Reliability Calculation of Power Generation Systems Using Generalized Expansion

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Abstract

This paper presents a generalized expansion method for calculating reliability index in power generation systems. This generalized expansion with a gamma distribution is a very useful tool for the approximation of capacity outage probability distribution of generation system. The well-known Gram-Charlier expansion and Legendre series are also studied in this paper to be compared with this generalized expansion using a sample system IEEE-RTS(Reliability Test System). The results show that the generalized expansion with a composite of gamma distributions is more accurate and stable than Gram-Charlier expansion and Legendre series as addition of the terms to be expanded.

I. Introduction

Reliability has a wide range of meanings and applications but the basic intent is to indicate the overall ability of the system to perform its function. Reliability is considered to be as important as economy and security of the power system. It is a decision tool for carrying out the trade-off between the reliable operation at an acceptable level and cost involved. Also, it provides means of efficient and optimum economic planning and operation of electric power systems either over the long or short term period. A power system can be divided into functional zones of generation, transmission and distribution. This paper is concerned with the development of reliability evaluation methods for generation systems. In generation system reliability evaluation, the most commonly used index is the Loss of Load Expectation (*LOLE*). For the computation of this index, three basic steps are required;

1. development of generation model which describes the probability and frequency of the capacity outage,
2. development of load model for daily peak or hourly load,
3. convolution of these two models to form a generation reserve model from which *LOLE* can be obtained.

The methods to obtain a generation system model which consists of a large number of units can be broadly classified into two categories. In the first category are the methods based on recursive algorithms to describe the discrete characteristics of generation capacity probability distribution. The recursive algorithms are theoretically accurate for calculating the discrete probability distributions of generation capacity outages. These discrete probability distributions result from the underlying discrete probability distributions used to describe individual generating units.

Such an approach is, however, computationally expensive, especially when it is used repetitively for large power systems. In the second category are the indirect analytical methods which use the first few terms of some infinite expansion to model the generation system as a continuous distribution approximation. These methods are computationally fast and simple. Hence, many continuous probability models have been proposed as efficient alternatives for the discrete probability models. Stremel and Rau[1] and Rau and Schenk[2] employed Gram-Charlier expansion based on the concept of cumulants to approximate the discrete distribution of the probability of capacity outage, and these were further improved by using Edgeworth type expansion obtained by rearranging the terms in Gram-Charlier series for the small power systems by Levy and Kahn[3]. These expansions using Normal distribution, however, as indicated by Mazumdar and Gaver[4], could result in inaccuracy and instability for approximating the distribution due to the inherent characteristics of Normal distribution.

To overcome of these poor distribution fitting, Gross, Garapic and McNutt have proposed the mixture of Normals approximation technique[5] in which the distribution curve was partitioned into several classes, and the whole distribution curve was reconstructed by the mixture of Normal distributions of each partitioned curve. In addition to the expansions of Gram-Charlier and Edgeworth type which employed Chebyshev-Hermit polynomials based on Normal distribution, the application of the method of cumulants

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using the other orthogonal polynomials has proposed by some authors; such as Laguerre polynomials by Tian, *et al.*[6] and Legendre series by Jorgensen[7]. These orthogonal polynomials are defined on the interval $[0,\infty)$ which is more appropriate to represent the load and generator outage distribution in stead of using the interval as appeared on the Chebyshev-Hermit polynomials.

This paper describes two basic methods to calculate these indices: recursive method and continuous distribution model method using Gram-Charlier expansion, Legendre series, and a generalized expansion method. It also provides some test results and comparisons using these methods with a sample system IEEE-RTS (Reliability Test System)[8]. The results show that the generalized expansion with a gamma distribution is more accurate and stable than Gram-Charlier expansion and Legendre series as addition of the terms to be expanded.

