철근 콘크리트 구조물의 비선형 동적 해석을 위한 성치 측정에 의한 예측 접근법

Prediction Approach with a Stiffness Measure in Nonlinear Dynamic Analysis of Reinforced Concrete Structures

김 교 진*
Kim, Kyo Jin

전 경 훈**
Chon, Kyung Hoon

국문요약

최근의 철근 콘크리트 구조물의 내진 설계 방식은 비탄성 거대 변형에 의한 에너지 방출에 의존하고 있다. 이러한 구조물의 거동에 대한 비선형 동적 해석은 특히 계산이 여러 번 반복되어 질 때 많은 시간과 비용이 요구된다. 그러므로 효율적이고 한편 정확한 계산 방법의 채택이 중요하게 되었다. 예측 접근방법(PASM) 이라 불리는 새로운 방법을 제시하는 것이 현 연구의 주목적이다. 일반적인 동적 해석 방법에서는 매 시간 단계 혹은 반복 계산 때마다 수식 계산을 위하여 메트릭스 삼각 분해가 요구되어지나, 예측 접근방법에서는 구조물이 정적 반복하중으로 비선형 범위로 변형되어졌을 때의 강성 상태에서 미리 얻어진 한정적 수의 분해된 메트릭스를 동적 해석에서 이용하게 된다. 이곳에서 제시될 접근 방법은 강성치를 매 시각 단계 혹은 반복 계산 단계마다 재산출해야 하는 다른 접근 방법들과 비교할 때 전체적 수치 해석 양을 줄이게 될 것이다.

주요어 : 지진 공학, 비선형 동적 해석, 철근 콘크리트 구조물, 시각 이력 해석, 강성치 측정

ABSTRACT

Current seismic design philosophy for reinforced concrete (RC) structures relies on energy dissipation through large inelastic deformations. A nonlinear dynamic analysis which is used to represent this behavior is time consuming and expensive, particularly if the computations have to be repeated many times. Therefore, the selection of an efficient yet accurate algorithm becomes important. The main objective of the present study is to propose a new technique herein called the prediction approach with stiffness measure (PASM) method. In the conventional direct integration methods, the triangular decomposition of matrix is required for solving equations of motion in every time step or every iteration. The PASM method uses a limited number of predetermined decomposed effective matrices obtained from stiffness states of the structure when it is deformed into the nonlinear range by statically applied cyclic loading. The method to be developed herein will reduce the overall numerical effort when compared to approaches which recompute the stiffness in each time step or iteration.

Key words : *earthquake engineering, nonlinear dynamic analysis, reinforced concrete structure, time history analysis, stiffness measure.*

1. Introduction

The amount of research being done and the body of literature extant on nonlinear dynamic analyses are large. According to the coordinate system for expressing the equations of motion, the solution procedures may be divided into two categories; direct and indirect integration methods. In direct integration methods, the equations of motion are expressed in real coordinates. Whereas the equations of motion are expressed in normal or alternatively in Ritz vector coordinates in indirect methods Nonlinear dynamic response of a structure is usually performed by means of a direct integration of the

* 정회원 · 미국 UCLA Post. Doc.,
** 대불대학교 건축공학과 조교수

equations of motion expressed in the real coordinates. Direct integration methods are well covered in the literature.[(1)~(7)]

In the direct method, the evaluation of the equation of motion in real coordinates, involves the triangular factorization or decomposition and the backsubstitution of the effective mass matrix in each time step, that will require a significant computational effort when a large and widely banded system is analyzed.

Let us consider the incremental equilibrium equation[(3)] expressed at time $t+\Delta t$.

$$[M]\{\Delta \ddot{v}(t+\Delta t)\}+[C]\{\Delta \dot{v}(t+\Delta t)\}+[K(t)]\{\Delta v(t+\Delta t)\}=\{F(t+\Delta t)\}-[M]\{\ddot{v}(t)\}-[C]\{\dot{v}(t)\}-\{R(t)\} \quad (1)$$

in which, $[M]$= mass matrix; $[C]$= damping matrix; $\{F(t)\}$= external loading vector; $\{R(t)\}$= restoring force vector corresponding to the displacement of $\{v(t)\}$; $\{\ddot{v}(t)\}$, $\{\dot{v}(t)\}$ and $\{v(t)\}$ are relative acceleration, velocity and displacement vectors at time t respectively; and $\{\Delta \ddot{v}(t+\Delta t)\}$, $\{\Delta \dot{v}(t+\Delta t)\}$ and $\{\Delta v(t+\Delta t)\}$, are relative incremental acceleration, incremental velocity and incremental displacement vectors at time $t+\Delta t$ respectively.

