dimensional analysis employed by Lu and Calladine [6], the tearing energy absorption is expressed in terms of the parameters considered as

$$\frac{W}{\sigma_y t^3} = C\left(\frac{l}{n}\right)^n \tag{10}$$

where n and *C* are constants to be determined,

Here *C* is considered asa constant which varises according to the slenderness ratio, which is expressed by cutting length *l*, plate thickness *t*, yield stress σ_y and elastic modulus *E*. This concept was derived from the nondimensional analysis for the evaluation of crushing energy of plate by Lee [5]

$$C = f(\beta), \qquad \beta = \frac{l}{t}\sqrt{\frac{\sigma_y}{E}} \tag{11}$$

Through the analysis of experimental results for Al-alloyed plates in thickness t=2.0,3.2 and 4.8*mm* with half value of wedge angle = 0, 15 and 22.5. n was found to be equal to the best fitting value n=1.5 in eq.[10], then a new empirical formula for the dissipation energy of plate due to cutting can be proposed as follows.

$$W = C \cdot t^{1.5} l^{1.5} \cdot \sigma_y \tag{12}$$

Based on the regression analysis of experimental results of Al-alloyed plates by Lee [4], the variable coefficient *C* can be formulated as a function of slenderness ratio , which is shown in Fig.2, namely,

$$C = \frac{1}{2}\left(\frac{1}{\beta} + 1\right) + \varepsilon \tag{13}$$

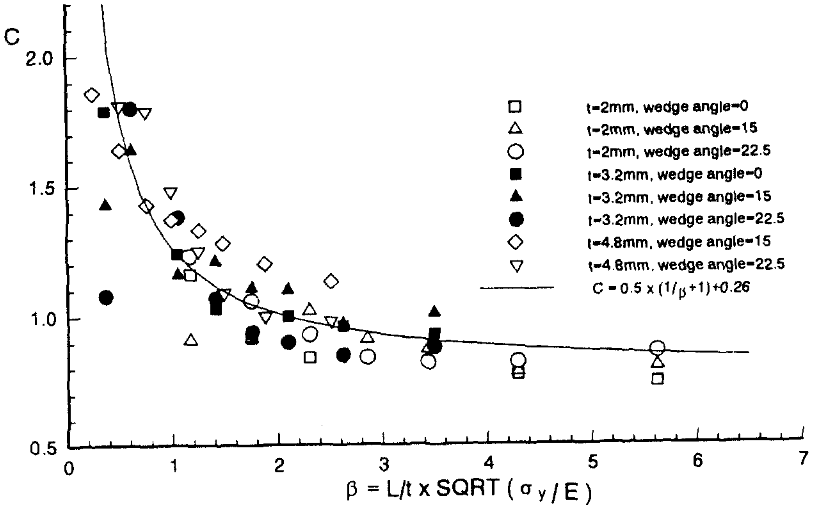

Figure 2. The coefficient of the proposed formula, C vs. slenderness ratio β

4. Comparision of the Proposed Formula with Other Existing Formulas and Experimental data

In Fig. 4, It is observed that two formulas; Lu & Calladine and the proposed new formula agree well with experimental results for Al-alloy plate. And in Fig. 5, the proposed formula is compared also with Lu and Calladines formula for copper plate, which are shown in good agreements.

In Fig. 6, the coefficient of new formula is compared with that of Lu and Calladine's formula, which contains a unvariable constant for certain metal plate. It is observed that the proposed new formula obtained from present study agrees well with Lu and Calladine's formula for the evaluation of dissipation energy of metal plates. For futher study, the dynamic effects of cutting forces on the dissipation energy should be considered in the proposed formula.

In Fig. 7, the proposed new formula is compared with the previous formulas derived by Lu and Calladine.

Table 1. Dimensional Analysis Results

(a) n=1.5, t=2.0 *mm*, SPEC:AL5086-H34

θ	$l(Cm)$	C	β
0°	4130	1.160	1.162
	6.200	0.920	1.745
	8.170	0.840	2.299
	10.130	0.840	2.850
	12.200	0.820	3.433
	15.260	0.770	4.294
	20.000	0.740	5.627
15°	4.130	0.910	1.162
	6.200	0.920	1.745
	8.170	1.020	2.299
	10.130	0.910	2.850
	12.200	0.870	3.433
	15.260	0.780	4.294
	20.000	0.800	5.627
22.5°	4.130	1.230	1.162
	6.200	1.060	1.745
	8.170	0.930	2.299
	10.130	0.840	2.850
	12.200	0.820	3.433
	15.260	0.820	4.294
	20.000	0.860	5.627

