PNU95 Geoid

Kwang-Sun Choi[1], Jeong-Hee Kim[2] and Chul-Soo Yang[3]

[1] Pusan National University, Kumjeong-ku,
Pusan 609-735, Korea

[2] Kyungnam University, 449 Wolyoung-dong,
Masan 631-701, Korea

[3] Cadastral Technology Research Institute, 624-1
Ungak-dong, Yongin 449-800, Korea

(Manuscript received 19 September, 1997)

Abstract

This paper presents a precise geoid (PNU95 Geoid) over the southern half of the Korean peninsula. The geoid using the OSU91A reference gravity model to degree and order of 167 with an integration radius of 27 km fits to the well-distributed 71 GPS data best. The RMS difference between the modeled geoid and the GPS/leveling-implied geoid is 0.15 m, and its accuracy with respect to the GPS/leveling distance is 1.1 ppm. The resulting PNU95 Geoid varies from 21.8 m at the western part of the peninsula, Kanghwa-Do, to 30.2 m at the south-eastern part, Kyungju, generally increasing eastward.

1. Introduction

The purpose of this study is to determine precise geoid undulation over the southern half of the Korean peninsula. The geoid has traditionally been of main interests to geodesists and geophysicists. Nowadays, the need for precise geoid has been more important as the orthometric height can be calculated when the geoid is combined with the ellipsoidal height produced by the ever widely used GPS technology (Leick, 1990; Turner, 1987). For better determination of the orthometric height, more precise undulation is necessary. For this purpose, global gravity models with good integrities in their long wave-length components are essential. But most models do not have short wave-length components and they are not accurate enough to apply to the orthometric height. Further, the local geoid must be calculated for the orthometric height application.

There are a few global earth gravity models, for example, the OSU91A model (Rapp, Wang and Pavlis, 1991) and the most recent EGM96 model (Lemoine et al., 1996). Long wave-length components (lower degrees and orders) of these models are mostly based on the analysis of satellite motions. Both the OSU91A and the EGM96 have higher degrees and orders to 360. Their coefficients are calculated with additional data of altimetry observations and numerous surface gravity data to recover shorter gravity potential components. However, this resolution is equivalent to about 55 km by 45 km in meridian and the parallel direction, respectively, over the study area, and it is not good enough to calculate orthometric heights with GPS measurements in most cases. Meanwhile, the accuracy of these models is estimated about 1 m, and substantial improvement in the accuracy is desired for various geophysical, geodetic, and surveying purposes.

In this study, we used a remove-restore procedure(Rapp, 1986). The reference geoid undulation is taken from the OSU91A model. Numerous surface gravity data were used to calculate the residual geoid undulation through Stokes′ integral applied for the flat boundaries. The resulting geoid undulation was compared with 71 well-controlled GPS stations over bench marks. The best fitting integral radius and the degrees and orders of the reference model were also investigated.

2. Computational Method

We chose the remove-restore method for the calculation of the detailed geoid. Firstly, we adopted the OSU91A model as the reference gravity field. We believed the model had very reliable long wave components which were originally adopted from the GEM-T2 gravity model based on numerous data and analysis of the satellite trackings. A more recent model of the EGM96 was developed from the OSU91A model and considered as a more precise one, but at the time of our study it was not available. Geoid undulation can be calculated using the well-known following formula (Heiskanen and Moritz, 1987):

$$N=\frac{GM}{r}\sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}(C_{nm}\cos m\lambda+S_{nm}\sin m\lambda)P_{nm}(\cos\theta), \qquad (1)$$

where standard notations are used. Fig. 1 shows geoid undulation of the southern Korean peninsula from the OSU91A model to degree and order 360.

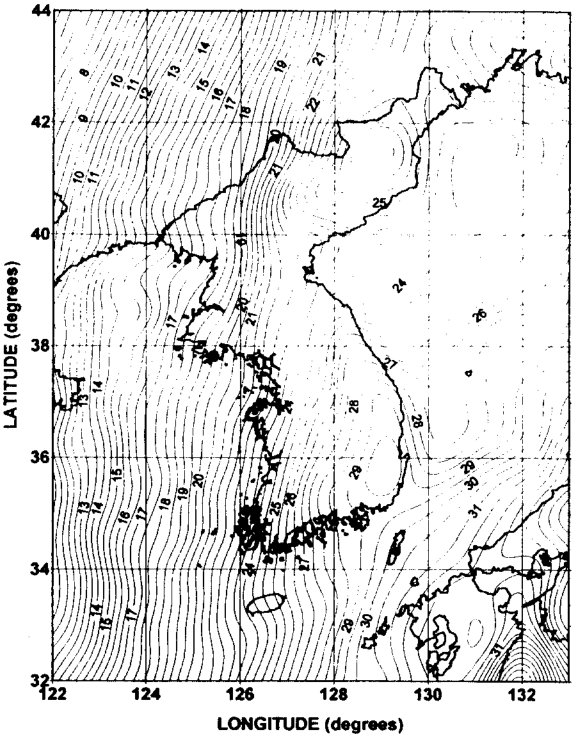

Fig. 1 Geoid undulation map of in and around the Korean peninsula with respect to the GRS80 ellipsoid using the OSU91A Earth gravity model of degree and order to 360. The contour interval is 0.5 m.

