Journal of the Korean
Statistical Society
Vol. 26, No. 3, 1997

Generalized Durbin-Watson Statistics in the Nonstationary Seasonal Time Series Model †

Sinsup Cho[1], Byungsoo Kim [2] and Young J. Park [3]

Abstract

In this paper we study the behaviors of the generalized Durbin-Watson (DW) statistics when the nonstationary seasonal time series regression model is misspecified. It is observed that when the series is seasonally integrated the generalized DW statistic for the seasonal period order autocorrelation converges in probability to zero while the generalized DW statistic for the first order autocorrelation has non-degenerate asymptotic distribution. When the series is regularly and seasonally integrated the generalized DW for the first order autocorrelation still converges in probability to zero.

Key Words : Total misspecification; Partial misspecification; Seasonal cointegration; Partially cointegrated; Spurious regression; Generalized Durbin-Watson statistics.

---

†The study is supported by the academic research fund(BSRI-97-1418) of Ministry of Education, Republic of Korea.

[1]Department of Statistics, Seoul National University, Seoul, 151-742, Korea. sinsup@stats.snu.ac.kr

[2]Department of Statistics, Inje University, Kimhae, 621-749, Korea. kbs@stat.inje.ac.kr

[3]Department of Statistics, North Carolina State University, Raleigh, USA.

1. INTRODUCTION

In the analysis of time series we usually assume that a time series under study is stationary, but in business and economics nonstationary time series arise frequently. The nonstationary part of the time series can be modeled by either a deterministic trend or a stochastic function and the behaviors and the forecasts from these two types of functions are quite different. Hence for a proper analysis of the time series it is important to determine whether the series is trend stationary or difference stationary.

Since Granger and Newbold (1974) examined some of the likely empirical consequences of nonsense or spurious regression in economic time series, Phillips (1986) provided the analytical solution to the asymptotic behaviors of spurious regression. Durlauf and Phillips (1988) have derived the limiting properties of the least squares estimators in the linear trend regression model when the true model is a random work or a random work with drift.

It is also important to determine if the seasonal regression is properly modeled as in the nonseasonal case. Abeysinghe (1991) studied some consequences of deterministic modelling of seasonally integrated series through a Monte Carlo simulation study. Abeysinghe (1994) also examined effects on the sample autocovariance by inappropriate use of seasonal indicator variables. Ahn *et al.* (1996) analytically studied the behaviors of parameter estimates, ordinary t statictic, F statistic, and R^2 when the model misspecification arises in regression of nonstationary seasonal time series.

When a spurious regression occurs, in many cases ordinary t statistic and F statistic diverge, and R^2 converges to a non-degenerate distribution, hence inference using these usual regression statistics can be often misleading. Large values of t, F, and R^2 statistics are the phenomenons of the model misspecification, but these statistics do not directly indicate a spurious regression. Thus we need another statistics to discriminate a spurious regression, one of which is the DW statistic.

Since Durbin and Watson (1950, 1951) proposed the DW statistic, it has long been used to detect the first order autocorrelation of the disturbances in the regression model

$$\mathbf{Y} = \mathbf{X}\beta + \epsilon \tag{1.1}$$

where $\mathbf{Y}$ is an $n \times 1$ vector of observations on the dependent variable, $\mathbf{X}$ is an $n \times k$ matrix of observations on k fixed regressors and ϵ is an $n \times 1$ vector of disturbances. The test has been generalized for the fourth order autocorrelation by Wallis (1972) and for autocorrelation of any order by Vinod

(1973). The generalized DW statistics are

$$d_k = \frac{\sum_{t=k+1}^{n}(\hat{\epsilon}_t - \hat{\epsilon}_{t-k})^2}{\sum_{t=1}^{n}\hat{\epsilon}_t^2}, \quad k = 1, \ldots, n-1, \tag{1.2}$$

where $\hat{\epsilon}_t$'s are the residuals of regression (1.1).

Shin and Sarkar (1995) indicated that a small DW statistic value may be taken as an indication of the possibility of model misspecification in the analysis of the nonstationary nonseasonal time series. In the nonstationary seasonal time series analysis the generalized DW statistics can also be used as an indication of the model misspecification.

In this paper almost in parallel with the study of Ahn *et al.* (1996), we analytically study the behaviors of the generalized DW statistics when the nonstationary seasonal time series regression model is misspecified. Two types of model misspecification will be considered as in Ahn *et al.* (1996), the first type is total misspecification where the predictors are deterministic functions or another statistically independent seasonally integrated process. The second type is partial misspecification where the predictor variable and the dependent variable are cointegrated at some, but not all of the seasonal frequencies.

2. PRELIMINARY RESULTS

For a nonstationary (seasonal) process Y_t and a process X_t, which can be nonstationary (seasonal) process or deterministic, consider a regression model

$$Y_t = \alpha X_t + \epsilon_t. \tag{2.1}$$

Two types of model misspecifications are considered in this paper. First is a totally misspecified type where $\alpha = 0$ so that $Y_t = \epsilon_t$. Second is a partially misspecified type where $\alpha \neq 0$ and ϵ_t is nonstationary. If X_t and Y_t are independent in (2.1), it is an example of a totally misspecified model. If seasonally integrated time series X_t and Y_t are cointegrated at frequency zero only, it is an example of a partially misspecified model. In this case the orders of X_t and ϵ_t are the same. See Section 5 and Ahn *et al.* (1996) for details.

