Journal of the Korean
Statistical Society
Vol. 26, No. 3, 1997

Sequential Confidence Interval with β-protection for a Linear Function of Two Normal Means†

Sung Lai Kim[1]

Abstract

A sequential procedure for estimating a linear function of two normal means which satisfies the two requirements, i.e. one is a condition of coverage probability, the other is a condition of β-protection, is proposed when the variances are unknown and not necessarily equal. We give asymptotic behaviors of the proposed stopping time.

Key Words : One-sided sequential confidence interval; Stopping time; Uniformly in; Asymptotic normality.

1. INTRODUCTION

Let $X_1,\ X_2,\ X_3,\ \cdots$ and $Y_1,\ Y_2,\ Y_3,\ \cdots$ be two independent sequences of random variables where the X_i's are i.i.d. $N(\mu_x,\ \sigma_x^2)$ and Y_i's are i.i.d. $N(\mu_y,\ \sigma_y^2)$. The four parameters $\mu_x,\ \mu_y,\ \sigma_x$ and σ_y are assumed unknown. Put parameter space $\Theta = \{\theta = (\mu_x,\ \mu_y,\ \sigma_x,\ \sigma_y); -\infty < \mu_x,\ \mu_y < \infty, 0 < \sigma_x,\ \sigma_y < \infty\}$. We want to find a confidence interval CI of the parameter of the form $\mu =$

†This research is supported from KOSEF 1992.

[1]Department of Mathematics, Chungnam National University, Taejon, 305-764, Korea.

$\lambda_1\mu_x + \lambda_2\mu_y$, where λ_1 and λ_2 are known nonzero real numbers, satisfying the two requirements. The first requirement is the coverage probability condition, i.e. for any given α $(0 < \alpha < 1)$

$$P_\theta\{\mu \in \mathrm{CI}\} \geq 1 - \alpha \qquad {}^\forall\theta \in \Theta \tag{1.1}$$

The second requirement concerns the precision of the confidence interval, which can be specified in various ways. For example, fixed-width confidence intervals(e.g. Chow and Robbins(1965), Starr(1966), Robbins, Simons and Starr(1967), Eisele(1990)) or minimum risk function(e.g. Wolfowitz(1950), Ghosh and Mukhopadhyay(1980)).

In this paper we adopt the "β-protection" as a measure of precision proposed by Wijsman(1981), i.e. for given a constant $d > 0$ and a probability β $(0 < \beta < 1)$

$$P_\theta(\mu - d \in \mathrm{CI}) \leq \beta \qquad {}^\forall\theta \in \Theta \tag{1.2}$$

For the β-protection problems, Wijsman(1982, 1983) studied the mean with known variance σ^2 and μ/σ for unknown σ^2. Juhlin(1985) studied the mean of scale parameter in the exponential distribution and Wijsman(1986) studied the 1-parameter problem in a rather setting, and was generalized to vector valued parameter by Fakhre-Zakeri(1989). Kim(1990) studied the mean in the presence of nuisance parameters.

A confidence interval CI for the linear parameteric function μ which satisfies (1.1) and (1.2) leads to the form $\mathrm{CI} = [L(X_1, X_2, \cdots, X_{N_x}, Y_1, Y_2, \cdots, Y_{N_y}), \infty)$ in the one-sided where L is some measurable function of the stopped sequences of random variables when employing stopping rule $N = (N_x,\ N_y)$. In contrast a two-sided situation arises if (1.2) is replaced ley $P_\theta(\mu - d \in \mathrm{CI}$ or $\mu + d \in \mathrm{CI}) \leq \beta \quad {}^\forall\theta \in \Theta$. In this case the confidence interval is a form of

$$[L_1(X_1, X_2, \cdots, X_{N_x},\ Y_1, Y_2, \cdots, Y_{N_y}),\ L_2(X_1, X_2, \cdots, X_{N_x},\ Y_1, Y_2, \cdots, Y_{N_y})]$$

where L_1 and L_2 are some measurable functions.