(b) $n=1.5$, $t=3.2$ *mm*, SPEC:AL5086-H16

θ	$l(Cm)$	C	β
0°	2.070	1.790	0.362
	3.490	1.800	0.610
	5.990	1.240	1.046
	8.060	1.030	1.408
	10.000	0.940	1.747
	11.990	1.000	2.095
	15.000	0.960	2.620
	20.000	0.930	3.494
15°	2.070	1.430	0.362
	3.490	1.640	0.610
	5.990	1.160	1.046
	8.060	1.210	1.408
	10.000	1.110	1.747
	11.990	1.100	2.095
	15.000	0.970	2.620
	20.000	1.010	3.494
22.5°	2.070	1.080	0.362
	3.490	1.800	0.610
	5.990	1.380	1.046
	8.060	1.070	1.408
	10.000	0.940	1.747
	11.990	0.900	2.095
	15.000	0.850	2.620
	20.000	0.980	3.494

(c) $n=1.5$, $t=4.8$ *mm*, SPEC:AL5086-H116

θ	$l(Cm)$	C	β
15°	2.000	1.860	0.250
	4.000	1.640	0.501
	6.000	1.430	0.751
	7.880	1.370	0.987
	10.000	1.330	1.252
	11.880	1.280	1.487
	15.000	1.200	1.878
	20.000	1.130	2.504
22.5°	2.000	2.780	0.250
	4.000	1.810	0.501
	6.000	1.790	0.751
	7.880	1.480	0.987
	10.000	1.250	1.252
	11.880	1.090	1.487
	15.000	1.000	1.878
	20.000	0.980	2.504

Conclusion

In this study, a series of cutting tests for mild steel plates by a rigid wedge were performed to confirm the applicability of an empirical new formula, which is derived from the dimensional and regression analysis from the experimental result of Al-alloy plates. From the present study, the following conclusions can be drawn;

- It is found that the dissipation energy of metal plate due to cutting can be formulated as a nonlinear function of slenderness ratio, which is expressed by cutting length, yield stress, plate thickness and elastic modulus.
- The proposed formula can be applied to the evaluation of dissipation energy mild steel and copper as well as aluminium plate.
- From the comparison with the previous formulas, we found that the new and Lu & Calladines formulas give a good agreements in the range of thin plates, whereas Paik's formula in the range of thick plates.

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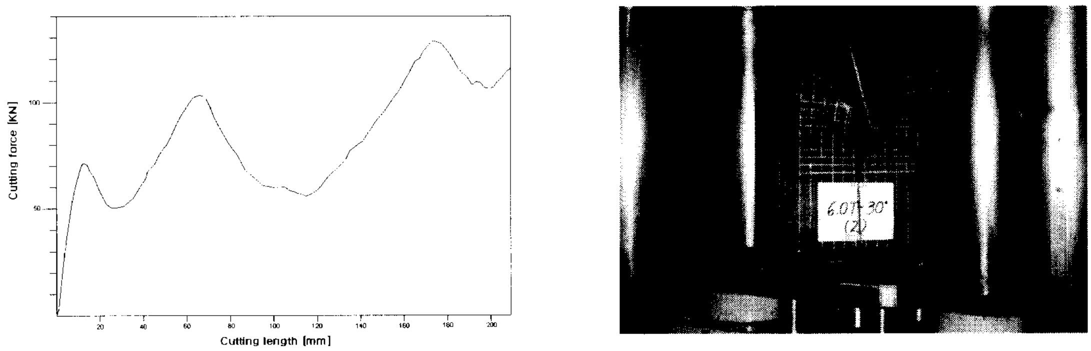
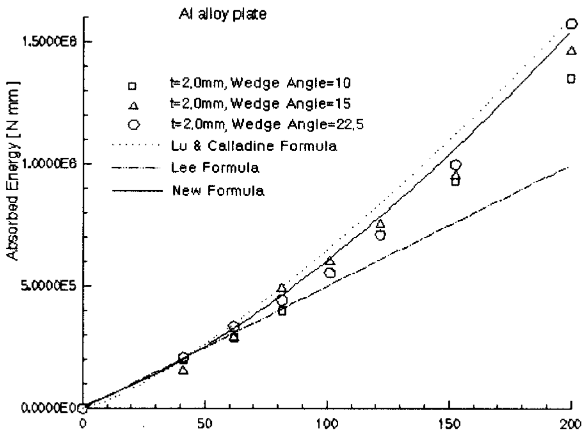