With the Newmarks constant average acceleration method for integration, Eq. (1) has the form

$$[\tilde{K}(t)]\{\Delta v(t+\Delta t)\}=\{F(t+\Delta t)\}+[M]\left(\frac{4}{\Delta t}\{\dot{v}(t)\}+\{\ddot{v}(t)\}\right)+[C]\{\dot{v}(t)\}-\{R(t)\} \quad (2)$$

where, $[\tilde{K}(t)]$ is the effective stiffness matrix.

$$[\tilde{K}(t)]=\frac{4}{\Delta t^2}[M]+\frac{2}{\Delta t}[C]+[K(t)] \quad (3)$$

The equation for the increments of velocity and acceleration are obtained in terms of initial conditions. The evaluation of the Eq. (2) involves the triangular factorization or decomposition of the effective stiffness matrix, $[\tilde{K}(t)]$, as follow[(8)]

$$[\tilde{K}(t)]=[\tilde{L}][\tilde{L}]^T \quad (4)$$

where $[\tilde{L}]$is the lower triangular matrix of $[\tilde{K}(t)]$ with the Cholesky factorization method. For the band symmetric matrix of, the number of scalar multiplications is $nb^2/2$ for triangular decomposition of the matrix and $2nb$ for the forward reduction of load vector and the back substitution of a triangular matrix.[(8)] If we can reduce the number of the triangular decomposition during the simulation, the efficiency of the dynamic analysis can be improved because the significant part of the computational effort is in performing triangular decomposition of the matrix.

The tangent stiffnesses of the hinge regions operating in nonlinear states can be linked directly to the overall stiffness of the structure. In other words, the nonlinear behavior of these structures is completely controlled by the behavior of the hinge regions. The overall stiffness can then be tied to the generated information, i.e., the decomposed matrix, from a certain stiffness state. As simulation time goes by, the nonlinear states of hinge stiffness undergo changes and therefore a different set of new generated information required to solve the equations of motion is needed each time the stiffness changes. The time varying EI for each hinge can be readily determined from the 4-branch relationship[(9)] given in Fig. 1 because the curvature history of each hinge zone is known.

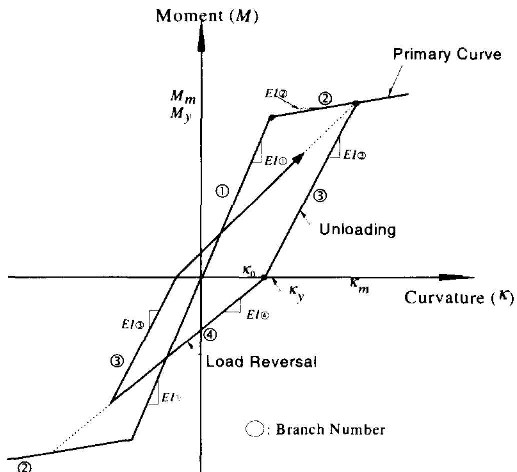

Fig. 1 Q-Hyst. Model (After Saiidi and Sozen[(9)])

Since there is the unique relationship between the stiffnesses in hinge zones and the overall stiffness of the structure, the unique relationship between the stiffnesses in hinge zones and the effective stiffness matrix exists. Thus, it is possible to store and reuse the previously decomposed matrix. In other words, if stiffness changes during the dynamic analysis, equation solving only requires backsubstitution of the decomposed matrix of the most appropriate set among the stored sets.