II. Generation Model

1. Development of Generation Model by the Recursive Method

In the recursive algorithm, the cumulative probability of capacity outage are calculated by unit addition algorithms. These algorithms proceed by updating the generation system model by adding one unit at a time. One form of such algorithms for two state units is the summation of conditional probability[12],

$$P_g(X) = (1-p_i)P_g^{-i}(X) + p_iP_g^{-i}(X-Q_i) \tag{1}$$

where $P_g^{-i}(X)$ and $P_g(X)$ represent probabilities of capacity outage greater than or equal to X, before and after unit i is added, and Q_i and p_i are capacity outage and forced outage rate of unit i, respectively. Figure 1 shows cumulative probabilities calculated by the recursive method for IEEE-RTS with the values of normal *FOR*(Forced Outage Rate) and *FOR*s divided into 2 and 4.

Therefore, the exact cumulative probability table $P_g(X)$ is obtained from the recursive method, or it can be approximated by the continuous distribution models as described on the next section. Once the cumulative outage table is determined, the reliability index *LOLE* can be evaluated easily.

LOLE is defined as the summation of the probability of generation deficiency for all hourly load. From the definition of cumulative probability $P_g(X)$, the annual index *LOLE* can be obtained by

$$LOLE = \sum_{j=1}^{YH} P_g(C-L_j) \tag{2}$$

where YH is total hours in a year (8760 hours), C is total installed capacity and L_j is hourly load during a year.

2. Calculation of Generation Model by Continuous Distribution Method

Many papers based on continuous distributions have appeared in the literature as an alternative to the recursive method to save computation time but maintain an acceptable accuracy. Probably the most widely used expression consists of an expansion in terms of Normal distribution. There are several variations on this approach such as Gram-Charlier[1], Edgeworth[3], mixture of Normals[5] and the modified version of the Gram-Charlier expansion[2]. Although they improve the throughput of indices calculation, they have some common drawbacks.

In this section, Gram-Charlier expansion and the recently published Legendre series method[7] are discussed and the results from these are shown in comparison with the results from the recursive method used as reference values.

Gram-Charlier Expansion

The standard Normal (or Gaussian) probability density function and its derivatives are used in the Gram-Charlier expansion as its basic elements and expansion terms. A density function $p(x)$ can be expanded in a series of derivative of Gaussian density function $g(x)$,

$$p(x) = \sum_{k=0}^{\infty} c_k H_k(x) g(x) \tag{3}$$

which is referred to Gram-Charlier series. In this equation, $H_k(x)$ are known as Chebyshev-Hermit polynomials and c_k is the coefficients of Gram-Charlier series, which are given as,

$$H_k(x) = \sum_{i=0}^{r} A(k,i)x^{k-2i} \tag{4}$$

$$c_k = \frac{1}{k!}\sum_{i=0}^{r} A(k,i)m_{k-2i} \tag{5}$$

where

$$A(k,i) = (-1)^i \frac{k!}{(k-2i)!2^i i!} \tag{6}$$

r = rounded-off integer value of $k/2$

In the expression of the coefficients c_k, m_i's are initial moments of the density function $p(x)$ of the capacity outage of system with $m_0 = 1$, and therefore, it is seen from (3) that any discrete density function $p(x)$ can be expressed by Chebyshev-Hermit polynomials and the moments.

If $p(x)$ is standardized, then $m_1 = 0$, $m_2 = 1$ and the first three coefficients in (5) are given as $c_0 = 1$ and $c_1 = c_2 = 0$. Therefore, Gram-Charlier series (3) can be simplified as

$$p(x) = g(x)[1 + \sum_{k=3}^{\infty} c_k H_k(x)] \tag{7}$$

Then, the cumulative probability is given by

$$P_g(X) = \int_X^{\infty} g(x)dx + g(x)\sum_{k=3}^{\infty} c_k H_{k-1}(x) \quad (8)$$

using the equations (5) and (7).

Legendre Series

It has been observed[7] that when the original random variables are bounded on the finite interval, i.e., the minimum and the maximum values for generation or load, it is better to use the random variables bounded on the finite interval rather than to use the ones defined for an infinite interval such as Gram-Charlier expansion. The Legendre series bounded on the interval (0,1) has been proposed recently to calculate the production cost and reliability indices[7]. The basic idea of the method of the Legendre series is as follows.

