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◉ 論　文

Robust Autopilot Design for Submarine Vehicles

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강인제어법을 통한 잠수함의 자동항법장치 설계

유 삼 상*

Key Words : Submarine vehicle (잠수함), Depth/pitch (심도 및 피치), Robust autopilot (강인 자동항법장치), Mixed H_2/H_∞ control synthesis (혼합 H_2/H_∞ 제어기 설계법), Linear matrix inequality (선형 행렬 부등식).

초 록

잠수함이 저심도 조건에서 특수임무 수행시, 표면파도 및 조류등 외란의 영향하에서 요구심도 및 위치 유지를 위한 강인한 자동항법장치 설계는, 첨단 군용 잠수함 개발에 필수적이다. 본 연구는 잠수함 조종 운동역학계에 기초하여, 정확한 심도 및 피치 운동 제어를 위해 선형 행렬 부등식을 이용한 혼합 H_2/H_∞ 설계법을 사용, 다중 목적 함수로 표현된 잠수함의 조종 성능들을 개선하였다. 또한, 제어기 설계법의 타당성을 수치 시뮬레이션을 통하여 검증하였다. 결과적으로, 본 제어법은 각종 외란 및 계의 불확실성하에서 잠수정의 만족스러운 과도 상태 응답과 일정 심도 유지 및 피치 각도 변동 최소화에 적합한 강인한 방법임이 검증되었다.

NOTATION

(Others defined in the main text)

A, B	system matrix, input matrix
$A>0 (A<0)$	positive-definite (negative-definite) matrices
h_∞, h_2	vectors of controlled outputs
I_n	$n \times n$ identity matrix (the subscript is omitted when the size can be determined from context)
y	measurement output vector
z	commanded depth
R	fields of real number
R^+	set of positive real numbers in $[0,\infty)$ over R
μ	vehicle surge rate (forward speed) along with the longitudinal axis $\bar{x}$
ω	heave rate (vertical speed) along with the axis $\bar{z}$
θ, q	vehicle pitch angle, pitch angular velocity

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δ_b, δ_s bow hydroplane angle, stern hydroplane angle

$\bar{\omega}$ angular frequency

$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ state-space realization of the transfer matrix $C(sI-A)^{-1}B+D$

$\| w(t) \|_2 = (\int_0^{\infty} w^T w \, dt)^{1/2} < \infty$: the bounded L_2 norm of the Hilbert space of signals $w(t) \in L_2$

$\| A(s) \|_{\infty} = ess \sup_{0<w<\infty} \bar{\sigma}[A(j\bar{w})]$; the H_{∞} norm of the stable transfer function matrix $A(s)$

$\| A(s) \|_2 = \{(1/2\pi) \int_{-\infty}^{\infty} Trace[A^*(j\bar{w})A(j\bar{w})d\bar{w}]\}^{1/2}$; the H_2 norm of the matrix $A(s)$

$\bar{\sigma}(A)$ largest (or maximum) singular value of A

1. INTRODUCTION

The design of accurate maneuvering control law, commonly called autopilot for submarine vehicle, is challenging because the system dynamics are highly nonlinear in nature and contain various uncertainties. In fact, a combat submarine when performing a specific mission near the sea surface is subject to external perturbations caused by sea conditions such as underwater currents and waves. Therefore, the vehicle autopilot must be capable of exhibiting considerable robustness to all disturbance effects and keeping the vehicle in the desired depth.

A comprehensive design methodology has been developed over the past decades for dynamics, control synthesis, and stability analysis of submarine systems[6),8),13)] in which the PID-type and optimal controllers are widely used in calm sea conditions. In order to further improve the autopilot's performance, multivariable methods have been applied to the depth-keeping control of modern submarines[11)]. Earlier works in multivariable control synthesis have mainly utilized a quadratic cost function to minimize the 2-norm of a system response to white noise inputs. As shown in references [1),14)], the linear quadratic Guassian (LQG) type of cost function is often a practical criterion for minimizing tracking errors or control signal variations. Although the H_2 approach[4)] is well suited to many real systems, it is known that its stability and performance cannot be guaranteed in the presence of various uncertainties. As is in case with any submarine, the vehicle system is expected to operate in a highly variable environment and will be effected by some fluctuations at shallow depth. One of the most important advances in the past decades on the multivariable control is the development of H_{∞} control theory[4),7)]. It has been recognized for long that the H_{∞} synthesis guarantees the robust stability and disturbance rejection performance of the closed loop system in the presence of uncertainty, but that the H_{∞}-optimal controller typically leads to an intolerably large control effort.