In view of the computational effort of the direct integration methods, the application of the modal superposition methods to nonlinear analysis has been an attractive idea, mainly because of the ability of the method to give fairly accurate solutions with only a few modes. The modal superposition technique has already been applied in nonlinear structural dynamic analysis. In the eigenvalue method[(10)-(16)], the eigenvalues and eigenvectors of the structure are updated at every instant. This method can require heavy effort because of repeated calculation of eigenvectors. This recomputation of eigenvectors is one of the major disadvantages of the eigenvalue method. Chang[(17)] proposed a new approach that uses a limited number of sets of predetermined eigenvalues and eigenvectors from the nonlinear states of a structure in the cyclic incremental static analysis. However, it is difficult to predict the correct mode shapes of complex structures by cyclic incremental static analyses, because the response cannot be known in advance. This problem becomes more difficult as the ground acceleration record becomes stronger. Chang presented results which show a discrepancy between the direct integration method and the approximate eigenvalue method. Assuming that the solution from the direct integration method is exact, the error in the approximate eigenvalue method is due to the error caused when incorrect truncated mode shapes are used for the transformation of the nodal response to the real response. In order to have truncated mode shapes available for the stiffness state representing the stiffness states for highly nonlinear deformation, a static loading used in the deformed shape analysis should resemble the incremental inertia force shape, which is almost impossible to anticipate.

Unlike the approximate eigenvalue, i.e., Changs, method, the proposed method performs a dynamic analysis in the real coordinates system, hence, illuminates the incomplete problems of indirect methods.

2. Analysis Algorithm

The analysis procedure is divided into two phases; 1) the static phase, and 2) the dynamic phase. In the static phase, the decomposed effective stiffness matrixes are calculated based on the predicted stiffness states by incremental static analysis and thereafter in the dynamic phase, the predetermined decomposed effective stiffness matrixes in the static phase are utilized in calculating the response of the structures. Since the information necessary to perform the dynamic analysis is predicted in static phase, the proposed method will be called a Prediction Approach with a Stiffness Measure (PASM) method. The computation procedure in each phase is summarized below;

Static Phase (Prediction Phase)

1. Apply static load incrementally
2. Calculate decomposed effective stiffness matrix for new stiffness state.
3. Store stiffnesses of hinge zones and corresponding decomposed effective stiffness matrix. This combined set wil be referred as hinge zone stiffness (HZS) set.

Dynamic Phase

1. Solve dynamic equations in real coordinates, i.e., Eq. (2).
2. If stiffnesses change, select a HZS set predetermined in the static phase.
3. Continue response calculation with the selected HZS set.

2.1 Static Phase

Structural behavior to an earthquake is generally in a cyclic fashion. The nonlinear behavior of the structure to cyclic load or deformation reversals can be determined through a incremental step-by-step procedure with the, tangent stiffness iteration method.

$$[K]_t\{\delta v_n\}^i = \{\Delta P_n\}-\{\Delta R_n\}^i \quad (5)$$

and,

$$\{v_n\}^i = \{v_{n-1}\} + \sum_{j=1}^{i}\{\delta v_n\}^j \quad (6)$$

where, subscript n=loading step number; superscript i=iteration number; $[K]_t$=tangent stiffness matrix; $\{\Delta P_n\}$=incremental static load vector, $\{v_n\}^i$=total displacement of loading step n after i-th iteration; $\{\delta v_n\}^i$=incremental displacement vector for the i-th iteration; and $\{\Delta R_n\}^i$ = incremental restoring force.

To produce sets of stiffness states that will closely represent the stiffness states in dynamic analysis, the pattern of static load vector is the critical factor of this method. The static load vector $\{\Delta P\}_n$ used in Eq. (5) is expressed as

$$\{\Delta P_n\} = \Delta L(n)\{S\} \quad (7)$$

where, $\Delta L(n)$ = Static Load History (incremental load scalar for load increment n); and $\{S\}$ = Static Load Shape.

In order to find out whether load increment $\{\Delta P_n\}$ causes changes in the stiffness of the structure, the curvature increment in moment-curvature relation at the plastic hinge zones of each element needs to be determined. When changes in the stiffness occurs after a loading step, triangular decomposition of effective stiffness matrix (Eq. (4)) is performed for each new stiffness set. The calculated decomposed matrix is stored with corresponding stiffnesses of hinge zones in order to be used during the dynamic analysis.

The objective of doing static analysis is to predetermine the HZS sets for the dynamic analysis. Thus, the following requirements for the static load analysis should be satisfied.