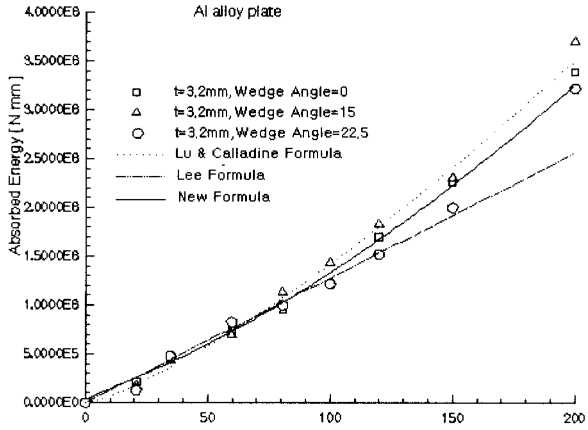
(a) Plot of cutting force vs. cutting length

(b) Photograph of the cutting process

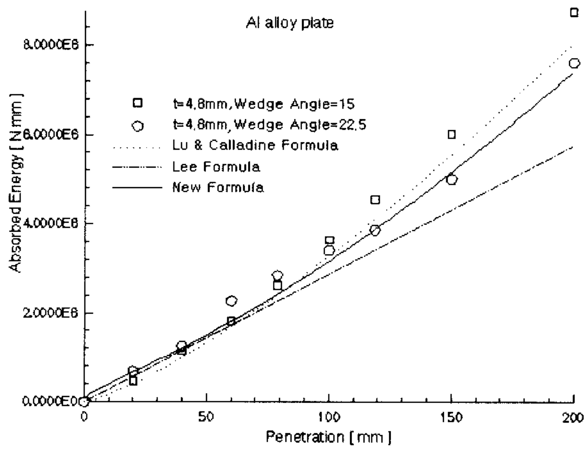
Figure 3. Experiment of cutting process of mild steel (t=6.0mm , $\theta = 15^\circ$)



(a) t=2.0 *mm*

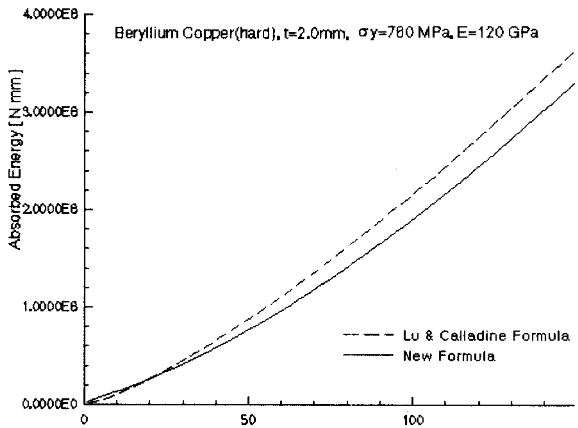


(b) t=3.2 *mm*

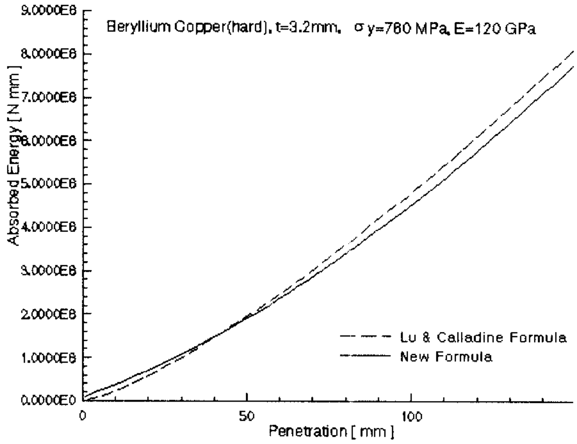


(c) t=4.8 *mm*

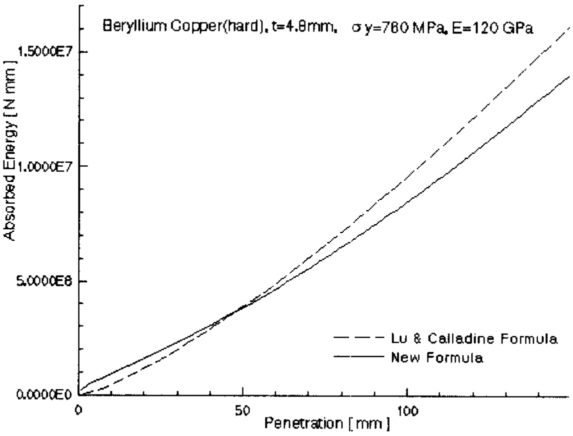
Figure 4. Comparison of the proposed formula with the previous formulas and the experimental results for aluminium alloy plate