To quantitatively demonstrate design trade-offs, the mixed H_2 and H_{∞} performance criteria become indispensable. Researchers have devoted considerable attention over the past several years to the mixed H_2/H_{∞} control method for uncertain dynamical systems[1),10)]. On the other hand, it must be noted that many control problems can be cast into multi-objective characteristics and readily solved by linear matrix inequality (LMI) approach[2)]. In addition, several authors, for example, Chilali and Gahinet[3)], and Iwasaki and Skelton[9)], have shown that an LMI synthesis is powerful and useful tool for multiobjective control problems.

To date, there has been no paper considering the LMI-based H_2/H_∞ approach for submarine vehicles. This study is to design the robust autopilot of submarines for vertical plane motions; H_2 optimal control with a given H_∞ norm bound of disturbance attenuation via LMIs. As a result, the submarine vehicle maintains a nearly constant depth relative to the sea surface and has minimal angular pitch motions as well in the presence of the uncertainties.

The paper is organized as follows. Section 2 describes the vertical dynamics of the submarine vehicle. In Section 3, we present a class of robust linear controllers for the pitch/depth maneuver. In Section 4, the autopilot performance has been extensively assessed through a series of numerical simulations. Finally, the contributions and conclusions of the work are summarized in Section 5.

2. SYSTEM DYNAMICS

The standard submarine being considered in the study is a realistic one but does not represent any particular model in use. Figure 1 depicts two orthonormal coordinate systems in a right-handed sense[14]; ($\bar{O}-\bar{X}$, $\bar{Y}$, $\bar{Z}$) is the inertial reference frame fixed on sea surface with $\bar{Z}$ pointing "down"; ($\bar{o}-\bar{x}$, $\bar{y}$, $\bar{z}$) is the body-fixed (moving) frame with its origin located at the vehicle's center of gravity (or c.g.). As is usual, the depth/pitch guidance and control surfaces for a standard submarine include a set of stern hydroplanes and bow hydroplanes. Since we are concerned only with depth and pitch equations of motions, the roll/yaw plane dynamics will not be considered in this paper. It is beyond the scope of this paper to review the vast literature associated with the highly nonlinear dynamics of the submarine vehicles. The interested reader refers to the references[6),8),11),13),14)] for details.

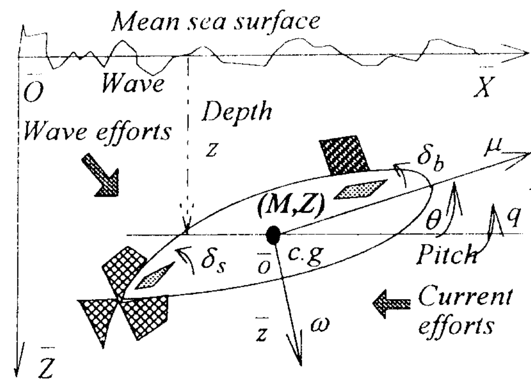

Fig. 1 Submarine system configuration (lateral view)

To obtain a linear model, we can define the system state-vector as $x=[w,q,z,\theta]^T$, the control input variables as $u=[\delta_b,\delta_s]^T$, and the uncertainty (or disturbance) vector as $\mathrm{w}=[\mathrm{w}_1,\mathrm{w}_2]^T$. Considering the standard maneuvering conditions along with the desired depth z and nominal speed $\bar{\mu}$, the linearized form of the state-variable model can be expressed as:

$$\dot{x}=Ax+B_1\mathrm{w}+B_2u \tag{1}$$

which can be rewritten in a system of four, first-order, linear, and coupled ordinary differential equations:

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{z}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}a_{11}\bar{\mu} & a_{12}\bar{\mu} & 0 & a_{14}\bar{z}_g\\ a_{21}\bar{\mu} & a_{22}\bar{\mu} & 0 & a_{24}\bar{z}_g\\ 1 & 0 & 0 & -\bar{\mu}\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ z\\ \theta\end{bmatrix}+\begin{bmatrix}b_1 & 0\\ 0 & b_2\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\mathrm{w}_1\\ \mathrm{w}_2\end{bmatrix}+\bar{\mu}^2\begin{bmatrix}b_{11} & b_{12}\\ b_{21} & b_{22}\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_b\\ \delta_s\end{bmatrix} \tag{2}$$

where $\bar{z}_g$ is the coordinate of c.g. in the body frame $\bar{z}$; $a_{ij}\bar{\mu}$ are the submarine stability derivatives; and b_{ij} are parameters associated with the lift forces of the hydroplanes[13]. It is worth noting that the vector w represent all system uncertainties such as external disturbances due to the sea states, higher-order unmodelled dynamics, and others. For accurate pitch/depth maneuver, the robust autopilot must compensate the uncertainty effects (w) and keep the vehicle in the desired depth with a minimal pitch angular motion. The control synthesis issues of the submarine will be given in the next section.