For more detailed and more precise geoid calculations, surface gravity measurements are essential and the well known Stokes' formula (Moritz, 1980) is used for the local geoid of limited boundaries. In this case, we can use Stokes' formula at a point $(x_p, y_p)$from the gravity point (x, y) as:

$$N(x_p, y_p) = \frac{1}{2\pi\gamma} \int\int \frac{\Delta g(x, y)}{s} dxdy, \qquad (2)$$

where s is the distance between (x_p, y_p) and (x, y). The integration radius in equation (2) has to be decided since we can not extend the calculation area over the whole Earth and we do not need to do so since we use a reference gravity field. As matter of fact this is the whole idea of the remove-restore procedure.

The GPS measurement gives the ellipsoidal height(h) which has only a geometrical meaning. The orthometric height(H) is the distance, along the normal line, from an adopted equi-gravity potential surface, usually the mean sea level and supposedly the geoid. Now the geoid undulation(N) is the separation between geoid and the reference ellipsoid. Therefore, in theorey, the geoid can be calculated from the GPS measured ellipsoidal height and the orthometric height as:

$$N = H - h. \tag{3}$$

Errors in H and h are a few cm levels that are fairly smaller than expected errors in the geoid undulation calculation. Therefore, these GPS and leveling data can be used to verify the accuracy of the calculated geoid.

3. Computation

The surface gravity data used for the local geoid calculation over the land of this study were those measured by Choi et. al. (1995) at more than 4,500 points. Fig. 2 shows the distribution of original data points. More surface gravity data over the sea were taken from Sandwell(1993). These surface gravity data were then transformed into 1′ by 1′ mean data. To calculate the mean data, we firstly changed them into Bouguer anomalies with a 15″ by 15″ digital terrain model to avoid height effects on the free-air anomalies when interpolating them. We also corrected the data with terrain effects. The terrain effects were calculated using the same digital terrain data over 2° by 2° around of the gravity point. Then, we interpolated resulting data and re-calculated condensed free-air anomalies of 1′ by 1′ means. Fig. 3 is the resulting mean anomaly map. Data in the northern Korean peninsula were geophysical-implied data. Accuracies were around 2 mgals over the southern peninsula and 8 to 10 mgals over the northern peninsula and the seas, respectively.

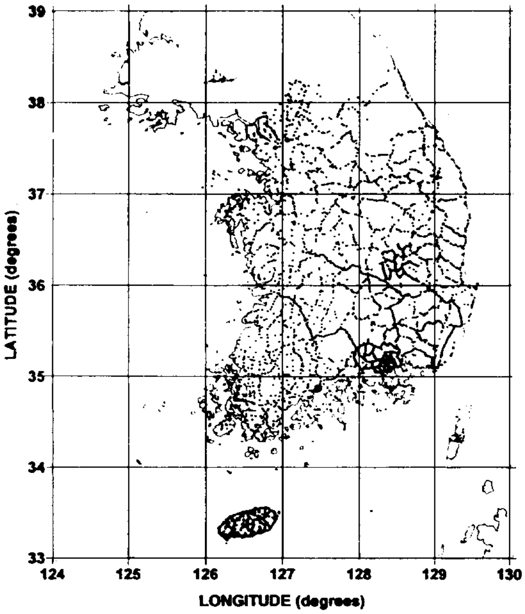

Fig 2. Distribution of surface gravity data points in the southern half of the Korean peninsula.