For X_t and ϵ_t in (2.1) we assume that $X_n = O_p(a_n)$ and $\epsilon_n = O_p(b_n)$. Also we assume that $X_n - X_{n-k} = O_p(c_{k,n})$ and $\epsilon_n - \epsilon_{n-k} = O_p(d_{k,n})$. Note that the orders of $c_{k,n}$ and $d_{k,n}$ are same or lower than those of a_n and b_n, respectively. We note that for a totally misspecified model $Y_n = O_p(b_n)$ and for a partially misspecified model $Y_n = O_p(max\{a_n, b_n\})$. In the above partially misspecified regression example, X_n, ϵ_n, and Y_n are all $O_p(n^{1/2})$.

Ahn *et al.* (1996) showed that when the model (2.1) is fitted by the least squares method $\hat{\alpha} - \alpha = O_p(b_n/a_n)$ and $\hat{\epsilon}_n = O_p(b_n)$. The order of $\hat{\alpha} - \alpha$ depends on the orders of ϵ_t and X_t. Regardless of the order of X_t, the order of $\hat{\epsilon}_t$ is the same as the order of ϵ_t. From these result, we have the following asymptotic results for least squares regression of (2.1).

Lemma 1. If the model (2.1) is fitted by least squares method, then, under the assumption stated above,

$$\begin{aligned}\sum_{t=1}^{n} \hat{\epsilon}_t^2 &= O_p(nb_n^2),\\ \sum_{t=k+1}^{n} (\hat{\epsilon}_t - \hat{\epsilon}_{t-k})^2 &= O_p(max\{nd_{k,n}^2, nc_{k,n}^2 b_n^2/a_n^2\}),\\ d_k &= O_p(max\{d_{k,n}^2/b_n^2, c_{k,n}^2/a_n^2\}).\end{aligned}$$

Proofs of this lemma is based on the Lemma 1 of Ahn *et al.* (1996). From this lemma, we can obtain that d_k converges to zero in probability if and only if $c_{k,n}$ and $d_{k,n}$ are of lower orders than a_n and b_n simultaneously. If either the order of $c_{k,n}$ is the same as a_n or the order of $d_{k,n}$ is the same as b_n, d_k converges to a non-degenerate limiting distribution. When X_t and Y_t are independent and regularly integrated time series, a_n and b_n are $O_p(n^{1/2})$ and $c_{k,n}$ and $d_{k,n}$ are $O_p(1)$, so that $d_k = O_p(1/n)$ for all $k \geq 1$. Thus, if the model of (2.1) is fitted by the least squares method, d_1 converges to zero and nd_1 converges to a non-degenerate limiting distribution, as shown by Phillips (1986) and Durlauf and Phillips (1988). This lemma is useful for proofs of the theorems 1 to 3.

3. SEASONALLY INTEGRATED PROCESSES

Let Y_t be a seasonally integrated process with period s, and be generated by

$$(1 - B^s)Y_t = \omega_t, \tag{3.1}$$

where ω_t are stationary random variables with $E(\omega_t) = 0$, $Var(\omega_t) = \sigma_\omega^2$, and $sup_t\ E(|\omega_t|^{2+\delta}) < \infty$ for some $\delta > 0$. For this nonstationary series, one may consider a deterministic linear time trend. If starting values of Y_t contain a strong seasonality, a realization may be consistently repetitive, and one is subject to use seasonal indicators or another seasonal time series X_t to account for the seasonal behavior. Here we assume that X_t are generated from

$$(1 - B^s)X_t = \nu_t, \tag{3.2}$$

where ν_t satisfy the above conditions of ω_t, and are independent of ω_t, so that X_t and Y_t are independent. Thus, for Y_t, we consider the following regression models, as in Ahn et al. (1996):

$$Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + \epsilon_t, \tag{3.3}$$

$$Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + \gamma t + \epsilon_t, \tag{3.4}$$

$$Y_t = \alpha X_t + \epsilon_t, \tag{3.5}$$

$$Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + \alpha X_t + \epsilon_t, \tag{3.6}$$

$$Y_t = \sum_{j=1}^{s} \beta_j \delta_{jt} + \gamma t + \alpha X_t + \epsilon_t, \tag{3.7}$$

where $\delta_{jt} = 1$ if $j \equiv t \ (mod\ s)$ or 0 otherwise. All the models are totally misspecified for Y_t.

For these totally misspecified regression models, Ahn *et al.* (1996) showed that $\hat{\beta}_j$'s diverge, $\hat{\gamma}$ converges to zero in probability, and $\hat{\alpha}$ converges to a non-degenerate limiting distribution. Note that $\hat{\beta}_j$'s and $\hat{\alpha}$ are not consistent, only $\hat{\gamma}$ is a consistent estimator. These results indicate that in all models a spurious relationship between Y_t and the predictors occurs.

In Addition to these results, by noting Y_n and X_n is $O_p(n^{1/2})$ and using Lemma 1, we can obtain the asymptotic properties of the generalized DW statistics in the following theorem.

Theorem 1. If the regression models of (3.3) through (3.7) are fitted by the least squares method for Y_t generated by (3.1) and X_t by (3.2), then d_k is $O_p(1.1)$ for $k < s$, and d_s is $O_p(n^{-1})$.

For regularly integrated processes ($s = 1$), the results of the above theorem are equivalent to those of Theorem 1 of Phillips (1986) and Theorems 2.2 of

Durlauf and Phillips (1988). On the other hand, for seasonally integrated processes ($s \neq 1$), if $k < s$, d_k does not converge to zero, but d_s converges to zero in probability. Theorem 1 indicates that, as Durlauf and Phillips (1988) considered d_1 for nonseasonal cases, the generalized DW statistics can be a useful tool to determine whether the series contain a seasonal unit root. For this purpose, unlike nonseasonal cases, we have to check the lowest order of the generalized DW statistics that goes to zero, which is s in this case, thus we have to carefully examine d_s instead of d_1.