The orthogonal polynomials $J_n(t)$ of the Legendre series are defined by

$$J_n(t) = \sum_{k=0}^{n} B(n,k)t^{n-k} \quad (9)$$

where

$$B(n,k) = (-1)^k \frac{(2n-k)!}{k![(n-k)!]^2} \quad (10)$$

Any density function $p(t)$ can then be expressed using the Legendre series,

$$p(t) = \sum_{r=0}^{\infty} a_r J_r(t) \quad (11)$$

The coefficients a_r's are determined by multiplying $J_r(t)$ on both sides of (11) and integrating,

$$a_r = (2r+1)\sum_{j=0}^{r} B(r,j)m_{r-j} \quad (12)$$

Applying these probability tables or functions obtained from either recursive, Gram-Charlier expansion or Legendre series, the *LOLE* can be calculated using equation (2).

III. Generalized Expansion

It was shown[9] that any discrete distribution can be expressed in terms of another distribution and its expansion using Fourier transformation. This generalized expansion is a very useful tool for the approximation of capacity outage probability distribution of generation system. The well-known Gram-Charlier expansion becomes a special case of this generalized expansion. In this section, the relationships for determining the distribution parameters from the generating unit parameters for two, three and generalized 2*n*-parameter gamma distributions are derived and these distributions are tested on a sample systems with normal and low *FOR*s.

1. Generalized Expansion

The method described in this section is a generalization of the expansion methods. This method provides a generalized approach for expanding a given distribution in terms of another distribution.

The discrete probability distribution of a generation system can be approximated by a continuous probability distribution using Fourier and inverse Fourier transformations which give a general formula for the expression of any discrete distribution in terms of any continuous distribution and its derivatives[9],

$$p(x) = \sum_{i=0}^{\infty} (-1)^i \frac{D_i}{i!} f^{(i)}(x) \quad (13)$$

where

$p(x)$ = discrete density function of generation system

$f(x)$ = continuous density function to be used for expanding $p(x)$

$f^{(i)}(x)$ = $i-$th derivative of $f(x)$

and

$$D_i = m_i - \mu_i - \sum_{r=1}^{i-1} \binom{i}{r} \mu_{i-r} D_r \quad (14)$$

where m_i and μ_i are i-th moments of $p(x)$ and $f(x)$, respectively. It can be shown that the coefficients D_i's have recurrence relationships as

$$D_0 = 1$$
$$D_1 = \zeta'_1 \quad (15)$$
$$D_j = \zeta'_j + \sum_{r=1}^{j-1} \binom{j-1}{r} \zeta'_{j-r} D_r, \quad j \geq 2$$

where

$\zeta'_i = \zeta_i^p - \zeta_i^f$

$\zeta_i^p,\ \zeta_i^f$ = $i-$th cumulant of $p(x)$, $f(x)$, respectively.

If the continuous density function $f(x)$ contains r parameters and if these r parameters can be estimated by matching the first r moments of $p(x)$, then the first r cumulants of $f(x)$ and $p(x)$ are equal, i.e.,

$$\zeta_i^p = \zeta_i^f, \quad i = 1, \cdots, r \quad (16)$$

and therefore the first r coefficients should be zero,

$$D_0 = 1, \quad D_1 = \cdots = D_r = 0 \quad (17)$$

It can be seen that the well-known Gram-Charlier expansion is only a special case of this general formula using Normal distribution,

$$p(x) = \sum_{i=0}^{\infty} \frac{D_i}{i!} H_i(x) g(x) \tag{18}$$

where $g(x)$ and $H_i(x)$ are Normal density function and Chebyshev-Hermit polynomials, respectively, and D_i's are the same with (15) except that $D_1 = D_2 = 0$ and $\zeta_i^f = 0$ $(i \geq 2)$.

2. Application to the Gamma Distributions

The generalized expansion described above is applied to the gamma distributions with two, three, and multi-parameters in this section.

(A) Two Parameter Gamma Distribution

The two parameter gamma density function is

$$f_2(x) = \frac{1}{\Gamma(\alpha)} \rho^{\alpha} e^{-\rho x} x^{\alpha-1}, \quad \rho > 0,\ \alpha > 0,\ \text{and } x \geq 0 \tag{19}$$

where

ρ = scale parameter of the gamma distribution,

α = shape parameter,

$\Gamma(\cdot)$ = gamma function.