In section 2, we propose sequential procedures with β-protection for the linear function of two means and in section 3, we give some limiting behaviors of the stopping time.

Throughout this paper, let $\Phi(x)$ be the distribution of the standard normal and z_α its upper α point, i.e. $1 - \Phi(z_\alpha) = \alpha$ $(0 < \alpha < 1)$. We use the following notations;

$$\overline{X}_m = \frac{1}{m}\sum_{i=1}^{m} X_i, \quad \overline{Y}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

$$
\begin{aligned}
Z_{x,m} &= \frac{1}{\sigma_x}(\overline{X}_m - \mu_x), \quad Z_{y,n} = \frac{1}{\sigma_y}(\overline{Y}_n - \mu_y) \\
S_{x,m}^2 &= \frac{1}{m-1}\sum_{i=1}^{m}(X_i - \overline{X}_m)^2, \quad S_{y,n}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y}_n)^2, \ m,\ n \geq 2 \\
S_{x,m}^{\prime 2} &= \frac{1}{\sigma_x^2}S_{x,m}^2, \quad S_{y,n}^{\prime 2} = \frac{1}{\sigma_y^2}S_{y,n}^2
\end{aligned}
$$

and define

$$
\begin{aligned}
\alpha(c,\ \theta) &= P_\theta(\mu \notin \mathrm{CI}) \\
\beta(c,\ \theta) &= P_\theta(\mu - d \in \mathrm{CI}).
\end{aligned}
$$

2. CONFIDENCE INTERVAL ESTIMATION FOR μ.

We wish to construct a confidence interval for $\mu = \lambda_1\mu_x + \lambda_2\mu_y$ which satisfies the two requirements (1.1) and (1.2) with unknown parameters. Now there is no loss of generality if we assume that $\lambda_1 = 1$ and $\lambda_2 = -1$, i.e. the difference of two means. In order to arrive a sequential procedure we pretend first that there is a fixed sample size confidence interval CI of the form

$$
\mathrm{CI} = [\overline{X}_m - \overline{Y}_n - \rho d,\ \infty) \tag{2.1}
$$

for m observations on X and n observations on Y, where $\rho (0 < \rho < 1)$ is still to be chosen. Then by two requirements (1.1) and (1.2), for all $\theta \in \Theta$

$$
\begin{aligned}
&P_\theta\{\mu_x - \mu_y \in \mathrm{CI}\} = P_\theta\left\{\left[\overline{X}_m - \overline{Y}_n - (\mu_x - \mu_y)\right] / \left(\tfrac{\sigma_x^2}{m} + \tfrac{\sigma_y^2}{n}\right)^{\frac{1}{2}}\right. \\
&\left.\leq \rho d / \left(\tfrac{\sigma_x^2}{m} + \tfrac{\sigma_y^2}{n}\right)^{\frac{1}{2}}\right\} \geq 1 - \alpha
\end{aligned} \tag{2.2}
$$

and

$$
\begin{aligned}
&P_\theta\{\mu_x - \mu_y - d \in \mathrm{CI}\} = P_\theta\left\{\left[\overline{X}_m - \overline{Y}_n - (\mu_x - \mu_y)\right] / \left(\tfrac{\sigma_x^2}{m} + \tfrac{\sigma_y^2}{n}\right)^{\frac{1}{2}}\right. \\
&\left.\leq -(1-\rho) d / \left(\tfrac{\sigma_x^2}{m} + \tfrac{\sigma_y^2}{n}\right)^{\frac{1}{2}}\right\} \leq \beta
\end{aligned} \tag{2.3}
$$

If σ_x and σ_y were known, then we get

$$\frac{\sigma_x^2}{m} + \frac{\sigma_y^2}{n} \le \Big(d/(z_\alpha + z_\beta)\Big)^2 \tag{2.4}$$