3. MULTI-OBJECTIVE CONTROL SYNTHESIS VIA LMIS

3.1 Standard regulator problem

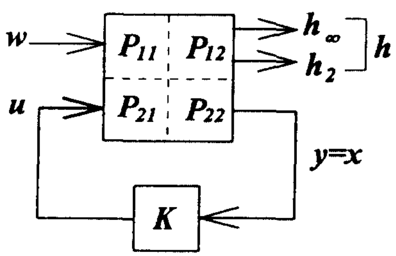

Fig. 2 The multi-objective optimization framework with state-feedback regulator

Consider the general mixed-norm control synthesis for the submarine autopilot shown in Fig. 2. First, the generalized plant P (or Σp) to be controlled by the state-feedback gain matrix $K(\in \mathcal{K})$ is given by the state-space realization with assuming full measurements of its state vector:

$$\Sigma p : \begin{cases} \dot{x} = Ax + B_1 \mathrm{w} + B_2 u \\ h_\infty = C_1 x + D_{11} \mathrm{w} + D_{12} u \\ h_2 = C_2 x + D_{21} \mathrm{w} + D_{22} u \\ y = x \end{cases} \tag{3}$$

with a control law $u = Kx$

where A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21} and D_{22} are all real matrices of compatible dimensions; $\mathcal{K}$ denotes the set of all real proper controllers which achieve internal stability of the closed-loop system. Moreover, the state-space realizations of the closed-loop systems are described by

$$\Sigma c : \begin{bmatrix} \dot{x}_c \\ h_\infty \\ h_2 \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ C_{c\infty} & D_{c\infty} \\ C_{c2} & D_{c2} \end{bmatrix} \begin{bmatrix} x_c \\ \mathrm{w} \end{bmatrix} \tag{4}$$

where the closed-loop matrices with the corresponding state vector x_c have appropriate sizes and are further given as

$$\begin{bmatrix} A_c & B_c \\ C_{c\infty} & D_{c\infty} \\ C_{c2} & D_{c2} \end{bmatrix} = \begin{bmatrix} A + B_2 K & B_1 \\ C_1 + D_{12} K & D_{11} \\ C_2 + D_{22} K & D_{21} \end{bmatrix} \tag{5}$$

Let $\Sigma c\infty(K) = \{A_c, B_c, C_{c\infty}, D_{c\infty}\}$ and $\Sigma c2(K) = \{A_c, B_c, C_{c2}, D_{c2}\}$ denote the state-space realizations of $T_{h_\infty w}$ and $T_{h_2 w}$, respectively, where the corresponding operators are given by

$$T_{h_\infty w} = \left[\frac{A_c | B_C}{C_{c\infty} | D_{C\infty}} \right] = \left[\frac{A + B_2 K | B_1}{C_1 + D_{12} K | D_{11}} \right] \tag{6}$$

$$T_{h_2 w} = \left[\frac{A_c | B_C}{C_{c2} | D_{C2}} \right] = \left[\frac{A + B_2 K | B_1}{C_2 + D_{22} K | D_{21}} \right] \tag{7}$$

Then the closed-loop transfer matrix leading to the lower linear fractional transformation (LFT) of P and K[1], if well posed, is defined as

$$h = T_{hw} w \tag{8}$$

where $T_{hw}(P, K) = [T^T_{h_\infty w}, T^T_{h_2 w}]^T$ is the matrix objective function with $h = [h^T_\infty, h^T_2]^T$. In this autopilot synthesis, the maps $T_{h_\infty w}$ and $T_{h_2 w}$ more specifically represent the closed-loop transfer function matrices from the inputs w to the controlled performance outputs $h_\infty = [z, \theta]^T$ and $h_2 = [w, q, \delta_b, \delta_s]^T$, respectively.