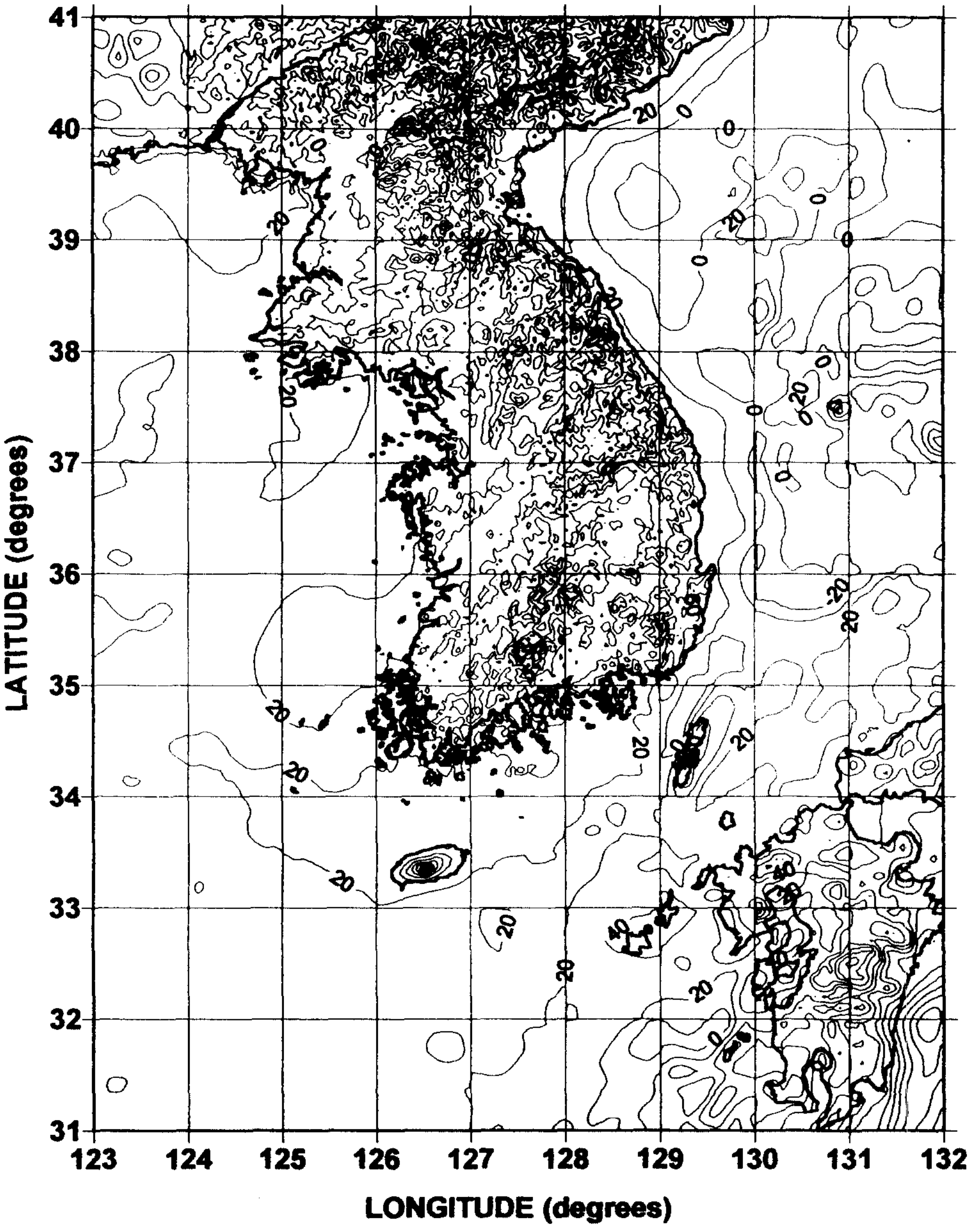

Fig. 3. Condensed free-air anomaly. The contour interval is 20 mgals.

GPS measurements were carried out at 71 well-distributed bench marks of the first or the second order. At the bench marks, well-controled spirit leveling data were available. The measurements were done with the SST models of the Trimble company. Fig. 4 shows the data distribution. The figure also shows difference contours which will be discussed in a later section.