Regarding m and n as continous variables, then the pair $(m^*,\ n^*)$ which satisfies (2.4) and for which the total observation $T = m + n$ is a minimum is given by

$$m^* = \sigma_x(\sigma_x + \sigma_y) \cdot \Big[(z_\alpha + z_\beta)/d\Big]^2, \quad n^* = \sigma_y(\sigma_x + \sigma_y) \cdot \Big[(z_\alpha + z_\beta)/d\Big]^2 \tag{2.5}$$

and the total sample size is

$$T^* = m^* + n^* = \Big[(\sigma_x + \sigma_y)(z_\alpha + z_\beta)/d\Big]^2 \tag{2.6}$$

When both σ_x and σ_y are unknown there is no fixed sample size procedure which satisfies the two requirements (1.1) and (1.2). So we shall give a sequential procedure which consists of

(1) A sampling scheme which tells us at stage whether to take the next observation of X or Y

(2) A stopping rule which determines observations m and n and therefore terminal decision rule as confidence interval CI

Now let $S_{x,m}^2$ and $S_{y,n}^2$ be the estimates of σ_x^2 and σ_y^2 respectively. For the scheme for sampling we take n_0 ($n_0 \ge 2$) observations on X and on Y to begin with. Then if at any stage we have taken m observations on X and n observations on Y. We take the next additional observation

(a) on X if $m/n \le S_{x,m}^2/S_{y,n}^2$

(b) on Y if $m/n > S_{x,m}^2/S_{y,n}^2$

This procedure generates an infinite sequence of observations on X and Y and does not depend on the value of $\alpha,\ \beta,\ d$ or ρ.

For the stopping rule we can give four more or less equivalent stopping rules (refer Eisele(1990)) but we adopt one of the four as follows.

Stop with the first $m,\ n$ ($m,\ n \ge n_0$) such that if N_x observations on X's and N_y observations on Y's have been taken with

$$\begin{aligned} N_x &= N_x(c) = \inf\{m : m \ge c^2 S_{x,m}(S_{x,m} + S_{y,n})\} \\ N_y &= N_y(c) = \inf\{n : n \ge c^2 S_{y,n}(S_{x,m} + S_{y,n})\} \\ N &= N_x + N_y \end{aligned} \tag{2.7}$$

in which the constant $c > 0$ is still to be chosen. Using the above stopping rule (2.7), we propose the confidence interval CI for the $\mu = \mu_x - \mu_y$ as

$$CI = [\overline{X}_{N_x} - \overline{Y}_{N_y} - \rho d,\ \infty) \tag{2.8}$$

in which $0 < \rho < 1$ is still to be chosen. From our sequential procedure we obtain the result $N_x/N_y \to \sigma_x/\sigma_y$ a.s. as $N = N_x + N_y \to \infty$ by Robbins, Simons and Starr(1967). The next theorem shows that our sequential procedure satisfies the two requirements (1.1) and (1.2).

Theorem 2.1. For any given α, β, ρ $(0 < \alpha,\ \beta,\ \rho < 1)$ there exists $c_0 > 0$ such that for $c > c_0$ our sequential procedure satisfies (1.1) and (1.2).

Before proving the theorem, we need Lemmas.

Lemma 2.2. For arbitrary given $\sigma_0 > 0$

(a) $N_x \to \infty$ a.s. as $c \to \infty$ uniformly in $\Theta_1 = \{\theta \in \Theta;\ \sigma_x > \sigma_0 \text{ or } \sigma_x \cdot \sigma_y > \sigma_0\}$

(b) $N_y \to \infty$ a.s. as $c \to \infty$ uniformly in $\Theta_2 = \{\theta \in \Theta;\ \sigma_y > \sigma_0 \text{ or } \sigma_x \cdot \sigma_y > \sigma_0\}$

Proof. Define a new stopping time N_x' as follows;

$$N_x' = N_x'(c) = \inf\{m \geq n_0 :\ m \geq c^2 \min(S_{x,m}^2,\ S_{x,m} \cdot S_{y,n})\}$$

Then we see that $N_x(c) \geq N_x'(c)$ a.s. and $N_x'(c) \to \infty$ a.s. as $c \to \infty$ uniformly in Θ_1 by Chow and Robbins(1965). Similar argument proves (b).