With these notations and assumptions in mind, we are now interested in synthesizing a multi-objective (sub)optimization for the submarine autopilot: Given the augmented system P and a predetermined scalar number $\gamma_\infty > 0$, find a stabilizing controller K for all admissible uncertainties w to solve the following control synthesis problem via LMI optimization[1),10)]:

minimize $\| T_{h_2 w} \|_2$ subject to $\| T_{h_\infty w} \|_\infty < \gamma_\infty$

3.2 H_∞-(sub)optimal compensator design

The standard H_∞ suboptimal synthesis is to find a proper stabilizing controller $K_\infty (\in K)$ such that the closed-loop L_2-induced norm gain can be formalized as $\| T_{h_\infty w} \|_\infty = \sup_w \| T_{h_\infty w}(j\overline{w}) \|_\infty < \gamma_\infty$ for a given $\gamma_\infty \in R^+$, where L_2 is the Hilbert space of square-integrable signals defined over R^+. To maximize performance, γ_∞ should be made as small as possible. In this case, we can obtain the pure H_∞ optimal problem with $K_{\infty opt} (\in K)$ such that $\| T_{h_\infty w} \|_\infty$ is minimized; or $\inf_{K_{\infty opt} \in K} \| T_{h_\infty w} \|_\infty = \gamma_{\infty opt}$, where $\gamma_{\infty opt}$ $(0 \le \gamma_{\infty opt} < \gamma_\infty)$ is the minimal attainable scalar value for $\| T_{h_\infty w} \|_\infty$. Thus, by minimizing the infinity-norm which is defined as the supremum over all frequencies of its largest singular value, the performance outputs due to disturbance inputs are obviously minimized.

Lemma 1: Consider a LMI given by

$$\begin{bmatrix} J_1(x) & J_2(x) \\ J_2^T(x) & J_4(x) \end{bmatrix} < 0 \tag{9}$$

where $J_1(x) = J_1^T(x)$, $J_4(x) = J_4^T(x)$, and $J_2(x)$ depend affinely on x. Then the inequality is equivalent to

$$J_4(x) < 0 \text{ and } J_1(x) - J_2(x) J_4^{-1}(x) J_2^T(x) < 0 \tag{10}$$

or, equivalently,

$$J_1(x) < 0 \text{ and } J_4(x) - J_2^T(x) J_1^{-1}(x) J_2(x) < 0$$

Proof: See Boyd et al.[2)] for more details.

Based on the generalized Bounded Real Lemma[9)], the H_∞ performance constraint of the closed-loop system can be converted to the following LMIs[2),10),12)].

Theorem 1: There exists a stabilizing state-feedback control gain K_∞ which guarantees the closed-loop H_∞ norm of the performance constraint $\| T_{h_\infty w} \|_\infty < \gamma_\infty$ in (6) if and only there exist a Lyapunov matrix $X_\infty > 0$ such that the following LMI formulation holds:

$$\begin{pmatrix} A_c X_\infty + X_\infty A_c^T & B_c & X_\infty C_{c\infty}^T \\ B_c^T & -\gamma_\infty I & D_{c\infty}^T \\ C_{c\infty} X_\infty & D_{c\infty} & -\gamma_\infty I \end{pmatrix} < 0 \tag{11}$$

Proof: To establish the global stability for the closed-loop system (6), we first make use of a quadratic Lyapunov function candidate

$$V(t, x_c) = x_c^T S x_c \text{ with } S = S^T > 0 \tag{12}$$

where V is a positive-definite for all nonzero vector x_c and at least one-time differentiable function as well. Taking its derivative (12) for some $\gamma_\infty > 0$ with all $w \in L_2[0, \infty)$[2)] yields

$$\dot V + h_\infty^T h_\infty - \gamma_\infty^2 w^T w < 0 \text{ for all } t \in R^+ \tag{13}$$

Substituting the closed-loop system $\Sigma_{c\infty}$ in (6) into the inequality (13) gives

$$\begin{aligned} &[A_c x_c + B_c w]^T S x_c + x_c^T S [A_c x_c + B_c w] \\ &+ [C_{c\infty} x_c + D_{c\infty} w]^T [C_{c\infty} x_c + D_{c\infty} w] - \gamma_\infty^2 w^T w < 0 \end{aligned} \tag{14}$$

which can be rewritten in a compact LMI form

$$\begin{pmatrix} U_1 & U_2 \\ U_2^T & U_4 \end{pmatrix} < 0 \tag{15}$$

where $U_1 = A_c^T S + S A_c + C_{c\infty}^T C_{c\infty}$,

$U_2 = SB_c + C^T_{c\infty}D_{c\infty}$, $U_2^T = B_c^T S + D^T_{c\infty}C_{c\infty}$, and $U_4 = -\gamma_\infty^2 I + D^T_{c\infty}D_{c\infty}$. After some algebraic manipulations with Lemma 1 by defining a new matrix variable $X_\infty(>0)$ as $X_\infty = S^{-1}$, we can further simplify the inequality (15) immediately as

$$\begin{pmatrix} A_c X_\infty + X_\infty A_c^T & B_c \\ B_c^T & -\gamma_\infty I \end{pmatrix} + \frac{1}{\gamma_\infty}\begin{pmatrix} X_\infty C^T_{c\infty} \\ D^T_{c\infty} \end{pmatrix}(C_{c\infty}X_\infty \quad D_{c\infty}) < 0 \tag{16}$$

Clearly, through the use of Schur complement formula, the transformed version of the above inequalities is now equivalent to (11) formulated in terms of the state-space matrices A_c, B_c, $C_{c\infty}$ and $D_{c\infty}$.