Example 1. Y_t are generated from (3.1) by simulation, where ω_t are independently and identically distributed normal random variables with mean zero and variance 1. The seasonal period is $s = 4$, the sample size is $n = 100$, and the initial values are $Y_{-3} = 2$, $Y_{-2} = 5$, $Y_{-1} = -2$, and $Y_0 = -5$. When the regression model (3.4) is fitted by the least squares method, the t statistics of β_j's are -0.3, 3.8, 1.0, and -4.0, respectively. The t statistics of γ is -3.0 and $R^2 = 0.33$. The true values of the above statistics are all zeros. Thus inference using these statistics may often lead wrong conclusions. The first four orders generalized DW statistics are $d_1 = 1.99$, $d_2 = 3.06$, $d_3 = 2.12$, and $d_4 = 0.73$. It is observed that d_4 has relatively small value, on the other hand d_k ($k < 4$) have relatively large value. Therefore d_4 can be a discriminant tool for the model misspecification and the seasonal unit root in the disturbances.

4. REGULARLY AND SEASONALLY INTEGRATED PROCESSES

Both regularly and seasonally integrated time series models are found to be useful to explain variety of seasonal time series. See the airline data of Series G in Box et al.(1994) for example. Let Y_t be a regularly and seasonally integrated process with period s, and be generated by

$$(1 - B)(1 - B^s)Y_t = \omega_t. \tag{4.1}$$

For Y_t in (4.1) one may consider regression models of (3.3) through (3.7). Here we assume X_t is also a regularly and seasonally integrated process generated by

$$(1 - B)(1 - B^s)X_t = \nu_t. \tag{4.2}$$

Assume that ω_t and ν_t are independent and satisfy the conditions of Section 3.

For these Y_t and X_t the regression models of (3.3) through (3.7) are totally misspecified. For these models, Theorem 2 in Ahn *et al.* (1996) shows that $\hat{\beta}_j$'s and $\hat{\gamma}$ diverge, and $\hat{\alpha}$ converges to a non-degenerate limiting distribution. All the estimates are not consistent. These results indicate that in all models a spurious relationship between Y_t and the predictors occurs.

Then, by noting Y_n and X_n are $O_p(n^{3/2})$, we have the following theorem.

Theorem 2. If the regression models of (3.3) through (3.7) are fitted by the least squares method for Y_t generated from (4.1) and X_t from (4.2), then d_1 is $O_p(n^{-2})$.

Unlike the results of Theorem 1, d_1 converges to zero in probability. This is because that $\hat{\epsilon}_n - \hat{\epsilon}_{n-1}$ is of lower order than $\hat{\epsilon}_n$. The order of $\hat{\epsilon}_n$ is $n^{3/2}$, while the order of $\hat{\epsilon}_n - \hat{\epsilon}_{n-1}$ is $n^{1/2}$. Theorem 2 indicates that the generalized DW statistics will provide a simultaneous test of both regular and seasonal unit roots. For this purpose, we note that d_1 goes to zero in probability and its order is $O_p(n^{-2})$.

Example 2. The airline passenger data of Series G in Box *et al.* (1994) are considered. Let Y_t be the log transformed data of the original series. When the regression model (3.4) is fitted with $s = 12$, most of the t statistics of β_j's and γ are significant and $R^2 = 0.98$. The first order generalized DW statistic is $d_1 = 0.42$, which indicates the possibility of the model misspecification and the existence of regular and seasonal unit roots.

5. PARTIALLY COINTEGRATED SEASONAL PROCESSES

For two seasonally integrated processes

$$Y_t = Y_{t-s} + \omega_t \quad \text{and} \quad X_t = X_{t-s} + \nu_t, \tag{5.1}$$

we consider partially cointegrated cases where Y_t and X_t are cointegrated at some of but not all of the frequencies corresponding to the roots of $1 - B^s = 0$, so that ω_t and ν_t are not independent. For partially cointegrated seasonal processes one may consider regression models of (3.5), (3.6), or (3.7). For partially cointegrated processes Y_t and X_t, $\alpha \neq 0$ and ϵ_t is nonstationary. We consider a seasonal time series of $s = 4$, in which the roots of $(1 - B^s) = 0$ are 1, -1, and $\pm i$, which correspond to frequencies zero, 1/2, and 1/4, respectively.

If Y_t and X_t are cointegrated at frequency zero, there exists a constant $\alpha \neq 0$ such that $Y_t - \alpha X_t$ dose not have a root of one in its autoregressive representation. That is,

$$(1+B)(1+B^2)(Y_t - \alpha X_t) = u_t, \tag{5.2}$$

where u_t is a stationary process. Then (5.2) can be rewritten as in (3.5) with ϵ_t satisfying $(1+B)(1+B^2)\epsilon_t = u_t$. For more partial cointegration, see Ahn *et al.* (1996).

For the regression models of (3.5), (3.6), and (3.7), Ahn *et al.* (1996) shows in Theorem 3 that $\hat{\beta}_j$'s diverge, $\hat{\gamma}$ converges to zero in probability, and $\hat{\alpha} - \alpha$ converges to a non-degenerate limiting distribution. These results indicate that in all models a model misspecification occurs. Then, by noting X_n and ϵ_n are all $O_p(n^{1/2})$, we have the following results.

Theorem 3. If the regression models of (3.5), (3.6) and (3.7) are fitted by the least squares method for Y_t and X_t generated from (5.1) and partially cointegrated, then d_k is $O_p(1)$ for $k < s$, and d_s is $O_p(n^{-1})$.