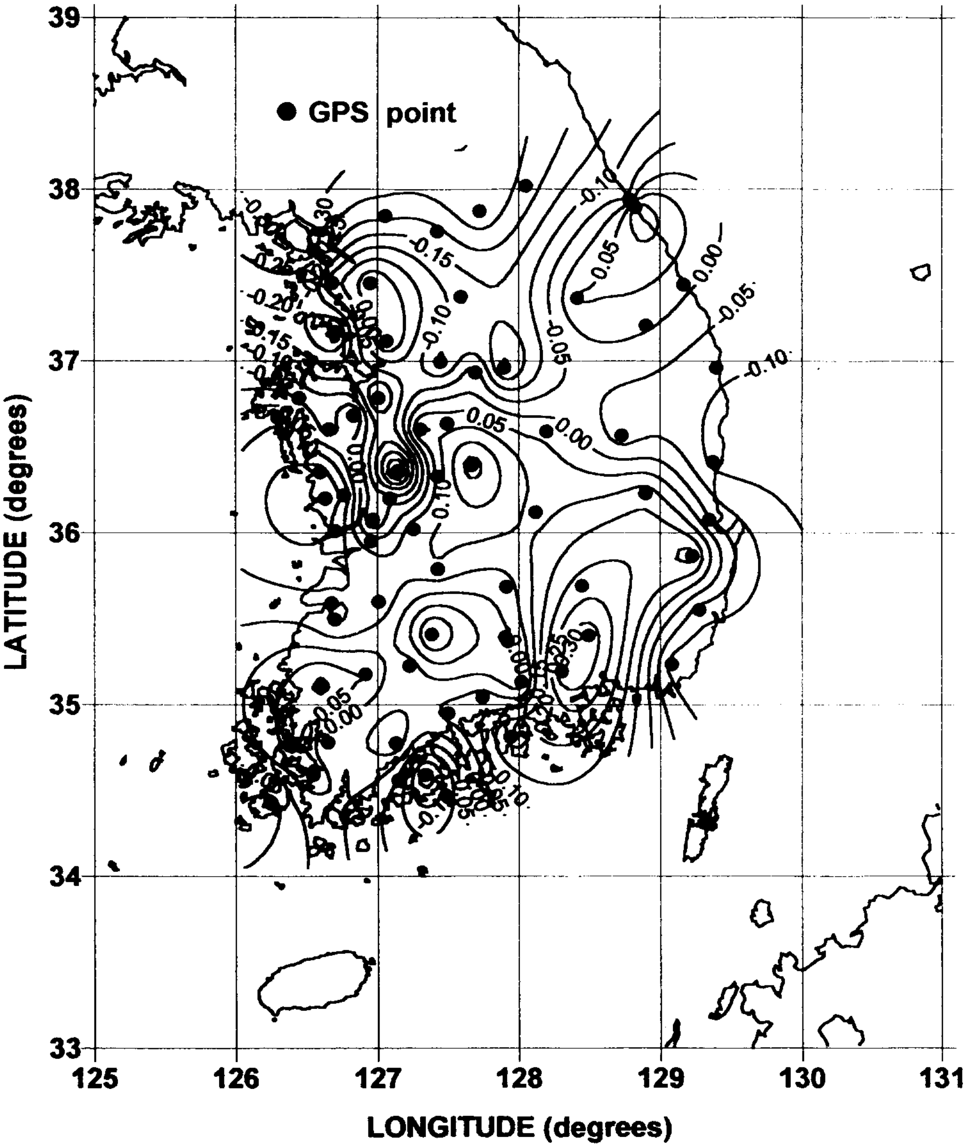

Fig. 4. Distribution of 71 GPS measurement bench marks and map of differences between the PNU95 geoid and the GPS/leveling implied geoid. The contour interval is 0.05 m.

We might use all degrees and orders of the OSU91A for the reference geoid calculation, but we thought that all coefficients were not equally precise. Therefore, we decided to use limited lower coefficients, cutting higher coefficients off at certain degrees and orders. With different degrees, slightly different N values would be calculated through equation (1). They were then compared with the GPS-leveling geoid and the best fitting degree and order were turned out to be 167. Fig. 5 shows this geoid undulation.

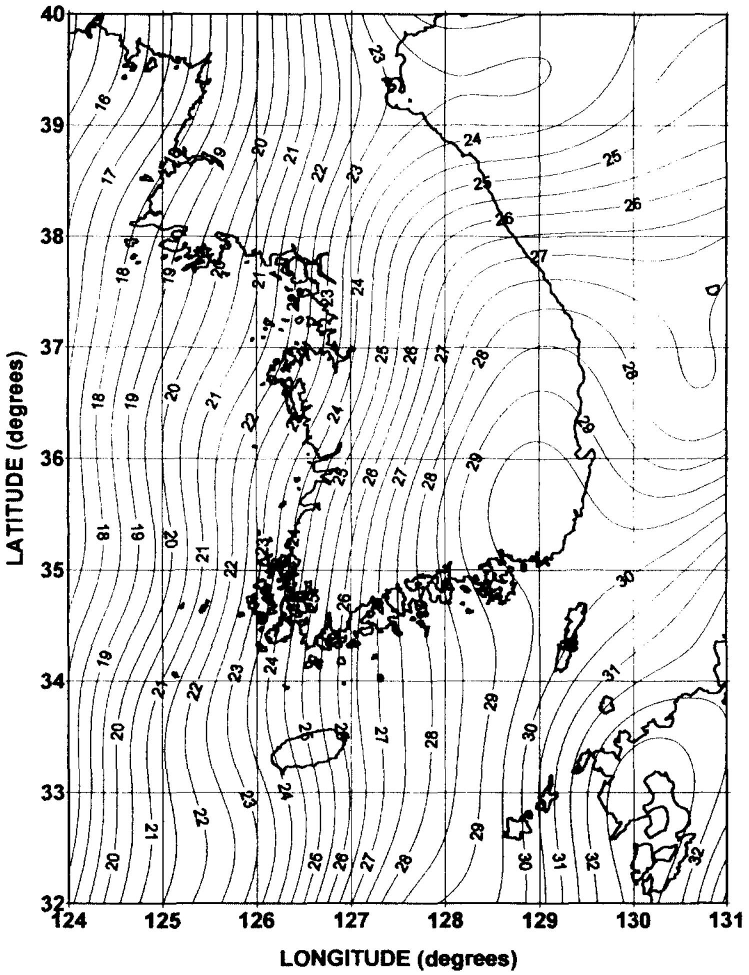

Fig. 5. Geoid undulation map of the southern half of the Korean peninsula with respect to the GRS80 ellipsoid using the OSU91A Earth gravity model of degree and order to 167. The contour interval is 0.5 m.

For the calculation of the local geoid, we firstly, subtracted the reference free-air gravity anomalies from free-air anomalies calculated from the OSU91A gravity potential coefficients to certain degrees and orders, resulting in residual free-air anomalies. Then, using equation (2), we calculated the local residual geoid. To determine the best fitting integration radius, we changed the radius from 1 km up to 60 km. Then, we compared the resulting geoid with the GPS-leveling driven geoid as suggested in equation (3). We found out that the best fitting radius was 27 km. The maximum, minimum, and RMS (Root Mean Square) values on land were 2.13 m, -1.53 m, and about 0.4 m, respectively. This best fitting residual geoid undulation is shown in Fig. 6.