Requirement 1. Maximum displacement of static history must agree with dynamic; when initial static histories disagree with the dynamic result they are revised in each trial so that the static maximum agrees with the approximate dynamic result.

Requirement 2. Numbers of static loop; Loops which in three or more incremental steps reach the maximum excursion must be used in order to accommodate the possible nonlinear state before the maximum dynamic occurs.

As a static load history, $\Delta L(n)$, a cyclic static loading procedure is employed in which each cycle will allow more members to yield until the desired ductility is reached. Recommend to reach μ (critical story ductility) = 4. The critical story, i.e., the story at which the most nonlinearity occurs, can be predetermined by applying one directional static load until the failure occurs, up to a specific ductility level. The yield drift is found from the static phase of analysis by observing the branch number of the Saiidi Rule Fig. 1 of the members of which a story is composed. When any one member reaches the branch #2 of Saiidi Rule, i.e., the plastic region, for the first time, the story drift at that point is determined as the yield drift for that story. The story yield drift could be defined as the drift after which the stiffness of that story is starting to decrease (not the stiffness of the whole frame).

2.2 Dynamic Phase

The only difference in the dynamic solution procedure between this proposed method and the conventional step-by-step dynamic analysis methods, is the equation solving. The triangular decompositions of effective stiffness matrixes are not required to be performed during the solution procedure by using the best available decomposed effective stiffness matrix of the best suitable HZS set among the predetermined sets. The selection of the most appropriate HZS set at each new stiffness states of the structure while performing dynamic analysis can be achieved by the use of the stiffness ratio technique[17][18] In order to select a proper HZS set among the stored sets, member stiffnesses of stored HZS sets are compared to the current member stiffnesses using the relation

$$SM^k = \frac{1}{N}\sum_{i=1}^{N}\frac{MAX({}_{dyn}(EI)_i^t\ ,\ {}_{sto}(EI)_i^k)}{MIN({}_{dyn}(EI)_i^t\ ,\ {}_{sto}(EI)_i^k)} \qquad (8)$$

where, SM^k is a stiffness measure for HZS set *k; N* is total number of hinge zones; ${}_{dyn}(EI)_i^t$ = the dynamic *EI* of hinge zone *i* at time *t;* and ${}_{sto}(EI)_i^k$ = the stored *EI* of hinge zone i for stored HZS set *k* obtained from deformed shape analyses. The dynamic stiffness state is indicated by ${}_{dyn}(EI)_i^t$, i.e., the slope of the moment curvature relation for each hinge zones in the structure. Since the larger *EI* value in Eq. (8) is in the numerator and the smaller *EI* value is in the denominator, the ratio will exceed unity except when the stored and dynamic *EIs* are equal. Thus, SM^k represents the average value of the stiffness ratios of the hinge zones. The SM value will always be equal to or greater than unity. When the SM value is closer to unity, the stiffness state of the HZS set is closer to the dynamic stiffness state. The HZS set with the smallest SM value corresponds to the best available set among the stored HZS sets, i.e.,

$$SM_{MIN} = MIN(SM^1, SM^2, \cdots, SM^M) \qquad (9)$$

where, SM_{MIN} is the smallest SM value and *M* is a total number of stored HZS sets.

According to the moment-curvature relation, e.g., Q-Hyst Model[9] Fig. 1, the stiffnesses of hinge zones can be traced for displacement, $\{v(t+\Delta t)\}$. As the stiffness changes, the SM values (Eq. (8)) between the current stiffness state and stiffness states of the stored decomposed matrix sets are evaluated. The decomposed effective mass matrix of the HZS set with SMMIN will be chosen and be used for the next time step. In other words, if stiffness changes during the dynamic analysis, equation solving only requires backsubstitution of the decomposed matrix of the most appropriate HZS set among the stored sets.