Like Theorem 1, if $k < s$, d_k converges to a non-degenerate limiting distribution, while d_s converges to zero in probability. This is because the order of $\hat{\epsilon}_n - \hat{\epsilon}_{n-k}$ is the same as $\hat{\epsilon}_n$ if $k < s$, while $\hat{\epsilon}_n - \hat{\epsilon}_{n-s}$ is of lower order than $\hat{\epsilon}_n$, i.e., the orders of $\hat{\epsilon}_n$ and $\hat{\epsilon}_n - \hat{\epsilon}_{n-k}$ are $n^{1/2}$ and the order of $\hat{\epsilon}_n - \hat{\epsilon}_{n-s}$ is 1.

6. CONCLUSION

When a nonstationary seasonal time series regression model is misspecified, the generalized DW statistics do not always converge to non-degenerate limiting distributions. For example, if a regression model of regularly integrated time series is misspecified, d_1 goes to zero in probability. When a regression model of seasonally integrated process is totally misspecified, d_s converges to zero in probability, while d_1 has a non-degenerate asymptotic distribution. When a regression model of regularly and seasonally integrated process is totally misspecified, d_1 converges to zero in probability.

These properties indicate that the generalized DW statistics may be used to determine a seasonal unit root. For this purpose, unlike nonseasonal cases, we have to check the lowest order of the generalized DW statistics that goes to zero, which is seasonal period s in this case, thus we have to consider d_s instead of d_1. Also, the generalized DW statistics may provide a method to discriminate regular and seasonal unit roots simultaneously. For this purpose, we note that the order of d_1 is $O_p(n^{-2})$.

When a regression model of seasonally integrated process is partially misspecified, the generalized DW statistics have the same behaviors of the totally misspecified cases, the only difference is the form of the limiting distribution.

TECHNICAL APPENDIX

In this appendix we provide a proof of Lemma 1 and explicit forms of the limiting distributions of the various estimators and the generalized DW statistics appeared in the text.

A1. Proof of Lemma 1: Since $X_n = O_p(a_n)$ and $\epsilon_n = O_p(b_n)$, $\sum X_t\epsilon_t = O_p(na_nb_n)$ and $\sum X_t^2 = O_p(na_n^2)$ by similar arguments used in (A.1) of Phillips (1986). Therefore,

$$\hat{\alpha} - \alpha = \frac{\sum X_t\epsilon_t}{\sum X_t^2} = O_p(b_n/a_n).$$

From $\hat{\epsilon}_t = \epsilon_t - (\hat{\alpha} - \alpha)X_t$ it follows immediately that

$$\hat{\epsilon}_t = O_p(b_n),$$

and

$$\sum \hat{\epsilon}_t^2 = O_p(nb_n^2).$$

Since $\hat{\epsilon}_t - \hat{\epsilon}_{t-k} = \epsilon_t - \epsilon_{t-k} - (\hat{\alpha} - \alpha)(X_t - X_{t-k})$,

$$\sum(\hat{\epsilon}_t - \hat{\epsilon}_{t-k})^2 = O_p(max\{nd_{k,n}^2, nc_{k,n}^2b_n^2/a_n^2\}).$$

From these facts, the order of d_k can be obtain as

$$d_k = O_p(max\{d_{k,n}^2/b_n^2, c_{k,n}^2/a_n^2\}).$$

A2. Explicit formulae for limiting distribution:

Using functional central limit theorem and continuous mapping theorem, in most cases we can easily derive the limiting distributions of interest statistics. Some work related these theorems explored in Phillips (1986, 1987) for nonseasonal models, and in Park and Cho (1995) and Ahn et al.(1996) for seasonal models. Based on those theorems, we can derive the limiting distributions of the statistics in the context.

In the following, "$\Rightarrow$" means convergence in distribution, and "$\xrightarrow{p}$" means convergence in probability. Also $W_j(r)$, $V_j(r)$, and $U_j(r)$ are Brownian motions as limiting distributions of processes corresponding to the partial sums of j-th season, and $\hat{\epsilon}_t$ are the least squares residuals of the regression models of (3.3) through (3.7). For simple notation, we denote $\int_0^1 W_j(r)dr$ as $\int W_j$, and so on. We assume $n = ms$ where s is a seasonal period.

A2.1. Formulae for Theorem 1:

A2.1.0. Definition and notation:

data generating : $Y_t = Y_{t-s} + \omega_t, \quad X_t = X_{t-s} + \nu_t$

$$\begin{aligned}
(n/s)^{-1/2} \sum_{t=1}^{[\frac{n}{s}r]} \omega_{(t-1)s+j} &\Rightarrow W_j(r), \quad W_j^* \equiv W_j - \int W_j, \quad j = 1, \ldots, s \\
(n/s)^{-1/2} \sum_{t=1}^{[\frac{n}{s}r]} \nu_{(t-1)s+j} &\Rightarrow V_j(r), \quad V_j^* \equiv V_j - \int V_j, \quad j = 1, \ldots, s \\
\Omega_{\omega j} &\equiv \lim_{n\to\infty} n^{-1} \sum_{t=j+1}^{n} E(\omega_t \omega_{t-j}) \\
\Omega_{\nu j} &\equiv \lim_{n\to\infty} n^{-1} \sum_{t=j+1}^{n} E(\nu_t \nu_{t-j})
\end{aligned}$$

A2.1.1. Formulae for model (3.3):

$$\begin{aligned}
n^{-1/2}\hat{\beta}_j &\Rightarrow s^{-1/2} \int W_j \\
n^{-2} \sum_t \hat{\epsilon}_t^2 &\Rightarrow s^{-2} \sum_{j=1}^{s} \int W_j^{*2} \\
n^{-2} \sum_t \hat{\epsilon}_t \hat{\epsilon}_{t-1} &\Rightarrow s^{-2} \sum_{j=1}^{s} \int W_j^* W_{j-1}^* \\
d_1 &\Rightarrow 2 - 2\frac{\sum_{j=1}^{s} \int W_j^* W_{j-1}^*}{\sum_{j=1}^{s} \int W_j^{*2}} \\
n^{-1} \sum_t (\hat{\epsilon}_t - \hat{\epsilon}_{t-s})^2 &\xrightarrow{p} \Omega_{\omega 0} \\
nd_s &\Rightarrow \frac{s^2 \Omega_{\omega 0}}{\sum_{j=1}^{s} \int W_j^{*2}}
\end{aligned}$$