Lemma 2.3. For arbitrary given $\sigma_0 > 0$, both $N_x c^{-2}(\sigma_x(\sigma_x + \sigma_y))^{-1}$ and $N_y c^{-2}(\sigma_y(\sigma_x + \sigma_y))^{-1}$ go to 1 a.s. as $c \to \infty$ uniformly in

$$\Theta_3 = \{\theta \in \Theta;\ \min(\sigma_x,\ \sigma_y) > \sigma_0 \text{ or } \sigma_x\sigma_y > \sigma_0\}$$

Proof. By definition (2.7) of N_x we have the double inequalities

$$\frac{S_{x,N_x}'(\sigma_x S_{x,N_x}' + \sigma_y S_{y,N_x}')}{\sigma_x + \sigma_y} \leq \frac{N_x c^{-2}}{\sigma_x(\sigma_x + \sigma_y)} < \frac{1}{c^2 \sigma_x(\sigma_x + \sigma_y)} + \frac{S_{x,N_x-1}'(\sigma_x S_{x,N_x-1}' + \sigma_y S_{y,N_x-1}')}{\sigma_x + \sigma_y}$$

By Lemma 2.2 and SLLN, S_{x,N_x}', S_{x,N_x-1}', S_{y,N_y}' and S_{y,N_y-1}' go to 1 a.s. as $c \to \infty$ uniformly in Θ_3 and as $c \to \infty$ $c^2\sigma_x(\sigma_x + \sigma_y) \to \infty$ uniformly in Θ_3. Therefore $N_x c^{-2}(\sigma_x(\sigma_x + \sigma_y))^{-1} \to 1$ a.s. as $c \to \infty$ uniformly in Θ_3. $N_y c^{-2}(\sigma_y(\sigma_x + \sigma_y))^{-1} \to 1$ a.s. as $c \to \infty$ uniformly in Θ_3 by the same argument.

Proof Theorem 2.1.

$$\alpha(c,\ \theta) \ = \ P_\theta(\mu_x - \mu_y \notin CI) = P_\theta\{\overline{X}_{N_x} - \mu_x - (\overline{Y}_{N_y} - \mu_y) \geq \rho d\}$$

$$
\begin{aligned}
&\leq P_\theta\left\{\overline{X}_{N_x}-\mu_x \geq \frac{1}{2}\rho d\right\}+P_\theta\left\{\overline{Y}_{N_y}-\mu_y \leq -\frac{1}{2}\rho d\right\} \\
\beta(c,\ \theta) &= P_\theta\{\mu_x-\mu_y-d \in CI\} = P_\theta\{\overline{X}_{N_x}-\mu_x-(\overline{Y}_{N_y}-\mu_y) \leq -(1-\rho)d\} \\
&\leq P_\theta\left\{\overline{X}_{N_x}-\mu_x \leq -\frac{1}{2}(1-\rho)d\right\}+P_\theta\left\{\overline{Y}_{N_y}-\mu_y \geq \frac{1}{2}(1-\rho)d\right\}
\end{aligned}
$$

Let $t_0=\min\left\{\frac{1}{2}\rho d,\ \frac{1}{2}(1-\rho)d\right\}$, then

$$
\begin{aligned}
\alpha(c,\ \theta) &\leq P_\theta\{\overline{X}_{N_x}-\mu_x \geq t_0\}+P_\theta\{\overline{Y}_{N_y}-\mu_y \leq -t_0\} \\
\beta(c,\ \theta) &\leq P_\theta\{\overline{X}_{N_x}-\mu_x \leq -t_0\}+P_\theta\{\overline{Y}_{N_y}-\mu_y \geq t_0\}
\end{aligned}
$$

