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◉ 論　　文

Safety Assessment of Offshore Structural System Using the Response Surface Approach

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응답면 접근법을 이용한 해양구조물 시스템의 안전성 평가

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Key Words : Offshore Structure (해양구조물), Bearing Capacity (저항능력), Failure Mode (파괴모드), Reliability (신뢰도), Response Surface (응답면), Monte Carlo Simulation

초 록

본 논문에서는 해양구조물의 신뢰성을 평가할 수 있는 새로운 방법을 제시하였다. 우선 구조의 저항능력에 대한 응답면을 구축하였고, Pearson 곡선중 하나를 이용해서 응답면을 근사시킨 후 Monte-Carlo Simulation을 수행하였고, 최종적으로 수치적분법을 적용해서 파괴확률을 구하였다. 해양구조물에의 적용을 통해 본 논문의 방법이 갖는 정당성을 보였다.

1. INTRODUCTION

As a rule reliability of offshore structures is estimated on the basis of Level-II methods[1]. In the process of calculations a failure surface in n-dimensional space of basic variables is defined. Then, the reliability index β as the shortest distance from the origin of coordinates to the failure surface in a normalized space of basic variables is calculated. This reliability index is usually used as a measure of structural reliability. The above approach causes significant difficulties in calculations :

- The failure surface can not be the most commonly defined explicitly and therefore the derivatives essential for reliability index calculations can be determined only numerically.
- In the general case there is no one-to-one relationship between reliability index and failure probability, so the value of failure probability is estimated approximately.

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- The failure surface can be of such a form that several local minima are present. In this case if the process of calculations converges not to the global minimum structural reliability can be dangerously overestimated.
- To estimate the reliability of offshore structures many possible failure modes and their correlations should be taken into account.

The approach suggested in this paper makes it possible overcome the above drawbacks. The statement of the problem is as follows.

Basic load $L_1, L_2, ..., L_t$ and resistance $R_1, R_2, ..., R_n$ variables are assumed to be random values with known probability density functions. Arbitrary sets of realizations of basic variables are denoted by $l = (l_1, l_2, ..., l_t)$ and $r = (r_1, r_2, ..., r_n)$, respectively. A set $l = (l_1, l_2, ..., l_t)$ represents a load pattern.

It is assumed that for any l and r the load bearing capacity of the structure is determined by the load factor u, that is, one of the failure modes occurs when the load pattern is $u\,l = (u\,l_1, u\,l_2, ..., u\,l_t)$ and a set of resistance variables is fixed. Load factor u can be determined on the basis of any deterministic design method: a load pattern l is being increased proportionally until a failure mode occurs.

The probability of failure P_f is denoted as :

$$P_f = \text{Prob.}\ (u < 1) \tag{1}$$

and the reliability is $P_s = 1 - P_f$. The task of the reliability analysis of an offshore structures is to determine P_f. This task is solved in the following order.

First a response surface in the form of a polynomial for the load factor u is built. The polynomial is a function of realizations of basic variables. Then Monte Carlo simulation is performed : for random sets of basic variables the values of u are obtained from the equation of response surface. The next step is to fit a probability density function in the form of one of the Pearson's curves for the values of u thus obtained. Finally failure probability, Eq.(1) is computed by numerical integration. The main concepts of this approach are discussed at the next sections.

2. STRENGTH OF STRUCTURAL SYSTEM

The load factor of a structural system up to a particular failure stage can be calculated by using the extended incremental load method, which is one of the method for the structural system reliability analysis[2]. Let $u_1, u_2, ..., u_j$ be mean load factors corresponding to incremental stages. At the j th failure stage (incremental stage) the relation between component strengths and load factors for a failure mode is as follow :

$$\begin{Bmatrix} R_1 \\ R_2 \\ \cdot \\ \cdot \\ R_j \end{Bmatrix} = \begin{bmatrix} A_{11} & & & & \\ A_{21} & A_{22} & & & \\ \cdot & \cdot & \cdot & & \\ \cdot & \cdot & & \cdot & \cdot \\ A_{j1} & A_{j2} & \cdot & \cdot & A_{jj} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ u_j \end{Bmatrix} \tag{2}$$

where $R_1, R_2, ..., R_j$ are strengths of components $n_1, n_2, ..., n_j$ which are involved in the current faiure path, $n_1 - n_2 - ... - n_j$, and $u_1, u_2, ..., u_j$ are load factors up to a particular failure stage. For example u_1 is the load factor to fail the first component n_1, u_2 is that to fail component n_1 and n_2. For simplicity Eq.(2) is expressed in a matrix form :

$$\{R_e\} = [A]\ \{U\} \tag{3}$$

where $\{R_e\}$ and $\{U\}$ are vector of component strengths and load factors, respectively. Matrix [A] is called the total utilization matrix and its element A_{ki} is given as :

$$A_{ki} = \sum_{l=1}^{t} a_{ki}^{(l)} P^{(l)} \quad (4)$$

where t is the number of basic load cases applied to the structure and $P^{(l)}$ is the l-th load value. The vector of load factors for load increments can be obtained by .