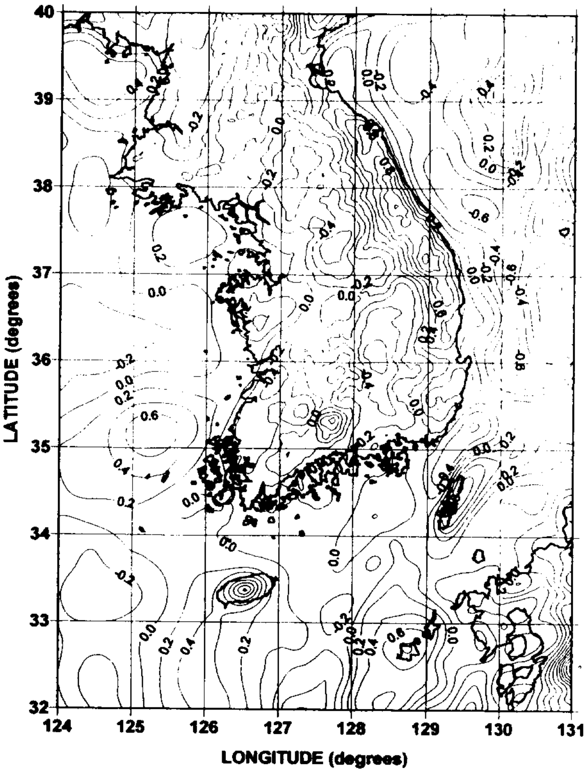

Fig. 6. Residual geoid undulation map of the southern half of the Korean peninsula calculated from the Stokes integration with a radius of 27 km. The contour interval is 0.5 m.

Fig. 7 shows the RMS differences between the calculated geoid undulation and the GPS/leveling-implied geoid as a function of degree (from 150 to 210) and integration radus. The locations, the GPS heights, the bench mark heights, the OSU91A geoid, the residual geoid, the total geoid, and the differences at 71 bench marks where GPS measurements were carried out, are presented in Table 1.

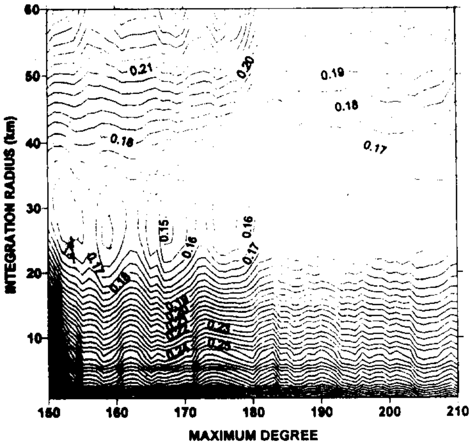

Fig. 7. RMS difference distribution when different degrees (from 150 to 210) and integration radii (1 to 60 km) were used.

Table 1. Comparison of the PNU95 geoid and the GPS/leveling geoid at 71 stations over bench marks.