A2.1.2. Formulae for model (3.4):

$$n^{1/2}\hat{\gamma} \Rightarrow 12s^{-3/2}\sum_j \int (r-\frac{1}{2})W_j \equiv \xi_{11}$$

$$n^{-1/2}\hat{\beta}_j \Rightarrow s^{-1/2}\int W_j - \frac{1}{2}\xi_{11}$$

$$n^{-2}\sum_t \hat{\epsilon}_t^2 \Rightarrow s^{-2}\sum_j \int W_j^{*2} - \frac{1}{12}\xi_{11}^2 \equiv \zeta_{11}$$

$$n^{-2}\sum_t \hat{\epsilon}_t\hat{\epsilon}_{t-1} \Rightarrow s^{-2}\sum_j \int W_j^* W_{j-1}^* - \frac{1}{12}\xi_{11}^2 \equiv \zeta_{12}$$

$$d_1 \Rightarrow 2-2\frac{\zeta_{12}}{\zeta_{11}}$$

$$n^{-1}\sum_t(\hat{\epsilon}_t-\hat{\epsilon}_{t-s})^2 \xrightarrow{p} \Omega_{\omega 0}$$

$$nd_s \Rightarrow \frac{\Omega_{\omega 0}}{\zeta_{11}}$$

A2.1.3. Formulae for model (3.5):

$$\hat{\alpha} \Rightarrow \frac{\sum_j \int W_j V_j}{\sum_j \int V_j^2} \equiv \xi_{12}$$

$$n^{-2}\sum_t \hat{\epsilon}_t^2 \Rightarrow s^{-2}\sum_j \int \{W_j^2 - \xi_{12}^2 V_j^2\}$$

$$n^{-2}\sum_t \hat{\epsilon}_t\hat{\epsilon}_{t-1} \Rightarrow s^{-2}\sum_j \int (W_j - \xi_{12}V_j)(W_{j-1}-\xi_{12}V_{j-1})$$

$$d_1 \Rightarrow 2-2\frac{\sum_j \int (W_j - \xi_{12}V_j)(W_{j-1}-\xi_{12}V_{j-1})}{\sum_j\{\int W_j^2 - \xi_{12}^2 \int V_j^2\}}$$

$$n^{-1}\sum_t(\hat{\epsilon}_t-\hat{\epsilon}_{t-s})^2 \xrightarrow{p} \Omega_{\omega 0}+\xi_{12}^2\Omega_{\nu 0}$$

$$nd_s \Rightarrow s^2\frac{\Omega_{\omega 0}+\xi_{12}^2\Omega_{\nu 0}}{\sum_j\{\int W_j^2 - \xi_{12}^2 \int V_j^2\}}$$

A2.1.4. Formulae for model (3.6):

$$\hat{\alpha} \Rightarrow \frac{\sum_j \int W_j^* V_j^*}{\sum_j \int V_j^{*2}} \equiv \xi_{13}$$

$$n^{-1/2}\hat{\beta}_j \Rightarrow s^{-1/2}\{\int W_j - \xi_{13}\int V_j\}$$

$$n^{-2}\sum_t \hat{\epsilon}_t^2 \Rightarrow s^{-2}\sum_j \int \{W_j^{*2} - \xi_{13}^2 V_j^{*2}\} \equiv \zeta_{13}$$

$$
\begin{aligned}
n^{-2}\sum_t \hat\epsilon_t\hat\epsilon_{t-1} &\Rightarrow s^{-2}\sum_j \int \left[W_j^* - \xi_{13}V_j^*\right]\left[W_{j-1}^* - \xi_{13}V_{j-1}^*\right] \equiv \zeta_{14} \\
d_1 &\Rightarrow 2 - 2\frac{\zeta_{14}}{\zeta_{13}} \\
n^{-1}\sum_t(\hat\epsilon_t - \hat\epsilon_{t-s})^2 &\xrightarrow{p} \Omega_{\omega 0} + \xi_{13}^2\Omega_{\nu 0} \\
nd_s &\Rightarrow \frac{\Omega_{\omega 0} + \xi_{13}^2\Omega_{\nu 0}}{\zeta_{13}}
\end{aligned}
$$

A2.1.5. Formulae for model (3.7):

$$
\begin{aligned}
n^{1/2}\hat\gamma &\Rightarrow \frac{s^{-\frac{3}{2}}}{ad - s^{-1}b^2}\sum_j\{d\int(r-\frac{1}{2})W_j - b\int W_j^*V_j^*\} \equiv \xi_{14} \\
\hat\alpha &\Rightarrow \frac{-1}{ad - s^{-1}b^2}\sum_j\{s^{-1}b\int(r-\frac{1}{2})W_j - a\int W_j^*V_j^*\} \equiv \xi_{15} \\
n^{-1/2}\hat\beta_j &\Rightarrow s^{-1/2}\{\int W_j - \xi_{15}\int V_j\} - \frac{1}{2}\xi_{14} \\
n^{-2}\sum_t \hat\epsilon_t^2 &\Rightarrow s^{-2}\sum_j\int\left[W_j^* - \xi_{15}V_j^* - s^{\frac{1}{2}}\xi_{14}(r-\frac{1}{2})\right]^2 \equiv \zeta_{15} \\
n^{-2}\sum_t \hat\epsilon_t\hat\epsilon_{t-1} &\Rightarrow s^{-2}\sum_j\int\left[W_j^* - \xi_{15}V_j^* - s^{\frac{1}{2}}\xi_{14}(r-\frac{1}{2})\right] \\
&\qquad \times\left[W_{j-1}^* - \xi_{15}V_{j-1}^* - s^{\frac{1}{2}}\xi_{14}(r-\frac{1}{2})\right] \equiv \zeta_{16} \\
d_1 &\Rightarrow 2 - 2\frac{\zeta_{16}}{\zeta_{15}} \\
n^{-1}\sum_t(\hat\epsilon_t - \hat\epsilon_{t-s})^2 &\xrightarrow{p} \Omega_{\omega 0} + \xi_{15}^2\Omega_{\nu 0} \\
nd_s &\Rightarrow \frac{\Omega_{\omega 0} + \xi_{15}^2\Omega_{\nu 0}}{\zeta_{15}}
\end{aligned}
$$