$$\{U\} = [A]^{1} \{R_e\} \quad (5)$$

If collapse of a structure occurs when j components $n_1, n_2, \ldots, n_j$ have failed, the total load factor, u corresponding to the structural collapse can be obtained by summing up all elements in the load factor $\{U\}$[2].

$$u = \sum_{i=1}^{j} u_i \quad (6)$$

This denotes the ratio of collapse load to applied load and it is closely related to the reserve strength factor (RSI)[3] defined as :

$$RSI = \frac{\text{system collapse load}}{\text{applied load}} \quad (7)$$

The deterministically important failure modes are found based on the deterministic criteria in reference [2]. The collapse of structural system theoretically occurs when the determinant of structural stiffness matrix is zero. This is, however, meaningless in the case of complex structural systems. Hence, the occurrence of the structural collapse is judged when the determinant is very small compared with that of initial state as component failure progresses. That is, the structural collapse occurs when following inequality is satisfied :

$$\frac{\det[K_j]}{\det[K_o]} \leq \varepsilon \quad (8)$$

where $\det[K_j]$ and $\det[K_o]$ are the determinant of stiffness of a structural system at the j-th failure stage and at the initial state, respectively. ε is a small number, say $\varepsilon = 10^{10}$.

3. RESPONSE SURFACE

To calculate P_f first a response surface in the form of a polynomial for the load factor u should be built. For this purpose a number of deterministic designs are carried out at different points of (n+t)-dimensional space of basic variables and load factor u is determined in each design. The response surface is built in (n+t+1)-dimensional space and passes through the values of u calculated at the above points. These points can be fixed in various ways.

Bucher and Bourgund[4] have suggested to use a response surface in the form of the following polynomial :

$$u(l_1, \ldots, l_t, r_1 \ldots, r_n) = k_0 + \sum_{j=1}^{2} \sum_{i=1}^{t} a_{ij} l_i^j + \sum_{j=1}^{2} \sum_{i=1}^{n} b_{ij} r_i^j \quad (9)$$

To obtain the values of unknown factors k_0, a_{ij}, b_{ij} deterministic designs are carried out at the points ($\mu_{l_1}, \ldots, \mu_{l_t}, \mu_{r_1}, \ldots, \mu_{r_n}$), ($\mu_{l_1} \pm \delta_{l_1}\sigma_{l_1}, \mu_{l_2}, \ldots, \mu_{l_t}, \mu_{r_1}, \ldots, \mu_{r_n}$), ($\mu_{l_1}, \mu_{l_2} \pm \delta_{l_2}\sigma_{l_2}, \ldots, \mu_{l_t}, \mu_{r_1}, \ldots, \mu_{r_n}$), ..., ($\mu_{l_1}, \mu_{l_2}, \ldots, \mu_{l_t}, \mu_{r_1}, \ldots, \mu_{r_n} \pm \delta_{l_n}\sigma_{l_n}$) where, μ_{l_i}, μ_{r_i}, σ_{l_i}, σ_{r_i} are respectively mean values (μ) and standard deviation (σ) for i-th load (l_i) or resistance(r_i) basic variable and δ_{l_i}, δ_{r_i} are the number of standard deviation from the mean value μ to the design point $\mu \pm \delta\sigma$. Factors k_0, a_{ij}, b_{ij} are determined as the solution to a system of (t+n+1) linear equations. The right hand sides of the equations are the values of u calculated at the above points.

The investigation has shown that sometimes the accuracy of the approximation by polynomial (9) should be improved. For this purpose other polynomials can be used. First of all the largest power of polynomial (9) can be increased:

$$u(l_1, ..., l_t, r_1, ..., r_n) = k_0 + \sum_{j=1}^{2h}\sum_{i=1}^{t} a_{ij} l_i^j + \sum_{j=1}^{2h}\sum_{i=1}^{n} b_i^j r_i^j \quad (10)$$

where h is an integer positive number. In this case the number of deterministic designs (the total number of polynomial terms) is N = 2h(t+n)+1.