3.3 H_2-optimal control problem

The standard H_2 optimization problem is to synthesize a proper stabilizing controller $K_{2opt} \in K$ (admissible) such that $\|T_{h_2 u}\|_2$ is minimized. Without loss of generality, we assume that $D_{21} = 0$. Then it is known that the H_2 norm of the closed-loop transfer function matrix is given by

$$\|T_{h_2 w}\|_2^2 = Trace(C_{c2}XC_{c2}^T) \tag{17}$$

which is finite, where $X>0$ is the controllability Gramian of (A_c, B_c) and satisfies the following Lyapunov equation for the closed-loop system Σ_{c2} in (7):

$$A_c X + XA_c^T + B_c B_c^T = 0 \tag{18}$$

Let $X_2(>X)$ denote a positive-definite solution to the Lyapunov inequality:

$$A_c X_2 + X_2 A_c^T + B_c B_c^T < 0 \tag{19}$$

Then the following property yields:

$$\|T_{h_2 w}\|_2^2 < Trace(C_{c2}X_2C_{c2}^T) \tag{20}$$

Moreover, the matrices $X_2 = X_2^T$ and $R = R^T$ satisfy the following LMIs:

$$A_c X_2 + X_2 A_c^T + B_c B_c^T < 0 \tag{21}$$

$$R > C_{c2}X_2C_{c2}^T \tag{22}$$

Then we can readily obtain $\|T_{h_2 w}\|_2^2 < Trace(R)$. Also note that the above inequalities (21) and (22) are equivalent to

$$\begin{pmatrix} A_c X_2 + X_2 A_c^T & B_c \\ B_c^T & -I \end{pmatrix} < 0 \tag{23}$$

$$\begin{pmatrix} R & C_{c2}X_2 \\ X_2 C_{c2}^T & X_2 \end{pmatrix} > 0 \tag{24}$$

With the above statements, it is then straightforward to show the following theorem.

Theorem 2: There exists a stabilizing state-feedback gain $K_2opt \in K$ (admissible) such that the closed-loop norm $\|T_{h_2 w}\|_2$ is minimized if and if only there exists symmetric matrices $X_2 = X_2^T$ and $R = R^T$. Then $\|T_{h_2 w}\|_2^2$ is the minimum of *Trace*(R) subject to a set of LMIs (23) and (24).

Moreover, there exists a norm bound $\gamma_2 \in R^+$ such that $Trace(R) \leq \gamma_2^2$. Clearly, we obtain $\|T_{h_2 w}\|_2^2 = \inf\{Trace(C_{c2}X_2C_{c2}^T)\} = \gamma_{2opt}^2$, with a minimal desired upper bound $\gamma_{2opt} \geq 0$.

3.4 A mixed H_2/H_∞-suboptimal control synthesis

In the preceding subsections, we have independently formulated the control schemes involving LMIs: H_2 optimal and H_∞ suboptimal design. In what follows we will combine the multiple performance objectives into a single formulation, thus providing a more flexible tool for submarine autopilot synthesis. Unfortunately, the set of all matrices (X_∞, X_2, R, K) simultaneously satisfying the LMIs given in (11), (23), and (24) is not convex in general. For the convex optimization over a set of LMI constraints[2),9),10)], it requires that the Lyapunov matrices should be $X_\infty = X_2 = X_c$ in the mixed synthesis problem, where X_c is a common Lyapunov matrix. Then we consider a single Laypunov function $V(t, x_c) = x_c^T X_C x_c$, with $X_C = X_C^T > 0$ for the closed-loop stability. Defining a new matrix variable G as $G = K_{2\infty} X_C$ with $K_{2\infty} \in K$, we now proceed to formulate the mixed control criterion involving LMI based convex optimization.

Theorem 3: There exists a stabilizing state-feedback control law $u = K_{2\infty} x$, which can be incorporated into the mixed H_2/H_∞ control synthesis for (5), if and if only there exist the matrices $X_C = X_C^T$, G, and $R = R^T$ such that

inf{$Trace$(R)} subject to LMIs (11), (23), (24), and $X_C > 0$

with making a change of variable $G = K_{2\infty} X_C$.