where

$$
\begin{aligned}
a &= 1/12 \\
b &= \sum_j\int(r-\frac{1}{2})V_j \\
d &= \sum_j\int V_j^{*2}
\end{aligned}
$$

A2.2. Formulae of Theorem 2:

A2.2.0. Definition and notation:

data generating : $(1-B)(1-B^s)Y_t = \omega_t, \ (1-B)(1-B^s)X_t = \nu_t$

$$(n/s)^{-1/2} \sum_{t=1}^{[\frac{n}{s}]r} \omega_{(t-1)s+j} \ \Rightarrow \ W_j(r), \quad W_j^* \equiv W_j - \int W_j, \quad j = 1, \ldots, s$$

$$(n/s)^{-1/2} \sum_{t=1}^{[\frac{n}{s}]r} \nu_{(t-1)s+j} \ \Rightarrow \ V_j(r), \quad V_j^* \equiv V_j - \int V_j, \quad j = 1, \ldots, s$$

$$(n/s)^{-3/2} Y_{[nr]} \ \Rightarrow \ \sum_{j=1}^{s} \int_0^r W_j(r_1) dr_1 \ \equiv \ B_Y(r), \quad B_Y^* \equiv B_Y - \int B_Y$$

$$(n/s)^{-3/2} X_{[nr]} \ \Rightarrow \ \sum_{j=1}^{s} \int_0^r V_j(r_1) dr_1 \ \equiv \ B_X(r), \quad B_X^* \equiv B_X - \int B_X$$

A2.2.1 Formulae for model (3.3):

$$n^{-3/2} \hat{\beta}_j \ \Rightarrow \ s^{-3/2} \int B_Y$$

$$n^{-4} \sum_t \hat{\epsilon}_t^2 \ \Rightarrow \ s^{-3} \int B_Y^{*2}$$

$$n^{-2} \sum_t (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2 \ \Rightarrow \ s^{-2} \{ \sum_{j=1}^{s} \int W_j^{*2} + B_Y^2(1) \}$$

$$n^2 d_1 \ \Rightarrow \ s \frac{\sum_{j=1}^{s} \int W_j^{*2} + B_Y^2(1)}{\int B_Y^{*2}}$$

A2.2.2. Formulae for model (3.4):

$$n^{-1/2} \hat{\gamma} \ \Rightarrow \ 12 s^{-3/2} \int (r - \frac{1}{2}) B_Y \equiv \xi_{21}$$

$$n^{-3/2} \hat{\beta}_j \ \Rightarrow \ s^{-3/2} \int B_Y - \frac{1}{2} \xi_{21}$$

$$n^{-4} \sum_t \hat{\epsilon}_t^2 \ \Rightarrow \ s^{-3} \int B_Y^{*2} - \frac{1}{12} \xi_{21}^2$$

$$n^{-2} \sum_t (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2 \ \Rightarrow \ s^{-2} \{ \sum_j \int W_j^{*2} + (B_Y(1) - s^{\frac{3}{2}} \xi_{21})^2 \}$$

$$n^2 d_1 \ \Rightarrow \ \frac{s^{-2} \{ \sum_j \int W_j^{*2} + (B_Y(1) - s^{\frac{3}{2}} \xi_{21})^2 \}}{s^{-3} \int B_Y^{*2} - \frac{1}{12} \xi_{21}^2}$$

A2.2.3. Formulae for model (3.5):

$$
\begin{aligned}
\hat{\alpha} &\Rightarrow \frac{\int B_Y B_X}{\int B_X^2} \equiv \xi_{22} \\
n^{-4}\sum_t \hat{\epsilon}_t^2 &\Rightarrow s^{-3}\{\int B_Y^2 - \xi_{22}^2 \int B_X^2\} \\
n^{-2}\sum_t (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2 &\Rightarrow s^{-2}\sum_j \int (W_j - \xi_{22}V_j)^2 \\
n^2 d_1 &\Rightarrow s\frac{\sum_j \int (W_j - \xi_{22}V_j)^2}{\int B_Y^2 - \xi_{22}^2 \int B_X^2}
\end{aligned}
$$

A2.2.4. Formulae for model (3.6):

$$
\begin{aligned}
\hat{\alpha} &\Rightarrow \frac{\int B_Y^* B_X^*}{\int B_X^{*2}} \equiv \xi_{23} \\
n^{-3/2}\hat{\beta}_j &\Rightarrow s^{-3/2}\{\int B_Y - \xi_{23}\int B_X\} \\
n^{-4}\sum_t \hat{\epsilon}_t^2 &\Rightarrow s^{-3}\{\int B_Y^{*2} - \xi_{23}^2 \int B_X^{*2}\} \equiv \zeta_{21} \\
n^{-2}\sum_t (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2 &\Rightarrow s^{-2}\{\sum_j \int [W_j^* - \xi_{23}V_j^*]^2 + (B_Y(1) - \xi_{23}B_X(1))^2\} \equiv \zeta_{22} \\
n^2 d_1 &\Rightarrow \frac{\zeta_{22}}{\zeta_{21}}
\end{aligned}
$$