Sometimes the following polynomial appears to be useful (N=2t+n) :

$$u(l_1, ..., l_t, r_1, ..., r_n) = \sum_{i_{l1}=0}^{1} \cdots \sum_{i_{lt}=0}^{1} \sum_{i_{r1}=0}^{1} \cdots \sum_{i_{rn}=0}^{1} k_{i_{l1}} \cdots i_{lt}, i_{r1} \cdots i_m \, l_1^{i_{l1}} \cdots l_t^{i_{lt}} r_1^{i_{r1}} \cdots r_n^{i_{rn}} \quad (11)$$

If the accuracy of approximation by polynomial (11) is to be increased then the terms of type (5) can be added to polynomial (11) :

$$u(l_1, \cdots, l_t, r_1, \cdots, r_n) = \sum_{i_{l1}=0}^{1} \cdots \sum_{i_{lt}=0}^{1} \sum_{i_{r1}=0}^{1} \cdots \sum_{i_{rn}=0}^{1} k_{i_{l1}} \cdots i_{lt}, i_{r1} \cdots i_m \, l_1^{i_{l1}} \cdots l_t^{i_{lt}} r_1^{i_{r1}} \cdots r_n^{i_{rn}} + \sum_{j=2}^{2h+1}\sum_{i=1}^{t} a_{ij} l_i^j + \sum_{j=1}^{2h}\sum_{i=1}^{n} b_{ij} r_i^j \quad (h, t, r \geq 1) \quad (12)$$

In this case $N=2^{t+n} + 2h(t+n)$.

To increase the accuracy of approximation the (t+n)-dimensional space of basic variables can be divided into several points and different polynomials (9) - (12) can be used for different parts.

4. MONTE CARLO SIMULATION

After the response surface has been built the Monte Carlo Simulation for each load combination is carried out in the following order.

- Knowing the probability density functions for all basic variables and using Monte Carlo Simulation obtain m random sets (l_i, r_i) (i=1,.., m) of basic load and resistance variables.
- Using equation(s) of the response surface determine the values of u_i (i=1, ..., m) for each random set of basic variables.
- Assume the values of u_i thus obtained to be m realizations of the random variable U, i.e., the sample size is m. Calculate the first four statistical moments of the sample.
- Fit appropriate probability density function $f_i(u)$ from the family of Pearson's curves (the first four statistical moments of the distribution coincide with those of the sample).
- Using numerical integration calculate the probability of failure for i-th load combination, P_{fi} :

$$P_{fi} = \int_{-\infty}^{1} f_i(u)\, du \quad (13)$$

Taking into account all load combinations and using the total probability theorem calculate the probability of failure :

$$P_f = \sum_{i=1}^{S} P_i(Q_i) P_{fi} \quad (14)$$

The algorithm was implemented in the form of a computer program. Accuracy of the program results was proved by many test examples[5]. A possibility of using statistics of extreme values was demonstrated[6].

5. NUMERICAL EXAMPLE

The numerical example presented below illustrates the above approach. Consider a jacket-type offshore structure shown in Fig.1[7]. Two cases with unclamped (Case 1) and clamped (Case 2) joints 31, 43, 55 are analysed. Load and resistance basic variables are assumed to be normally distributed with mean values and coefficients of variation presented in Table 1[7].

Polynomial given as Eq.(11) was used for approximation. Deterministic designs described in the section 2 has been carried out at the corresponding points with $\delta_{l_i} = \delta_{r_i} = 3$. The results of the calculations have shown that only one failure mode occurs at all points of each case: the failure paths are 35-33-36 for Case 1 and 19-33-17-25 for Case 2. In Case 1 the load factor u is affected only by fully correlated strengths of the elements with end numbers 29, 31, 32, 33, 35, 36 and by the fully correlated loads L_1, L_2, ..., L_6, L_{12}, ..., L_{17}. These strengths and loads are denoted by r_1 and l_1, respectively.

In Case 2 the load factor u is affected also by fully correlated loads L_7, L_8 which are denoted by l_2. The approximating polynomials are:

$$u = 1.4322 - 0.5276\, l_1 + 0.1145\, r_1 + 0.0836\, l_1^2 \qquad (15)$$

for Case 1 and

$$u = 2.9010 - 0.6662\, l_1 - 0.1013\, l_2 + 0.00955\, r_1 + 0.0766\, l_1^2 + 0.035\, l_2^2 \qquad (16)$$

for Case 2.

Monte Carlo Simulation has shown that probability density functions can be represented by the Pearson's curves of type 7 :

$$f(u) = 0.7739\, [1 + (\frac{u - 1.5106}{1.7419})^2]^{6.4541} \qquad (17)$$

for Case 1 and

$$f(u) = 0.5921\, [1 + (\frac{u - 2.9848}{3.7017})^2]^{15.8386} \qquad (18)$$

for Case 2.