Since $\overline{Z}_{x,m}$ and $\overline{Z}_{y,n}$ go to zero almost surely uniformly in Θ as m and n go to ∞. So there is a constant $a>0$ such that for any given $\varepsilon>0$ $P_\theta\{|\overline{Z}_{x,m}|<a,\ m=1,\ 2,\ \cdots\}>1-\varepsilon/2$ and $P_\theta\{|\overline{Z}_{y,n}|<a,\ n=1,\ 2,\ \cdots\}>1-\varepsilon/2$ for all θ. Hence both $P_\theta\{|\overline{Z}_{x,N_x}|\geq a\}$ and $P_\theta\{|\overline{Z}_{y,N_y}|\geq a\}$ are less than $\varepsilon/2$ no matter what the stopping time N_x and N_y are. If we choose $\sigma_0=t_0/a$, then both $\alpha(c,\ \theta)$ and $\beta(c,\ \theta)$ are less than ε uniformly in θ if $\sigma_x<\sigma_0$ and $\sigma_y<\sigma_0$ as $c\to\infty$.

For $\sigma_x\geq\sigma_0$ and $\sigma_y\geq\sigma_0$

$$
\begin{aligned}
\alpha(c,\ \theta) \leq\ & P_\theta\left\{\sqrt{N_x}\overline{Z}_{x,N_x} \geq t_0\cdot c\left[S'_{x,N_x}\left(S'_{x,N_x}+\frac{\sigma_y}{\sigma_x}\cdot S'_{y,N_y}\right)\right]^{\frac{1}{2}}\right\} \\
&+P_\theta\left\{\sqrt{N_y}\overline{Z}_{y,N_y} \geq t_0\cdot c\left[S'_{y,N_y}\left(S'_{y,N_y}+\frac{\sigma_x}{\sigma_y}\cdot S'_{x,N_x}\right)\right]^{\frac{1}{2}}\right\} \to 0
\end{aligned}
$$

as $c\to\infty$ by Anscombe's theorem, SLLN and Lemmas 2.2, 2.3.

For $\sigma_x<\sigma_0$ and $\sigma_y\geq\sigma_0$

$$
\begin{aligned}
\alpha(c,\ \theta) \leq\ & P_\theta\{|\overline{Z}_{x,N_x}|\geq t_0/\sigma_x\} \\
&+P_\theta\left\{\sqrt{N_y}\ \overline{Z}_{y,N_y} \geq t_0\cdot c\left[S'_{y,N_y}\left(S'_{y,N_y}+\frac{\sigma_x}{\sigma_y}S'_{x,N_x}\right)\right]^{\frac{1}{2}}\right\}
\end{aligned}
$$

The first term of the right-hand side of above is less than ε since $t_0/\sigma_x\geq a$ and the second term go to zero as $c\to\infty$ by Anscombe's theorem, SLLN and Lemmas 2.2, 2.3 again.

For $\sigma_x\geq\sigma_0$, $\sigma_y<\sigma_0$, the similar argument proves $\alpha(c,\ \theta)\leq\varepsilon$.

Similar argument proves $\beta(c,\ \theta)\to 0$ as $c\to\infty$ uniformly in Θ.

Theorem 2.4. For any given $c>0$ as $\sigma_x\sigma_y\to\infty$

$$\alpha(c,\ \theta)\to 1-\Phi(c\rho d) \quad \text{and} \quad \beta(c,\ \theta)\to 1-\Phi(c(1-\rho)d)$$

Proof. We rewrite $\alpha(c,\ \theta)$ and $\beta(c,\ \theta)$ as follows

$$\begin{aligned}\alpha(c,\ \theta) &= P_\theta\{(\overline{X}_{N_x}-\mu_x-\overline{Y}_{N_y}+\mu_y)/(\sigma_x^2/N_x+\sigma_y^2/N_y)^{\frac{1}{2}} \\ &\geq \rho d/(\sigma_x^2/N_x+\sigma_y^2/N_y)^{\frac{1}{2}}\} \end{aligned} \quad (2.9)$$

and

$$\begin{aligned}\beta(c,\ \theta) &= P_\theta\{(\overline{X}_{N_x}-\mu_x-\overline{Y}_{N_y}+\mu_y)/(\sigma_x^2/N_x+\sigma_y^2/N_y)^{\frac{1}{2}} \\ &\leq -(1-\rho)d/(\sigma_x^2/N_x+\sigma_y^2/N_y)^{\frac{1}{2}}\} \end{aligned} \quad (2.10)$$