BM NO	LONGITUDE (degrees)	LATITUDE (degrees)	GPS HT (m) A	BM HT (m) B	OSU91A GEOID (m) C	RELATIVE GEOID (m) D	TOTAL GEOID (m) E=C+D	DIFFENANCE (m) F=B+E-A-MEAN
14-26	128.73091	36.56647	121.775	93.139	28.927	-.327	28.600	-.084
2-24	126.67768	35.59054	27.269	3.091	24.680	-.368	24.312	.086
10-1-3-3	126.76304	36.21772	52.112	28.165	24.069	.073	24.142	.147
BM19	127.71950	37.87426	98.817	73.467	25.596	-.496	25.100	-.298
8-15	128.44789	35.69019	58.198	29.422	29.443	-.395	29.048	.224
37-23	126.59732	36.35179	30.566	7.074	23.429	.290	23.719	.179
16-16	127.69093	36.93363	170.382	144.567	26.154	-.312	25.842	-.021
2-21	126.70070	35.49446	60.472	36.137	24.840	-.335	24.505	.122
5-5	126.96705	36.07793	44.534	19.852	24.877	-.258	24.619	-.111
2-2-0-0	127.42396	35.79071	319.244	292.589	26.744	.055	26.799	.096
5-10-0-0	128.19659	36.59043	118.598	91.078	27.968	-.364	27.604	.036
5-2-3-1	126.70049	36.01050	28.411	4.348	24.218	.036	24.254	.143
13-14	128.11940	36.11956	103.017	75.339	28.393	-.582	27.811	.085
37-17	126.63292	36.19932	52.649	28.960	23.751	.159	23.910	.173
38-10	126.45085	36.78438	59.604	36.995	22.509	.144	22.653	-.004
7-15-24-11	128.89853	37.20746	1006.710	977.421	28.155	1.190	29.345	.008
3-3-5-9	129.27426	35.55293	39.785	9.704	29.738	.368	30.106	-.026
8-5-9-10	127.58742	37.37498	68.022	42.949	25.637	-.627	25.010	-.111
25-10	126.83291	36.68302	49.425	25.707	23.611	.233	23.844	.078
5-1-0-0	128.01669	35.13360	70.006	41.559	28.693	-.242	28.451	-.044
16-22	127.89815	36.96236	98.172	71.611	26.768	-.378	26.390	-.219
12-1	129.22531	35.86738	67.790	38.210	29.630	.224	29.854	.226
12-40	129.39438	36.96260	34.774	5.873	28.457	.404	28.861	-.088
12-20	129.36904	36.41252	40.588	11.033	29.047	.440	29.487	-.116
BM24	126.66029	36.60125	52.900	29.362	23.239	.314	23.553	-.033
5-1	126.95529	35.95188	44.313	19.728	25.041	-.384	24.657	.024
2-10-0-0	127.90996	35.68581	225.542	197.540	28.312	-.237	28.075	.025
4-2-5-3	128.89769	36.22829	389.517	360.105	29.407	.183	29.590	.130
JNUGS	126.91147	35.17624	62.701	37.190	25.739	-.235	25.504	-.055
35-8	128.78325	37.94113	53.430	26.610	26.669	.141	26.810	-.058
4-19	127.38108	35.41131	124.228	97.494	27.015	-.409	26.606	-.176
7-9-31-1	128.41612	37.36574	334.675	307.059	27.508	.211	27.719	.055
12-9	129.34387	36.07609	54.406	24.833	29.380	.240	29.620	-.001
PNUGS	129.08105	35.23419	82.555	52.684	29.783	.061	29.844	-.075
MOKPOBM	126.38492	34.78216	27.047	2.891	24.278	-.146	24.132	-.072
3-33	127.49604	34.94943	36.347	9.015	27.497	-.049	27.448	.068
SNUGS	126.95014	37.45502	139.174	115.725	23.631	-.083	23.548	.051
6-2-0-0	127.44161	36.99825	122.192	96.950	25.308	-.191	25.117	-.173
25-4	127.00520	36.78291	45.715	21.367	24.026	.137	24.163	-.233
BM21	129.16502	37.44607	33.232	5.157	27.834	.300	28.134	.011
BM9	127.29891	36.60399	50.596	25.616	25.145	.000	25.145	.117
BM8	127.42178	36.32625	80.863	55.156	25.928	-.109	25.819	.064
5-5-4-7	127.66224	36.39451	111.880	85.576	26.630	-.047	26.583	.231
16-5	127.49151	36.63493	71.047	45.484	25.755	-.066	25.689	.078
10-7-12-5	127.14177	36.36129	86.502	61.324	24.962	-.092	24.870	-.356
5-10	127.08730	36.19956	43.070	18.241	25.046	-.220	24.826	-.051
35-6	128.82697	37.89226	29.317	2.613	26.809	.086	26.895	.143
6-8-1	128.30453	35.19379	64.540	35.876	29.209	-.143	29.066	.354
8-5-1	128.49259	35.40188	44.851	16.115	29.529	-.411	29.118	.334
4-36	127.91810	35.36818	133.758	105.070	28.497	.191	28.688	-.048
4-35	127.90297	35.39336	115.590	86.972	28.457	.198	28.655	-.011
14-5-11-11	127.94056	34.81195	36.303	8.271	28.414	-.114	28.300	.220
3-41	127.73927	35.04483	196.808	168.880	28.080	-.028	28.052	.076
1-2-7-11	127.22521	35.22812	161.796	135.308	26.665	-.099	26.566	.030
14-10-23-8	127.14698	34.56084	40.215	13.766	26.650	-.204	26.446	-.051
14-10-22-2	127.32893	34.59085	31.798	4.712	27.094	-.209	26.885	-.249
3-21	127.12794	34.77535	158.127	131.638	26.551	.082	26.633	.096
14-1-4-5	126.65073	34.77961	39.944	15.117	25.134	-.203	24.931	.056
14-6-12-18	126.25144	34.42186	65.138	41.185	24.107	.020	24.127	.126
14-3-14-3	126.54578	34.58989	30.504	5.758	24.913	-.155	24.758	-.036
13-1-3-6	126.60980	35.11288	41.666	17.105	24.821	-.335	24.486	-.123
1-11-23-13	127.00762	35.59804	60.292	35.058	25.681	-.307	25.374	.092
2-1-2-6	127.25201	36.02200	98.054	72.441	25.854	-.113	25.741	.080
12-1-1-6	126.29871	36.67189	37.785	15.565	22.296	.063	22.359	.091
24-11	127.06433	37.11802	48.043	24.068	24.035	.053	24.088	.065
12-9-22-4	126.70379	37.16895	35.261	11.846	22.919	.265	23.184	-.279
29-2	126.68072	37.45787	44.604	21.721	22.801	.072	22.873	-.058
11-3-4-3	126.55243	37.64538	33.473	10.704	22.394	.110	22.504	-.313
11-8-17-6	127.05411	37.84588	95.906	72.388	23.860	-.509	23.351	-.215
20-10-21-11	127.41876	37.75528	83.215	58.407	24.973	-.304	24.669	-.187
34-26	128.05210	38.01853	292.984	266.811	25.916	.075	25.991	-.230

NUMBER OF DATAS = 71

DIFFERANCE MEAN = 0.0483 meters
DIFFERANCE RMS = 0.1491 meters
HEIGHT DIFF/DISTANCE DIFF = 1.1377 PPM

4. Results and Discussions

The PNU95 geoid was calculated with numerous surface gravity data, the OSU91A, digital terrain model, as well as with analysis on the GPS measurements and bench mark height data. Careful processings of surface gravity anomaly data to produce reliable mean data for residual local geoid were carried out. To find out the best fitting geoid to the GPS/Leveling implied geoid, we applied different integration radii and different degrees and orders of the OSU91 global Earth gravity model.