Depending on the nonlinearities in the system and the length of the time step, Δt, the explicit methods may introduce serious errors.[19]
Equilibrium errors will occur in solving the equations of motion. These errors will accumulate from step to step. This will result in inaccurate dynamic analysis because response calculation at one time step is always dependent on the previous response history. The accumulated errors can be diminished by solving the equations of motion with iteration.[20] Since this proposed stiffness measure approach uses the precalculated decomposed matrix based on the stiffness states caused by static loading, the equilibrium iteration is necessary to calculate the correct dynamic response of a structure. The response from the explicit formulation, Eq. (1), can be a first guess of the iteration at each time step. The equations of motion in real coordinates for iteration are given as;

$$\left[\tilde{K}(t+\Delta t)\right]^i\{\delta\ddot{v}\}^i = \left\{\hat{R}(t+\Delta t)\right\}^{i-1} \qquad i>1 \qquad (10)$$

Where,

$$\left\{\hat{R}(t+\Delta t)\right\}^i = \{F(t+\Delta t)\} - [M]\{\ddot{v}(t+\Delta t)\}^i - [C]\{\dot{v}(t+\Delta t)\}^i - \{R(t+\Delta t)\}^i \qquad (11)$$

where, $\{\Delta v\}^i = \{v(t+\Delta t)\}^i - \{v(t+\Delta t)\}^{i-1}$; and superscript *i* denotes an iteration number and greater than 1. Eq. (10) can be solved for the incremental acceleration of the *i*-th iteration, $\{\delta v\}^i$. In order to evaluate the restoring force, the displacements in generalized coordinates need to be evaluated by

$$\{v(t+\Delta t)\}^i = \{v(t+\Delta t)\}^{i-1} + \{\delta v\}^i \qquad (12)$$

Then we can calculate the restoring force, $\{R(t+\Delta t)\}^i$, corresponding to the real displacements of the i-th iteration, $\{v(t+\Delta t)\}^i$.

As the stiffness changes, the SM value (Eq. (8)) between the current stiffness state and the stiffness state of the previous iteration is calculated. The decomposed effective mass matrix of the HZS set with SMMIN will be chosen and be used for the next iteration. The iteration process should be continued until the convergence tolerance meets the allowable value within a time step. The convergence tolerance used here is the ratio of absolute values of incremental displacements and total displacements,

$$\frac{\|\{\delta\nu\}^i\|}{\|\{\nu(t)\}^i\|} \leq \text{CONV} \tag{13}$$

in which CONV is the maximum allowable value for the convergence. When Eq. (13) is satisfied, continue the response calculation to the next time step. The process may be continued step-by-step until any desired time.

3. Numerical Example

The present research is not directly concerned with direct integration method except as a means of comparison for solution results and computational time. The response calculated by the direct integration with Newmarks average acceleration method is treated as a standard solution for the purpose of evaluations of other methods. A 5-story 2-bay planar moment resisting frame Fig. 2 given in Ref. (21) is used in this presentation for testing of the various step-forward integration methods.

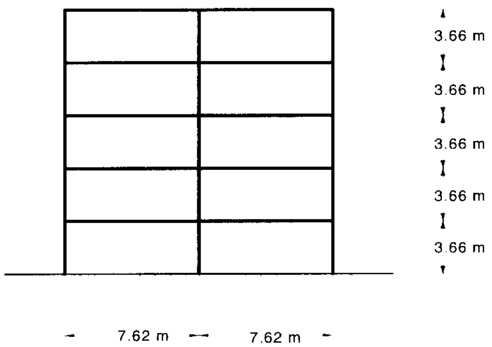

Fig. 2 Configuration of Ductile 5-Story 2-Bay Strong Column/Weak Beam Moment Resisting Frame[21]

The frame behaves as a strong column/weak beam structure, i.e., the preponderance of hinging occurs in the beams. The frame is mathematically modeled with elements which have hinge zones at each end

The moment curvature relation used for the hinges is based on the Q-hyst model[9] , and is shown in Fig. 1. The post-yield slope of the moment-curvature relation, i.e., branch ②, is assumed to be 0.5% of the elastic value. Lumped masses are placed at the beam-column joint coordinates. Viscous damping is assumed to be proportional to the mass. The proportionality coefficient was chosen so that a decay equivalent to 5% of critical damping of the first elastic mode was achieved.