From (2.7) of definition N_x and N_y, N_x and N_y go to ∞ a.s. as $\sigma_x\sigma_y \to \infty$ for any fixed $c > 0$. Therefore $\sigma_y N_y/\sigma_x N_x \to 1$ a.s. as $\sigma_x\sigma_y \to \infty$. By Anscombe's Theorem $(\overline{X}_{N_x}-\mu_x-\overline{Y}_{N_y}+\mu_y)/(\sigma_x^2/N_x+\sigma_y^2/N_y)^{\frac{1}{2}} \xrightarrow{L} N(0,\ 1)$ as $\sigma_x\sigma_y \to \infty$ for any fixed $c > 0$ and we can easily show that $\sigma_x^2/N_x+\sigma_y^2/N_y \to c^{-2}$ a.s. as $\sigma_x\sigma_y \to \infty$ for any fixed $c > 0$.

Therefore the right-hand sides of (2.9) and (2.10) converge to $1-\Phi(c\rho d)$ and $1-\Phi(c(1-\rho)d)$.

3. ASYMPTOTIC BEHAVIORS OF THE STOPPING TIME

In this section we show that asymptotic expectations of N_x and N_y as $c \to \infty$ and asymptotic normality of $N = N_x + N_y$ after suitable centering and scaling as $c \to \infty$.

Theorem 3.1. As $c \to \infty$, $E_\theta\ N_x/c^2\sigma_x(\sigma_x+\sigma_y) \to 1$ and $E_\theta\ N_y/c^2\sigma_y(\sigma_x+\sigma_y) \to 1$.

It is easily proved along the lines of Robbins, Simons and Starr(1967) and thus it is omitted.

Theorem 3.2. $(N-N_0)/(2N_0)^{\frac{1}{2}} \xrightarrow{L} N(0,\ 1)$ as $c \to \infty$ where $N = N_x + N_y$ and $N_0 = c^2(\sigma_x+\sigma_y)^2$.

Proof. By a straightforward two-sample extension of Anscombe's Theorem

$$N^{\frac{1}{2}}(S_{x,N_x}^2-\sigma_x^2,\ S_{y,N_y}^2-\sigma_y^2) \xrightarrow{L} N_2(0,\ \Sigma) \quad \text{as} \quad c \to \infty \quad (3.1)$$

where $\Sigma = \begin{pmatrix} 2\sigma_x^3(\sigma_x+\sigma_y) & 0 \\ 0 & 2\sigma_y^3(\sigma_x+\sigma_y) \end{pmatrix}$

Since $N/N_0 \to 1$ a.s. as $c \to \infty$, (3.1) reduces to

$$N_0^{\frac{1}{2}}(S_{x,N_x}-\sigma_x,\ S_{y,N_y}-\sigma_y) \xrightarrow{L} N_2(0,\ \Sigma) \text{ as } c \to \infty \quad (3.2)$$

where $\Sigma = \begin{pmatrix} \frac{1}{2}\sigma_x(\sigma_x+\sigma_y) & 0 \\ 0 & \frac{1}{2}\sigma_y(\sigma_x+\sigma_y) \end{pmatrix}$

Applying the Taylor theorem to (3.2), then

$$N_0^{\frac{1}{2}}[(S_{x,N_x}+S_{y,N_y})^2-(\sigma_x+\sigma_y)^2] \xrightarrow{L} N(0,\ 2(\sigma_x+\sigma_y)^4) \text{ as } c\to\infty \quad (3.3)$$

A result analogous to (3.3) will hold with S_{x,N_x-1} and S_{y,N_y-1}. Now $N_x \geq c^2 S_{x,N_x}(S_{x,N_x}+S_{y,N_y})$ and $N_y \geq c^2 S_{y,N_y}(S_{x,N_x}+S_{y,N_y})$ a.s.