A2.2.5. Formulae for model (3.7):

$$
\begin{aligned}
n^{-1/2}\hat{\gamma} &\Rightarrow \frac{s^{-3/2}}{ad-b^2}\{d\int (r-\frac{1}{2})B_Y - b\int B_Y^* B_X^*\} \equiv \xi_{24} \\
\hat{\alpha} &\Rightarrow \frac{-1}{ad-b^2}\{b\int (r-\frac{1}{2})B_Y - a\int B_Y^* B_X^*\} \equiv \xi_{25} \\
n^{-3/2}\hat{\beta}_j &\Rightarrow s^{-3/2}\{\int B_Y - \xi_{25}\int B_X\} - \frac{1}{2}\xi_{24} \\
n^{-4}\sum_t \hat{\epsilon}_t^2 &\Rightarrow s^{-3}\int \left[B_Y^* - \xi_{25}B_X^* - s^{\frac{3}{2}}\xi_{24}(r-\frac{1}{2})\right]^2 \equiv \zeta_{23} \\
n^{-2}\sum_t (\hat{\epsilon}_t - \hat{\epsilon}_{t-1})^2 &\Rightarrow s^{-2}\{\sum_j \int [W_j^* - \xi_{25}V_j^*]^2 + (B_Y(1) - \xi_{23}B_X(1) - s^{\frac{3}{2}}\xi_{24})^2\} \equiv \zeta_{24} \\
n^2 d_1 &\Rightarrow \frac{\zeta_{24}}{\zeta_{23}}
\end{aligned}
$$

where

$$a = 1/12$$

$$b = \int (r - \frac{1}{2}) B_X$$
$$d = \int B_X^{*2}$$

A2.3. Formulae of Theorem 3:

We first express ϵ_t in terms of u_t, for example, if X_t and Y_t are cointegrated at frequency zero, then

$$\epsilon_{(t-1)s+j} = \sum_{k=1}^{t} u_{(k-1)s+j} - \sum_{k=1}^{t} u_{(k-1)s+j-1}, \quad j = 1, \ldots, s.$$

Here we present the foremulae for the partially cointegrated case at frequency zero. The formulae for the other cases can be obtained from the authors upon request.

A2.3.0. Definition and notation:

data generating : $(1 - B^s)Y_t = \omega_t$, $(1 - B^s)X_t = \nu_t$, $(1 - B^s)(Y_t - \alpha X_t) = u_t - u_{t-1}$

$$(n/s)^{-1/2} \sum_{t=1}^{[\frac{n}{s}r]} \nu_{(t-1)s+j} \Rightarrow V_j(r), \quad V_j^* \equiv V_j - \int V_j, \quad j = 1, \ldots, s$$

$$(n/s)^{-1/2} (\sum_{t=1}^{[\frac{n}{s}r]} u_{(t-1)s+j} - \sum_{t=1}^{[\frac{n}{s}]r} u_{(t-1)s+j-1}) \Rightarrow U_j(r) - U_{j-1}(r) \equiv \bar{U}_j(r), \quad \bar{U}_j^* \equiv \bar{U}_j - \int \bar{U}_j$$

$$\Omega_{\nu j} = \lim_{n \to \infty} n^{-1} \sum_{t=j+1}^{n} E(\nu_t \nu_{t-j})$$

$$\Omega_{uj} = \lim_{n \to \infty} n^{-1} \sum_{t=j+1}^{n} E(u_t - u_{t-1})(u_{t-j} - u_{t-j-1})$$

A2.3.1. Formulae for model (3.5):

$$\hat{\alpha} - \alpha \Rightarrow \frac{\sum_j \int \bar{U}_j V_j}{\sum_j \int V_j^2} \equiv \xi_{31}$$
$$n^{-2} \sum_t \hat{\epsilon}_t^2 \Rightarrow s^{-2} \sum_j \int \{\bar{U}_j^2 - \xi_{31}^2 V_j^2\}$$
$$n^{-2} \sum_t \hat{\epsilon}_t \hat{\epsilon}_{t-1} \Rightarrow s^{-2} \sum_j \int (\bar{U}_j - \xi_{31} V_j)(\bar{U}_{j-1} - \xi_{31} V_{j-1})$$

$$
\begin{aligned}
d_1 &\Rightarrow 2 - 2\frac{\sum_j \int (\bar{U}_j - \xi_{31}V_j)(\bar{U}_{j-1} - \xi_{31}V_{j-1})}{\sum_j \int \{\bar{U}_j^2 - \xi_{31}^2 V_j^2\}} \\
n^{-1}\sum_t (\hat{\epsilon}_t - \hat{\epsilon}_{t-4})^2 &\xrightarrow{p} \Omega_{u0} + \xi_{31}^2\Omega_{\nu 0} \\
nd_s &\Rightarrow s^2 \frac{\Omega_{u0} + \xi_{31}^2\Omega_{\nu 0}}{\sum_j \int \{\bar{U}_j^2 - \xi_{31}^2 V_j^2\}}
\end{aligned}
$$