The section properties (Table 1) are found while assuming well confined cross sections.[22] The frame has the elastic fundamental period of 0.84 sec. The seismic coefficient, i.e., the base story yield shear divided by the building weight, was 0.352. This coefficient is in the normal range. The frame was subjected to the S16E component of the 1971 Pacoima Dam record[23] (Fig. 3). The record is considered to be severe because of the high values of peak velocity and acceleration.

We chose it for the present study because strong nonlinear begavior was guaranteed to occur.

A cyclic static loading procedure is employed in which each cycle will allow more members to yield until the first story drift ductility is reached 4. The resulting hysteresis curve of the cyclic static loading is plotted on Fig. 4. The static loading produced 59 different HZS sets to be used in the dynamic phase.

Two time histories, i.e., base shear and roof displacement (Fig. 5) are selected and used for comparison. We see that the quantities in Fig. 5 obtained by direct integration and the proposed PASM method are practically identical. The maximum displacement envelops are also compared in Fig. 6. The numerical results by the proposed PASM method are compared with other methods in Table 2. The accuracy achieved in among the compared methods is comparable. When the dynamic analysis is performed with iterations, a larger time step can be used for a converged solution. There is a substantial reduction in computing effort achieved by this method when compared with some of the available analysis methods (Table 3). The proposed PASM method has approximately 9% to 18% less computing time required for the direct integration method.

Table 1 Section Properties for 5 story Strong Column/Weak Beam Frame

	Cross Section (cm × cm.)	Reinforcement	Axial Force (kN)	EI_{el} ($MN\text{-}m^2$)	M_y (MN-m)
1st ext. columns	55.9x55.9	12#10	397. 21	118.1	124.2
1st int. column	61.0x61.0	12#11	693.44	155.9	163.1
2nd ext. columns	55.9x55.9	12#10	311.80	115.9	122.0
2nd int. column	61.0x61.0	12#11	543.10	151.9	158.9
3rd ext. columns	55.9x55.9	12#10	225.96	113.8	119.9
3rd int. column	61.0x61.0	12#11	392.76	147.9	154.7
4th ext. columns	55.9x55.9	12# 9	140.11	93.5	96.1
4th int. column	61.0x61.0	12#10	242.86	122.9	123.7
5th ext. columns	55.9x55.9	12# 9	63.16	91.3	94.2
5th int. column	61.0x61.0	12#10	115.65	119.0	120.1
1-3 story beams	55.9x76.2	(top) 6#8, 2#7 (bottom) 4#8	0.0	189.8	103.2
4-5 story beams	50.8x63.5	(top) 5#8, 2#7 (bottom) 4#8	0. 0	111.5	80.5

For the columns: EI_{el} and M_y are found using the Mander et al.[22] idealization.

For the beams: EI_{el} and M_y are averaged values obtained from the Mander et al.[22] idealization

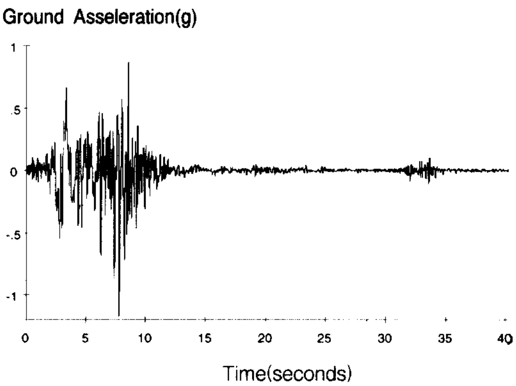

Fig. 3 1971 Pacoima Dam Earthquake Record

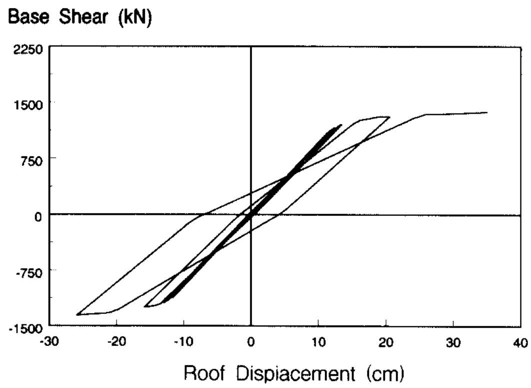

Fig. 4 Hysteresis curve by static loading

Table 2 Comparison of maximum response quantities

Analysis Type	Iteration	Base Shear		Roof Displacement	
		(kN)	%[1]	(cm)	%[1]
Direct Integration	Without	1551.46	0.00	31.45	0.00
	With	1573.26	1.67	31.45	0.00
Eigenvalue	Without	1556.80	0.60	31.12	0.89
	With	1576.37	1.87	31.37	0.24
PASM method	Without	1554.58	0.02	31.23	0.70
	With	1574.15	1.72	31.51	0.19