The best fitting geoid was achieved when the 27-km integration radius was chosen with the degrees and orders to 167. The RMS difference was 0.15 m with the 0.05 m mean value as shown in Fig. 7. In that figure, increasing the coefficient degrees does not produce better fitting geoid. This means commission errors in the coefficients are larger than the ommission errors in higher degrees. This analysis is only valid when the GPS measurements and the bench mark heights are relatively accurate. The bench mark heights are known very accurate to a few cm in most of the GPS points as provided by the Korean National Geography Institute, a government organization in charge of national surveyings, mappings, etc The GPS measurements are also fairly accurate to a few cm. Fig. 8 shows the comparison between geoid differences and distances between GPS points. The geoid difference increases with the GPS distance monotonically to 200 km. Analysis shows the accuracy of about 1 ppm. Therefore, we can conclude that GPS measurements are reliable and relatively accurate.

The optimal radius of 27 km does not seem to be consistent with the optimal degrees of 167. The resolution of the Earth gravity model to degrees and orders 167 is about 100 km which is much larger than the optimal radius of 27 km. This also suggests that the commission errors are larger than the ommision errors. Another possible explanation for the reason that larger integration radii do not improve the RMS difference is inaccurate surface gravity measurements and/or their weak long wave-length components.

From Fig. 4 and Table 1, we can find fairly large differences in the northern areas, the Mt. Jiri area of the south-middle of the peninsula, and a few isolated points such as the bench mark 10-7-12-5 and the bench mark 14-10-22-2. In the northen areas and the Mt. Jiri area it is reasonable to have larger differences because surface gravity data are relatively scarce and not evenly distributed. More surface gravity measurements in these areas are necessary to improve the residual geoid calculation. At two bench marks, we think there were problems in acquiring GPS measurements due to unspecified reasons. In other area, the difference is fairly even and much smaller.

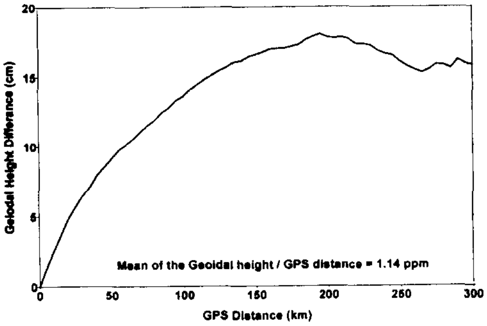

Fig. 8. Comparison between geoid differences and distances between GPS points.

The final geoid undulation, the summation of the residual geoid and the reference geoid of the OSU91A gravity model to degree 167 is shown in Fig. 9. Comparing this with Fig. 1 and Fig. 5 visualizes more detailed geoid. The highest geoid undulation of 30.2 m is found near Kyungju city at the southeastern part of the peninsula. The lowest geoid of 21.8 m locates at Kanghwa island of the most western part of the peninsula. The geoid undulation decreases westward in general with some local variations specially over the mountain areas.