Once solving a set of matrices (X_C, R, G), we obtain the control gain matrix $K_{2\infty}$ given by $K_{2\infty} = G X_C^{-1}$. Then the corresponding control law is finally given by $u(t) = K_{2\infty} x(t) = G X_C^{-1} x(t)$.

4. NUMERICAL SIMULATION

It is assumed that a typical operating speed of the length 80[m] of the standard submarine[11)] is $\bar{\mu} = 3.0867[m/s]$ for shallow submerged conditions. Then the corresponding state-space matrices for the depth and pitch model in (3) are given in Appendix by assuming $w = \omega$. All data given are calculated in dimensionless form with a nominal operating speed and length. Some numerical results are obtained by using LMI[5)] Control Toolbox in MATLAB.

To begin, we consider the open-loop system without control law. As shown in Fig. 3, the

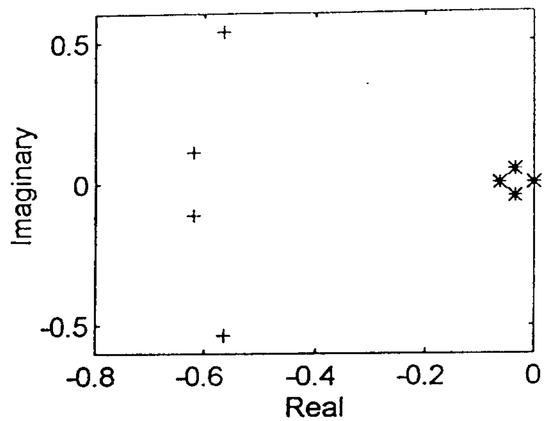

Fig. 3 Pole placement plots: open-loop(*), closed -loop(+)

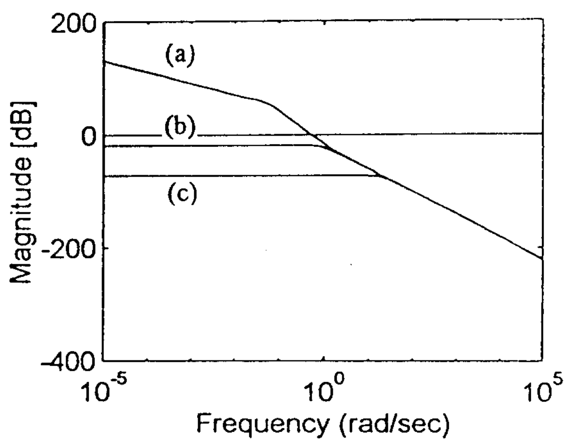

Fig. 4 Singular value plots between impulse input w and h_∞ for: (a) the open-loop, (b) the proposed H_2/H_∞ -(sub)optimal, (c) the H_∞ optimal