1. Difference in percent compared with corresponding values obtained using direct integration method without iteration

4. Conclusion

A method for nonlinear developed. With this method, it is not necessary to perform a triangular decomposition of the effective stiffness matrix whenever the change in the stiffness occurs. The advantage of this method is not only the improvement on the efficiency in computing, but also the ability to obtain the valuable information by performing static analysis; e.g., story yield displacement, recognizing the critical floor where most severe nonlinear behavior occurs. The stiffness measure compares stiffnesses of previously evaluated states of deformation with stiffnesses of the current dynamic state. The concept is useful for identifying and using previously generated information.

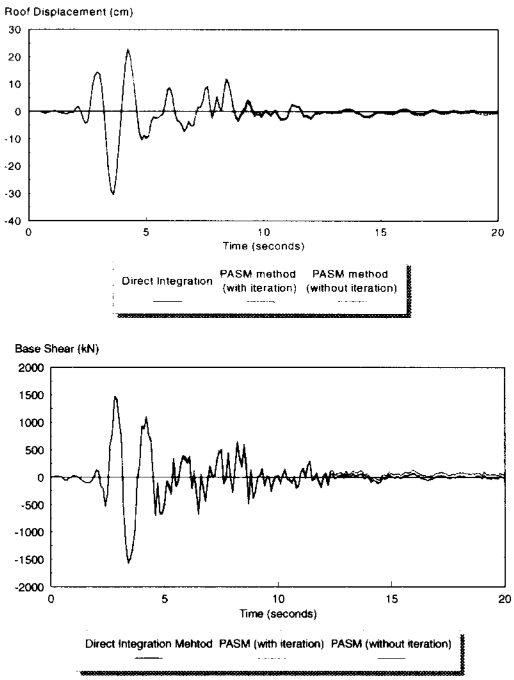

Fig. 5 Comparison of Response Time Histories

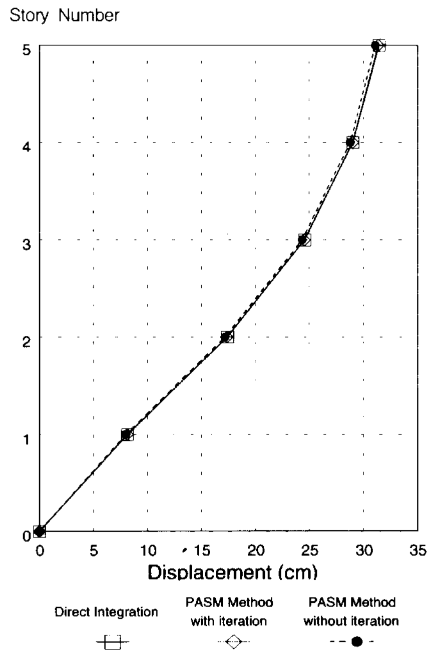

Fig. 6 Comparison of Maximum Displacement Envelopes

Table 3 Required time step and computing time for converged solutions

Analysis Type	Iteration	Δt (second)	Computing Effort(%)[1]
Direct Integration	Without	0.005	100.00
	With	0.01	113.00
Eigenvalue[2]	Without	0.005	233.96
	With	0.01	229.87
PASM method	Without	0.005	82.39
	With	0.01	91.51

1. Computing time in percent compared with corresponding values obtained using direct integration method without iteration
2. First five modes have been used for analyses