Probabilities of failure determined by numerical integration are P_f = 0.16593 for Case 1 and P_f = 0.002044 for Case 2. Failure probability P_f = 0.16593 is somewhat in excess of the similar failure probability P_f = 0.08758 determined in reference [7]. And in Case 2 (clamped joints) the failure probability is considerably less than in Case 1 (unclamped joints). These results indicates that usin the response surface approach the failure probability of a structural system can be efficiently calculated with retaing the accuracy. And the results of failure probability implies that the system safety can be much enhanced by strengthening the joints at the cross points of braces in case of jacket type offshore structural system.

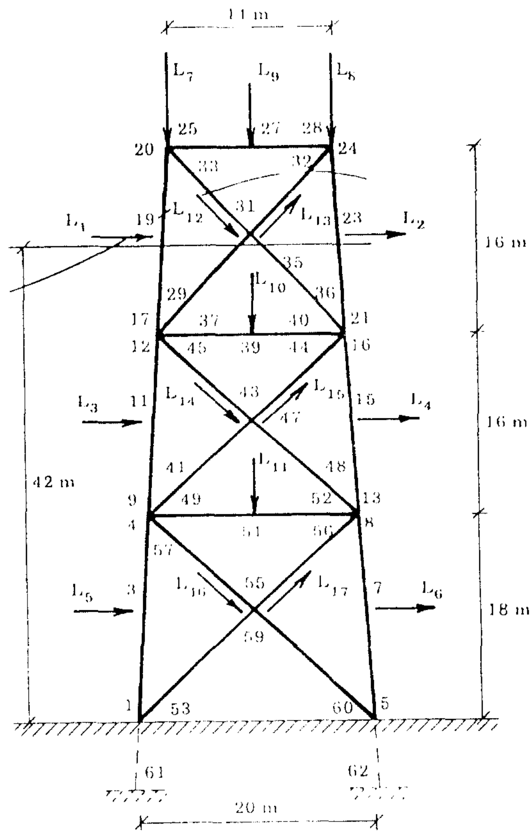

Fig. 1 Jacket-Type Offshore Structure[7]

6. CONCLUSIONS

From all said above one can see that the suggested approach makes it possible to overcome the difficulties mentioned in the introduction in this paper:

- There is no need to calculate derivatives essential for reliability index calculation. But the failure surface is expressed explicitly in the form of a polynomial.

Table 1 Data of Offshore Structure in Fig.1 (unit : m, KN)

(a) Load data

load no.	mean value	COV
wave load		
L_1	368.1	0.42
L_2	334.1	0.38
L_3	181.5	0.41
L_4	169.6	0.36
L_5	142.7	0.41
L_6	137.6	0.33
L_9	4.5	0.40
L_{10}	5.5	0.68
L_{11}	3.7	0.45
L_{12}	139.0	0.33
L_{13}	205.0	0.46
L_{14}	72.3	0.31
L_{15}	88.9	0.45
L_{16}	56.9	0.34
L_{17}	59.9	0.40
deck load		
L_7	2490.0	0.10
L_8	2490.0	0.10

o Correlation coefficient

ρ_{ij} = 1.0 for L_1 -L_6, L_{12} -L_{17}
= 1.0 for L_7 & L_8
= 1.0 for L_9 -L_{10}
= 0.0 for others

(b) Strength data

component no.	sectional area	reference strength*
1, 3, 4, 5, 7, 8	0.0810	5286.0
9, 11, 12, 13, 15, 16, 17, 19, 20, 21, 23, 24	0.0638	3842.0
25, 27, 28, 37, 39, 40	0.0154	476.9
29, 31, 32, 33, 35, 36, 41, 43, 44, 45, 47, 48	0.0200	798.5
49, 51, 52	0.0167	517.8
53, 55, 56, 57, 59, 60	0.0247	980.4

* reference strength : plastic bending moment
COV = 0.08

o Correlation coefficient

ρ_{ij} = 1.0 for the same component type
= 0.0 for the different component type

o Elastic Modulus = 210 GPa

o Mean yield stress = 276 MPa

- Failure probability is directly calculated. There is, therefore, no need for one-to-one relationship between reliability index and failure probability.
- Because of direct numerical integration of the probability density function the problem of many local minima does not arise.
- There is also no need to consider all possible failure modes or to single out stochastically dominant failure modes. All necessary failure modes as well as their correlations can be taken into account as the response surface is being built.

In authors' opinion all these advantages allow for simple and effective reliability analysis of offshore structures.

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