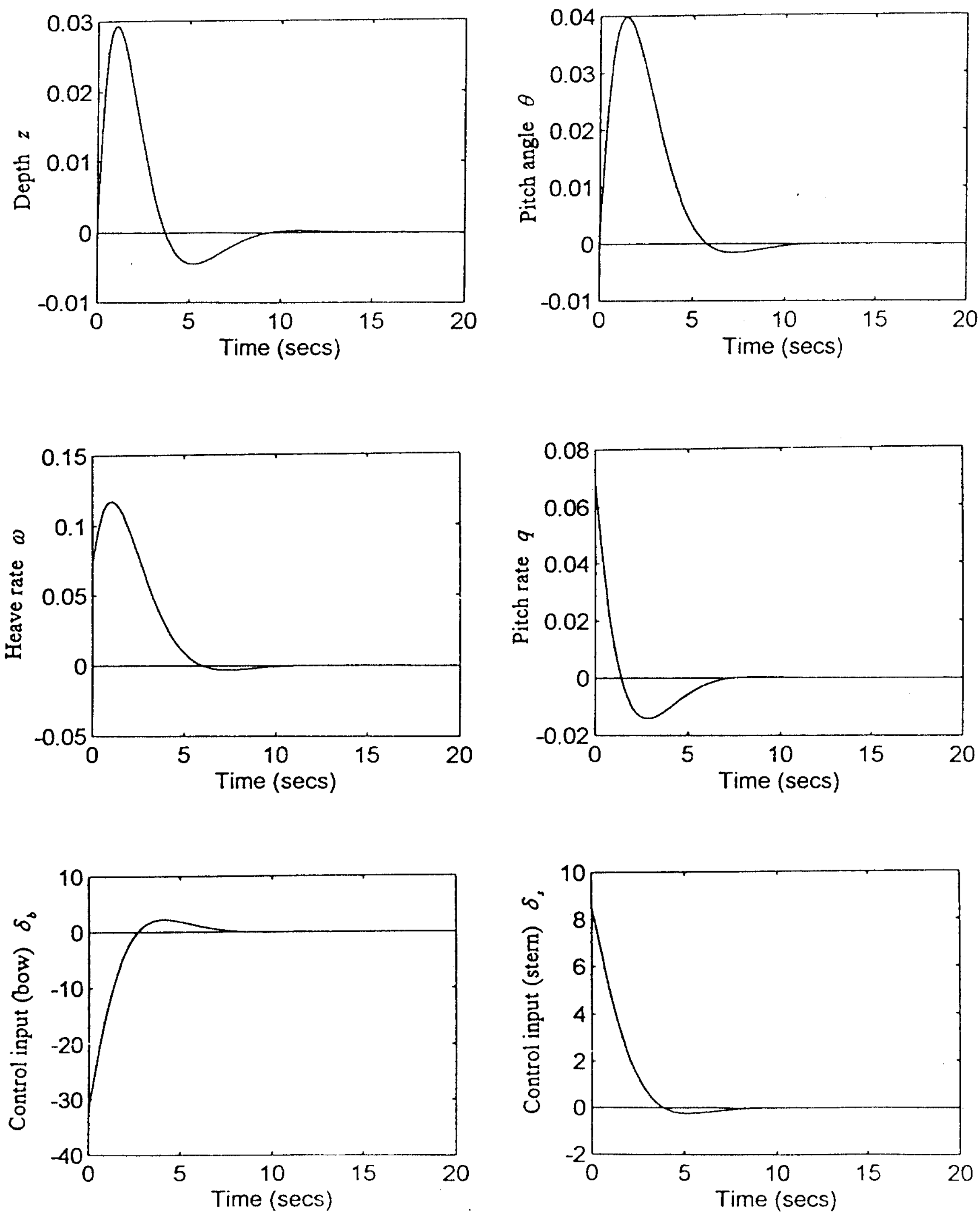

Fig. 5 Time responses of z, θ, ω, and q to impulse disturbance w for the closed-loop system and the corresponding hydroplane activities