(a) t=2.0 *mm*

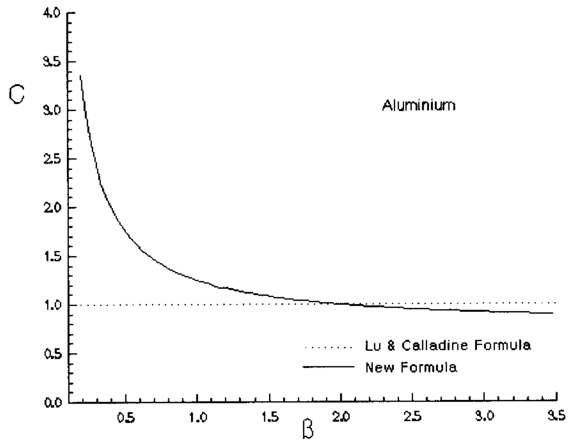


(b) t=3.2 *mm*

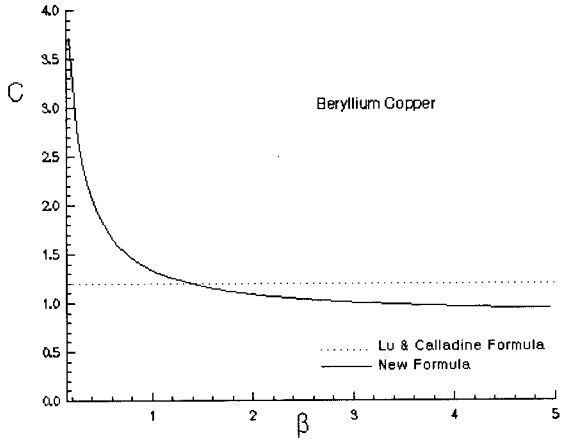


(c) t=4.8 *mm*

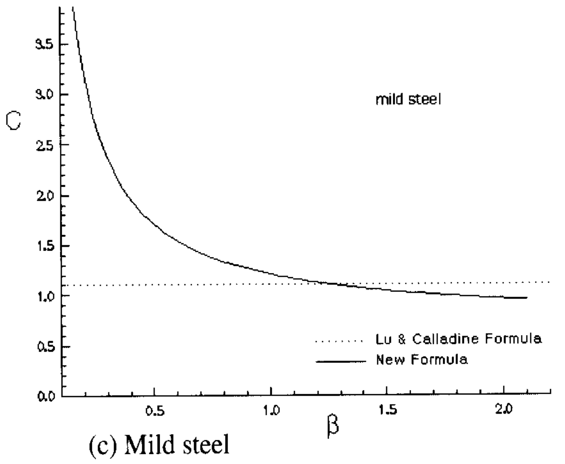
Figure 5. Comparison of the proposed formula with Lu & Calladine's formula for copper plate



(a) Aluminium

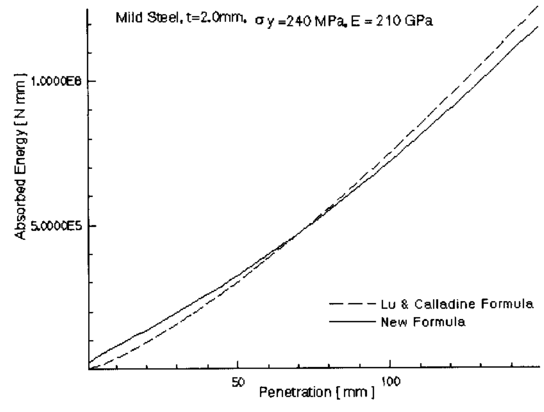


(b) Berylium Copper

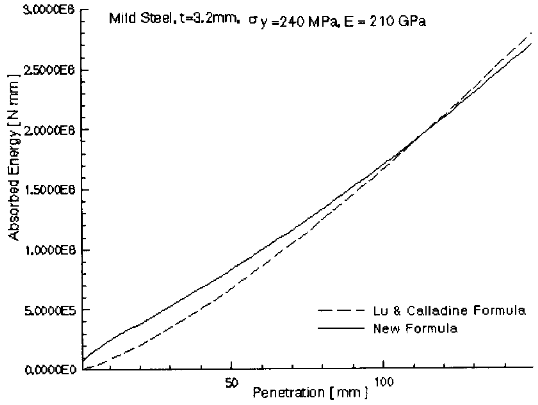


(c) Mild steel

Figure 6. Comparison of the proposed formula's coefficient with Lu & Calladine's coefficient



(a) t=3.2 *mm*



(b) t=4.8 *mm*

Figure 7. Comparison of the proposed formula with Lu & Calladine's formula for mild steel plate