$$P\{(N-N_0)/(2N_0)^{\frac{1}{2}} \leq x\} \leq P\left\{\frac{N_0^{\frac{1}{2}}[(S_{x,N_x}+S_{y,N_y})^2-(\sigma_x+\sigma_y)^2]}{\sqrt{2}(\sigma_x+\sigma_y)^2} \leq x\right\}$$

$$\limsup_{c\to\infty} P\{(N-N_0)/(2N_0)^{\frac{1}{2}} \leq x\} \leq \Phi(x) \quad (3.4)$$

Again it is easy to obtain

$$N_x \leq c^2 S_{x,N_x-1}(S_{x,N_x-1}+S_{y,N_y-1})+n_0 \text{ a.s.}$$

and

$$N_y \leq c^2 S_{y,N_y-1}(S_{x,N_x-1}+S_{y,N_y-1})+n_0 \text{ a.s.}$$

so that

$$P\{(N-N_0)/(2N_0)^{\frac{1}{2}} \leq x\} \geq P\Big\{N_0^{\frac{1}{2}}[(S_{x,N_x-1}+S_{y,N_y-1})^2-(\sigma_x+\sigma_y)^2]/$$

$$\sqrt{2}(\sigma_x+\sigma_y)^2+\frac{\sqrt{2}n_0}{c(\sigma_x+\sigma_y)} \leq x\Big\}$$

$$\liminf_{c\to\infty} P\{(N-N_0)/(2N_0)^{\frac{1}{2}} \leq x\} \geq \Phi(x) \quad (3.5)$$

Combining (3.4) and (3.5), we get the theorem.

REFERENCES

(1) Anscombe, F. J. (1952). Large sample theory of sequential estimation, *Proc. Cambridge Philos. Soc.*, **48**, 600-607.

(2) Chow, Y. S. and Robbins, H. (1965). On the asymptotic theory of fixed width confidence intervals for the means, *The Annals of Mathematical Statistics*, **36**, 456-462.

(3) Eisele, J. R. (1990). An adaptive biased coin design for the Behrens-Fisher problem, *Sequential Analysis*, **9**(4), 343-359.

(4) Fahker-Zakeri (1989). Sequential confidence sets with beta-protection in multiparameter families, *Sequential Analysis*, **8**, 1-26.

(5) Ghosh, M., and Mukhopadhyay, N. (1980). Sequential point estimation of the difference of two normal mean, *The Annals of Statistics*, **8**(1), 221-225.

(6) Juhlin, K. D. (1985). *Sequential and non-sequential intervals with guaranteed coverage probability and beta protection*, ph.D. Dissertation, Univ. of Illinois.

(7) Kim, S. L. (1990). Sequential confidence intervals for the mean with β-protection in a certain parameter space, *Journal of Korean Statistical Society*, **19**(2), 113-121.

(8) Robbins, H., Simons, G., and Starr, N. (1967). Sequential analogue of the Behrens-Fisher problem, *The Annals of Mathematical Statistics*, **38**, 1384-1391.

(9) Starr, N. (1966). The performance of a sequential procedure for the fixed-width interval estimation of the mean, *The Annals of Mathematical Statistics*, **37**, 36-50.

(10) Wijsman, R. A. (1981). Confidence sets based on sequential test, *Communications in Statistics-Theory and Method A*, **10**, 2137-2147.

(11) —— (1982). Sequential confidence sets; Estimation-oriented versus Test-oriented construction, *Statistical Decision Theory and Related Topics III*, **2**, 435-450.

(12) —— (1983). Sequential estimation and test-oriented confidence intervals for μ/σ in a normal population, *Sequential Analysis*, **2**, 149-174.

(13) —— (1986). Sequential confidence interval with beta-protection in one-parameter families, *Adaptive Stat. Proc. and Related Topics*, 96-107.

(14) Wolfowitz, J. (1950). Minimax estimates of the mean of a normal distribution with unknown variance, *The Annals of Mathematical Statistics*, **21**, 218-230.