참 고 문 헌

1. Newmark. N.M., "A method of computation for structural dynamics," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 85, No. EM5, July 1959, pp. 67-94.
2. Wilson, E.L., Farhoomand, I., and Bathe, K.J., "Nonlinear dynamic analysis of complex structures," *International Journal of Earthquake Engineering and Structural Dynamics*, Vol. 1, 1973, pp. 241-252.
3. Houbolt, J.C. "A recurrence matrix solution for the dynamic response of elastic aircraft," *Journal of the Aeronautical Sciences*, Vol. 17, 1950, pp. 540-550.
4. Clough, R.W., and Bathe, K.J., "Finite element analysis of dynamic response," *Proc. 2nd U.S.-Japan Conference on Recent Advances Comp. Meth. of Struct. Analy. Design, Univ.* Alabama Press, Huntsville, Alabama, 1972, pp. 153-179.
5. Goudreau G. L., and Taylor, R. L., "Evaluation of numerical integration methods in elasto-dynamics," Computer Methods in Applied Mechanics and Engineering, Vol. 2, 1972, pp. 69-97.
6. Hughes, T.J.R., and Hilber, H. M., "Step-by-step integration in linear structural dynamics," *Proceedings of Sixth Canadian Congress of Applied Mechanics*, Vancouver, 1977, pp. 1043-1044.
7. Timoshenko, S., Young, D.H., and Weaver, W., JR., *Vibration Problems in Engineering*, Fourth Edition, John Wiley & Sons, Inc., New York, 1974.
8. Bathe, K.J., *Finite Element Procedures in Engineering Analysis*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1982.
9. Saiidi, M., and Sozen, M.A., "Simple and complex models for nonlinear seismic response of reinforced concrete structures," *Structural Research Series No. 465,* Civil Engineering Studies, University of Illinois, Urbana, Aug. 1979.
10. Nickell, R.E., "Nonlinear dynamics by mode superposition," *Computer Methods in Applied Mechanics and Engineering*, Vol. 7, 1976, pp. 879-897.
11. Morris, N.F., "The use of modal superposition in nonlinear dynamics," *Computers and Structures*, Vol. 7, 1977, pp. 65-72.
12. Idelsohn, S.R., and Cardona, A., "A reduction method for nonlinear structural dynamic analysis," *Computer Methods in Applied Mechanics and Engineering*, Vol. 49, 1985, pp. 253-279.
13. Remseth, S.N., "Nonlinear static and dynamic analysis of frames structures," *Computers and Structures*, Vol. 10, 1979, pp. 879-897.
14. Maison, B.F., and Kasai, K., "Analysis for type of structural pounding," *Journal of Structural Engineering, ASCE*, Vol. 116, 1990, pp. 957-977.
15. Gillies, A.G., and Shepherd, R., "Prediction of post-elastic seismic response of structure by a mode superposition technique," *Bull. New Zealand National Society Earthquake Engineering*, Vol. 16, 1983, pp. 222-233.
16. Mohraz, B., Elghadamsi, F.E., and Chang, C.J., "An incremental mode superposition for non-linear dynamic analysis," *Earthquake Engineering and Structural Dynamics*, Vol. 20, 1991, pp. 471-481.
17. Chang, K.K., "Nonlinear Dynamic Response of Reinforced Concrete Structures Using Sets of Eigenvectors," *Ph.D. Dissertation,* Univ. of California, Los Angeles, 1992.
18. Kim, K.J., "Use of Stiffness Measure in Dynamic Analysis of Reinforced Concrete Frame Structures," *Ph.D. Dissertation,* Univ. of California, Los Angeles, 1996.
19. Humer, J.L., *Dynamics of Structures*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1990.
20. Zienkiewicz, O.C., and Taylor, R.L., *The Finite Element Method, Fourth Edition,* Vol. 2, McGraw-Hill, Inc., 1991.
21. Al-Haddad, M.S., and Wight, J.K., "Feasibility and consequences of moving beam plastic hinging zones for earthquake resistant design of R/C building," *Report No. UMCE 86-1,* Department of Civil Engineering, University of Michigan, Ann Arbor, 1986.

22. Mander, J.B., Priestley, M.J.N., and Park, R., "Theoretical stress-strain model for confined concrete," *Journal of Structural Engineering*, Vol. 114, No. 8, August 1988, pp. 1084-1826.

23. Trifunac, M.D., and Hudson, D.E., "Analysis of the pacoima dam accelerogram," *Engr. Features of the San Fernando Earthquake*, Calif. Inst. of Tech., Pasadena, 1971.