A2.3.2. Formulae for model (3.6):

$$
\begin{aligned}
\hat{\alpha} - \alpha &\Rightarrow \frac{\sum_j \int \bar{U}_j^* V_j^*}{\sum_j \int V_j^{*2}} \equiv \xi_{32} \\
n^{-1/2}\hat{\beta}_j &\Rightarrow s^{-1/2}\{\int \bar{U}_j - \xi_{32}\int V_j\} \\
n^{-2}\sum_t \hat{\epsilon}_t^2 &\Rightarrow s^{-2}\sum_j \int \{\bar{U}_j^{*2} - \xi_{32}^2 V_j^{*2}\} \equiv \zeta_{31} \\
n^{-2}\sum_t \hat{\epsilon}_t\hat{\epsilon}_{t-1} &\Rightarrow s^{-2}\sum_j \int (\bar{U}_j^* - \xi_{32}V_j^*)(\bar{U}_{j-1}^* - \xi_{32}V_{j-1}^*) \equiv \zeta_{32} \\
d_1 &\Rightarrow 2 - 2\frac{\zeta_{32}}{\zeta_{31}} \\
n^{-1}\sum_t (\hat{\epsilon}_t - \hat{\epsilon}_{t-s})^2 &\xrightarrow{p} \Omega_{u0} + \xi_{32}^2\Omega_{\nu 0} \\
nd_s &\Rightarrow \frac{\Omega_{u0} + \xi_{32}^2\Omega_{\nu 0}}{\zeta_{31}}
\end{aligned}
$$

A2.3.3. Formulae for model (3.7):

$$
\begin{aligned}
n^{\frac{1}{2}}\hat{\gamma} &\Rightarrow \frac{s^{-3/2}}{ad - s^{-1}b^2}\sum_j \{d\int (r - \frac{1}{2})\bar{U}_j - b\int \bar{U}_j^* V_j^*\} \equiv \xi_{33} \\
\hat{\alpha} - \alpha &\Rightarrow \frac{-1}{ad - s^{-1}b^2}\sum_j \{s^{-1}b\int (r - \frac{1}{2})\bar{U}_j - a\int \bar{U}_j^* V_j^*\} \equiv \xi_{34} \\
n^{-1/2}\hat{\beta}_j &\Rightarrow s^{-1/2}\{\int \bar{U}_j - \xi_{34}\int V_j\} - \frac{1}{2}\xi_{33} \\
n^{-2}\sum_t \hat{\epsilon}_t^2 &\Rightarrow s^{-2}\sum_j \int \left[\bar{U}_j^* - \xi_{34}V_j^* - s^{\frac{1}{2}}\xi_{33}(r - \frac{1}{2})\right]^2 \equiv \zeta_{33} \\
n^{-2}\sum_t \hat{\epsilon}_t\hat{\epsilon}_{t-1} &\Rightarrow s^{-2}\sum_j \int \left[\bar{U}_j^* - \xi_{34}V_j^* - s^{\frac{1}{2}}\xi_{33}(r - \frac{1}{2})\right] \\
&\quad \times \left[\bar{U}_{j-1}^* - \xi_{34}V_{j-1}^* - s^{\frac{1}{2}}\xi_{33}(r - \frac{1}{2})\right] \equiv \zeta_{34} \\
d_1 &\Rightarrow 2 - 2\frac{\zeta_{34}}{\zeta_{33}}
\end{aligned}
$$

$$n^{-1}\sum_t(\hat{\epsilon}_t - \hat{\epsilon}_{t-s})^2 \xrightarrow{p} \Omega_{u0} + \xi_{34}^2\Omega_{\nu 0}$$

$$nd_s \Rightarrow \frac{\Omega_{u0} + \xi_{34}^2\Omega_{\nu 0}}{\zeta_{33}}$$

where

$$\begin{aligned} a &= 1/12 \\ b &= \sum_j \int (r - \frac{1}{2})V_j \\ d &= \sum_j \int V_j^{*2} \end{aligned}$$

ACKNOWLEDGEMENT

The authors would like to thank the refrees for helpful comments and suggestions.

REFERENCES

(1) Abeysinghe, T. (1991). Inappropriate use of seasonal dummies in regression. *Economics Letters*, **36**, 175-179.

(2) Abeysinghe, T. (1994). Deterministic seasonal models and spurious regressions. *Journal of Econometrics*, **61**, 259-272.

(3) Ahn, S. K., Park, Y. J. and Cho, S. (1996). Model misspecification in nonstationary seasonal time series. Submitted for publication.

(4) Box, G. E. P., Jenkins, G. M. and Reinsel, G. C. (1994). *Time series analysis: Forecasting and control*, 3rd ed., Prentice-Hall, Englewood Cliffs, NJ.

(5) Durbin, J. and Watson, G. S. (1950). Testing for serial correlation in least squares regression I. *Biometrika*, **37**, 409-428.

(6) Durbin, J. and Watson, G. S. (1951). Testing for serial correlation in least squares regression II. *Biometrika*, **38**, 159-178.

(7) Durlauf, S. and Phillips, P. C. B. (1988). Trends versus random walks in time series analysis. *Econometrica*, **56**, 1333-1354.

(8) Granger, C. W. J. and Newbold, P. (1974). Spurious regression in economics. *Journal of Econometrics*, **2**, 111-120.

(9) Park, Y. J. and Cho, S. (1995). Lagrange multiplier test for both regular and seasonal unit roots. *The Korean Communications in Statistics*, **2**, 101-114.

(10) Phillips, P. C. B. (1986). Understanding spurious regressions in economics. *Journal of Econometrics*, **33**, 311-340.

(11) Phillips, P. C. B. (1987). Time series regression with a unit root. *Econometrica*, **55**, 277-301.

(12) Shin, D. W. and Sarkar, S. (1995). Model misspecification problems for nonstationary time series. Technical Report, Dept. of Statistics, Ewha Womans University.

(13) Vinod, H. D. (1973). Generalization of the Durbin-Watson statistic for higher order autoregressive processes. *Communication in Statistics*, **2**, 115-144.

(14) Wallis, K. F. (1972). Testing for fourth-order autocorrelation in quarterly regression equations. *Econometrica*, **40**, 617-636.