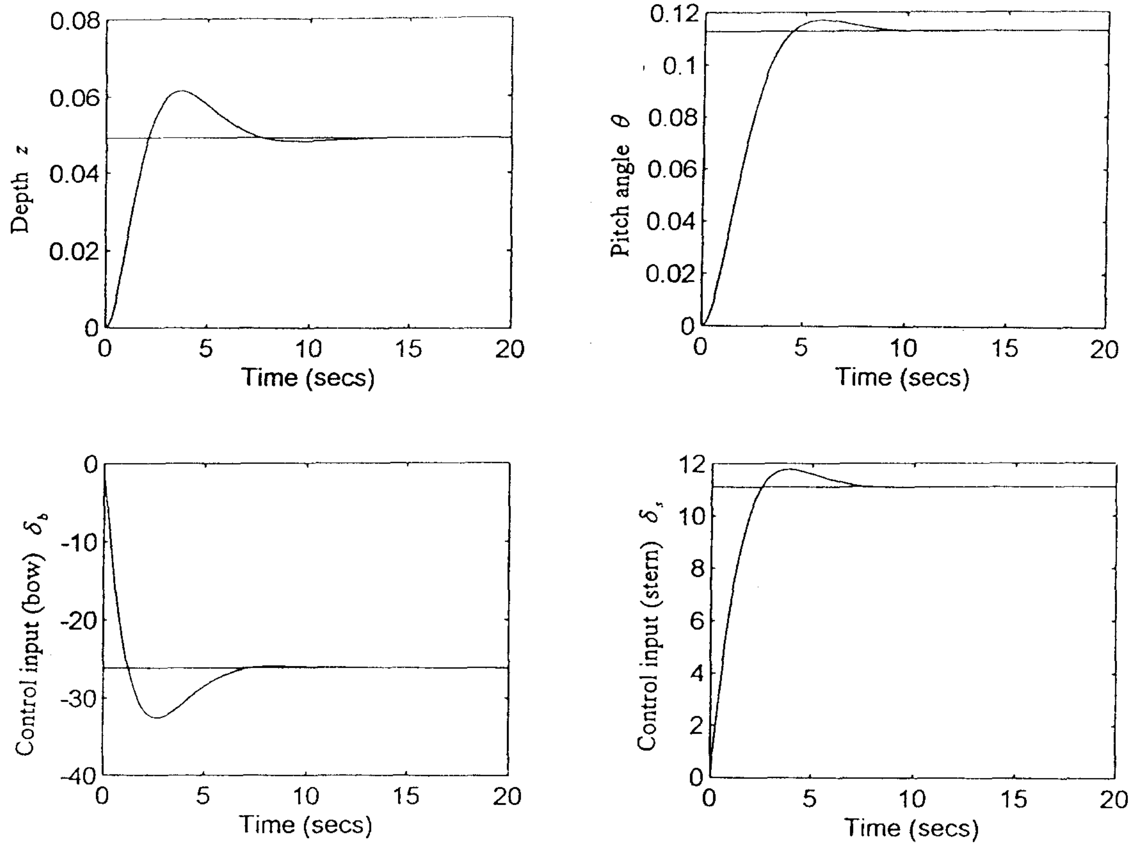

Fig. 6 Time responses of z and θ to step disturbance w for the closed-loop system and the corresponding hydroplane activities

open-loop poles are 0, $-0.0336\pm j0.0472$, and -0.0628. In this design, the control engineer is required to guarantee an acceptable level of disturbance attenuation ($\gamma_\infty=1$) while keeping the control effort acceptably low: minimization of the H_2 norm of T_{h_2w} subject to $\|T_{h_\infty w}\|_\infty < 1$ (or 0 dB). The closed-loop poles in the complex plane are now placed at $-0.6191\pm j0.11$ and $-0.5638\pm j0.5364$ (see Fig. 3). The resulting state-feedback law with $x\in R^4$ is given by $u(t)=Kx(t)$, where the control gain matrix is $K_{2\infty}=\begin{bmatrix}53.7 & -508.3 & 26.3 & -408.8\\ 32.8 & 89.8 & 6.9 & -5.9\end{bmatrix}$. For MIMO frequency response objectives, the singular value plots of the open-loop vs closed-loop system matrices are displayed in Fig. 4 for the comparison purpose. As expected, the singular value plots for the closed-loop system show that the disturbances are well rejected at the low frequencies. Furthermore, the measurement noise attenuation is reasonable at high-frequencies; or the roll-off is more than 40 *dB/decade*. The smaller the infinity norm γ_∞, the better the system is able to reject disturbances. In fact, the best performance level $\gamma_{\infty opt}$ that can be obtained for H_∞ alone (or H_∞-optimal) is $\gamma_{\infty opt}\approx 0$ (see Fig. 4) with the high control gain:

$$K_{\infty opt}=10^4\begin{bmatrix}0.0131 & -0.1172 & 0.0102 & -1.4946\\ 0.0082 & 0.0375 & 0.1617 & 0.4708\end{bmatrix}$$

Also note that we can decrease the attenuation level $\gamma_\infty(\gamma_\infty > \gamma_{\infty opt})$ at expense of a minimum gain γ_{2opt} in H_2/H_∞ method. In addition, for the given values $\gamma_\infty = 1$ and $\gamma_{2opt} = 30.2$, we evaluate the time responses of the closed-loop control system for the impulse- and step-type disturbances in Figs 5 and 6 along with reasonable control input activities. From the simulation results, we can see that the design objectives are certainly achieved.

5. CONCLUSIONS

This paper is concerned with the application of robust control thoery to design a submarine autopilot. First, we have reformulated the mixed H_2/H_∞ synthesis problem using LMIs to provide the convex suboptimal solution. Next, complete simulation analysis for the closed-loop performance in both the time and frequency domains is presented to explicitly evaluate the vehicle guidance and control performance. It has been shown that a set of robust controllers we selected give excellent performance and satisfy the control magnitudes. Finally, the LMI-based control approach provides tractable means to design the robust autopilot posed by several competing objective functions.

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APPENDIX

All system matrices are clearly defined here:

$$A=\begin{bmatrix}-0.038006 & 0.89604 & 0 & 0.0014673\\ 0.0017105 & -0.091976 & 0 & -0.0056093\\ 1 & 0 & 0 & -3.0867\\ 0 & 1 & 0 & 0\end{bmatrix}$$

$$B_1=\begin{bmatrix}0.07\\0.07\\0\\0\end{bmatrix},\quad B_2=\begin{bmatrix}-0.007542 & -0.022859\\ 0.001732 & -0.002221\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

$$C_1=\begin{bmatrix}0&0&1&0\\0&0&0&1\end{bmatrix},\quad C_2=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

$$D_{11}=\begin{bmatrix}0\\0\end{bmatrix},\quad D_{12}=\begin{bmatrix}0&0\\0&0\end{bmatrix},\quad D_{21}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix},\quad D_{22}=\begin{bmatrix}0&0\\0&0\\1&0\\0&1\end{bmatrix}$$