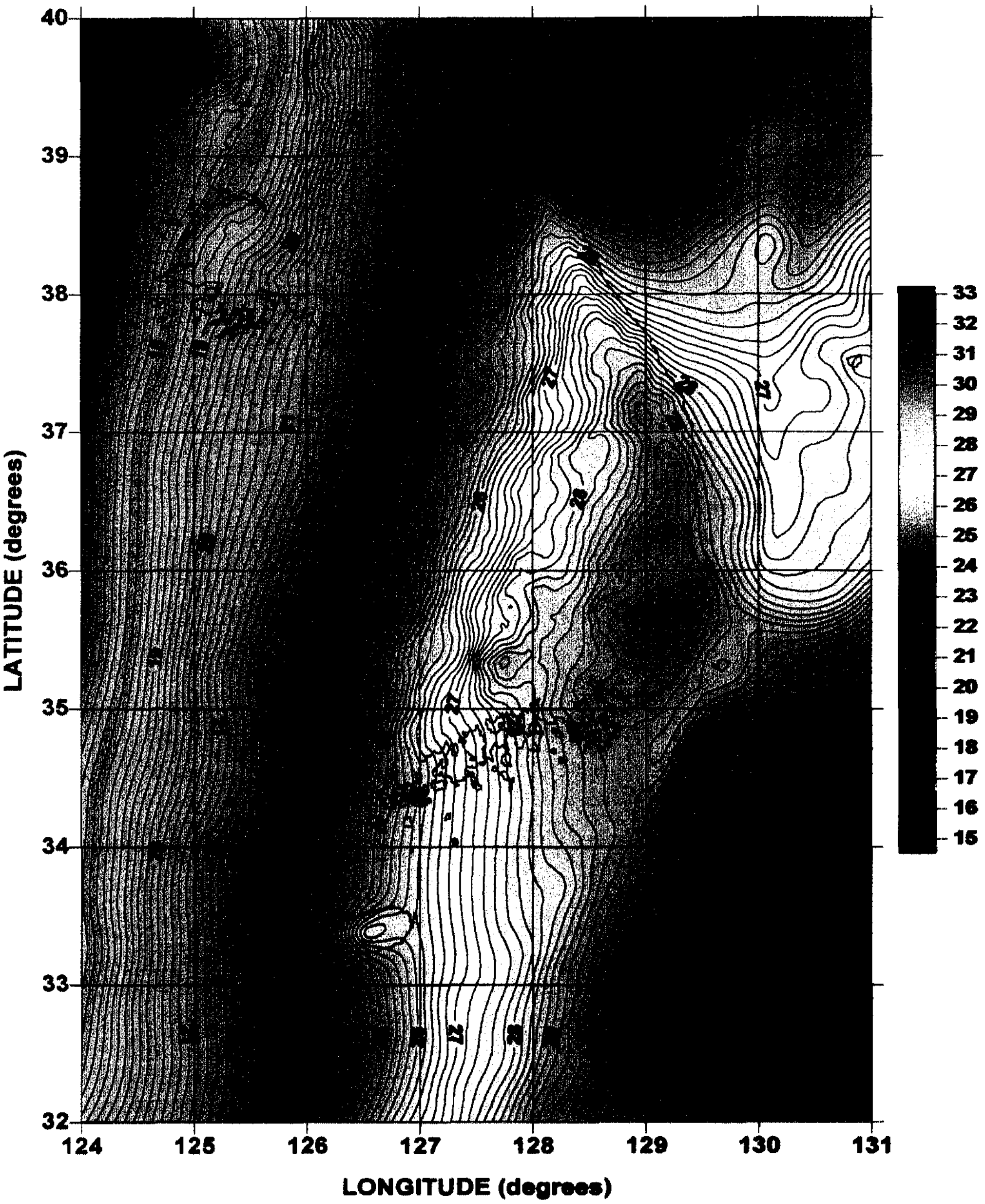

Fig. 9. The PNU95 geoid. The contour interval is 0.2 m

To improve this geoid, obviously, more and more evenly distributed surface gravity data are necessary. More gravity measurement projects over the land and the seas are going on at the present. Over 1,000 new measurements near Mt. Jiri and the northern mountainous areas have already been done by the authors. The Hydrographic Office, a government department, has carried out more precise ship-born gravity measurements over the East Sea area since March, 1996, and the authors are actively involved in that project. Altimetry derived marine gravity from Geosat and ERS-1 satellites have been being carried out by the authors, too. A new and much improved earth gravity model of the EGM96 came out in Nov, 1996. We intend to use that model and more surface gravity data mentioned above for the next local gravity model over the study area.

5. Conclusions

With the OSU91A Earth gravity model, numerous surface gravity data, GPS measurements, bench mark heights, and digital terrain model, the PNU95 geoid was calculated. Conclusions from this geoid and from the calculations of this geoid can be summarized as:

1. The best fitting geoid with respect to the 71-point well-distributed GPS/ leveling-implied geoid was produced when the OSU91A gravity model to degrees and orders 167 and the integration radius of 27 km were used. In that case, the RMS difference was 0.15 m.

2. The residual geoid varies from -1.6 m to 2.1 m with about 0.4 m RMS, and the mean with respect to the reference geoid is 0.05 m.

3. The resulting PNU95 Geoid varies from 21.8 m at the western part of the peninsula, Kanghwa-Do, to 30.2 m at the south-eastern part, Kyungju, generally increasing east-east-southward.

4. More dense and better distributed gravity measurements over the land and the seas are necessary for a better and more precise geoid.

5. The PNU95A geoid will be a good reference to produce GPS/geoid-implied heights that approximate the orthometric heights.

Acknowledgement

This study was carried out with the support of the KOSEF (95-0703-01-01-3). The authors also thank anonymous reviewers for helpful comments.

References

Choi, K.S., KiM, J.H., Yang, C.S., Min, K.U. and Kim, H.Y., A Study on the Gravitational Field and Space Environment(Korean), *Agency for Defense Development of Korea,* 1995.

Heiskanen, W.A. and Moritz, H., *Physical Geodesy*, Institute of Physical Geodesy, Technical University, Graz, Austria, 1987

Leick, A., *GPS Satellite Surveying*, John Wiley & Sons, 1990.

Lemoine, F.G., Smith, D.E., Smith, R., Kunz, L., Pavlis, E.C., Pavlis, N.K., Klosko, S.M., Chinn, D.S., Kenyon, S.C., Salman, R., Trimmer, R., Rapp, R.H. and Nerem, R.S., The Development of the NASA GSFC and DMA Joint Gepotential Model, *International Symposium on Gravity, Geoid, and Marine Geodesy 1996*, IAG (Tokyo, Japan), 1996.

Moritz, H., *Advanced Physical Geodesy*, Herbert Wichmann Verlag, 1980

Rapp, R.H., Global Geopotential Solutions, In Sunkel, H.,(ed), Mathematical and Numerical Techniques in Physical Geodesy, *Lecture Notes in Earth Science, vol 7, Springer-Verlag,* 366-415, 1986.

Rapp, R.H., Y. M. Wang, and N. K. Palvis, *The Ohio State 1991 Gravity and Sea Surface Topography Harmonic Coefficient Models, Rep. No. 410,* Dept. of Geodetic Sci. and Surveying, The Ohio Univ., Columbus, Ohio, 1991.

Sandwell, D., Global Marine Gravity Grid and Poster Developed, *EOS, 74* (3), 35, 1993.

Turner, S., *Applied Geodesy, Global Positioning System -Networks- Particle Accelerators -Mathematical Geodesy*, Springer-Verlag, 1987.

Announcement

The resulting PNU95 geoid is available to the public. Please contact the authors for that.