From the moment generating function of the two parameter gamma distribution, j-th moment is given by

$$\mu_j = \frac{\Gamma(\alpha+j)}{\rho^j \Gamma(\alpha)} \tag{20}$$

When two moments are matched, i.e., $\mu_1 = m_1$, $\mu_2 = m_2$ where m_i is i-th moment of generation system, then the two parameters of (19) and (20) can be determined by the system moments,

$$\alpha = \frac{m_1^2}{m_2 - m_1^2}, \qquad \rho = \frac{\alpha}{m_1} \tag{21}$$

Applying (19) and (21) to (13) gives the expression for cumulative probability of capacity outage of generation system using the two parameter gamma density function,

$$P_g(X) = \int_X^{\infty} p(x)dx = \int_X^{\infty} f_2(x)dx + \sum_{i=3}^{\infty} (-1)^i \frac{D_i}{i!} f_2^{(i-1)}(X) \tag{22}$$

since $D_1 = D_2 = 0$. The remaining coefficients D_j $(j \geq 3)$ can be determined with ζ_i^f and ζ_i^p by using (15). The cumulants of two parameter gamma density function ζ_i^f are obtained from the moments expression (20), equation (21), and the relationship between moments and cumulants. The cumulants ζ_i^p are obtained from the moments m_i of capacity outage of generation system.

The generalized expression of i-th derivative of $f_2(x)$ in (22) can be obtained as follows. The first derivative of $f_2(x)$ is

$$f_2'(x) = f_2(x) v(x) \tag{23}$$

where $v(x) = (\alpha-1)/x - \rho$. Generally, taking ($i$-1)-th derivative of (23) gives

$$f_2^{(i)}(x) = \sum_{r=0}^{i-1} \binom{i-1}{r} f_2^{(i-1-r)}(x) v^{(r)}(x) \tag{24}$$

where

$$v^{(0)}(x) = v(x) = \frac{(\alpha-1)}{x} - \rho, \qquad v^{(r)}(x) = (-1)^r \frac{r!(\alpha-1)}{x^{r+1}}, \quad r \geq 1 \tag{25}$$

(B) Three Parameter Gamma Distribution

The three parameter gamma density function is

$$f_3(x) = \frac{1}{\Gamma(\alpha)} \rho^{\alpha} e^{-\rho(x-d)} (x-d)^{\alpha-1} \tag{26}$$

where d is shifting parameter. When three moments are matched to determine these three parameters, it can be shown that

$$\rho = \frac{2M_2}{M_3}, \qquad \alpha = M_2 \rho^2, \qquad d = m_1 - \frac{\alpha}{\rho} \tag{27}$$

where M_i's are central moments of generation system, i.e.,

$$M_i = \sum_{r=0}^{i} \binom{i}{r} (-m_1)^r m_{i-r} \tag{28}$$

The cumulative probability $P_g(X)$ is the same as (22) except that $D_1 = D_2 = D_3 = 0$,

$$P_g(X) = \int_X^{\infty} f_3(x)dx + \sum_{i=4}^{\infty} (-1)^i \frac{D_i}{i!} f_3^{(i-1)}(X) \tag{29}$$

The cumulants ζ_i^f involved in the expression of D_i are determined using the moments μ_i of the three parameter gamma density function $f_3(x)$,

$$\mu_i = \sum_{r=0}^{i} \binom{i}{r} d^{i-r} \mu'_r \tag{30}$$

where

$$\mu'_r = \frac{\Gamma(\alpha+r)}{\rho^r \Gamma(\alpha)}$$

The expression for $f_3^{(i)}(x)$ is also the same as (24) except that $(x-d)$ should be used rather than x due to the shifting parameter d, i.e.,

$$f_3^{(i)}(x) = f_2^{(i)}(x-d) \quad (31)$$

3. Generalized Multi-Parameter Gamma Distribution

The proposed approach postulates multi-parameter distributions for the probability of generating capacity outage and then determines the parameters of these distributions from the generating unit parameters by using the moment matching technique. Theoretically if all the moments of any two distributions are equal, then the two distributions are identical. In practice by equating only a few moments, combinations of exponentials have been shown to reasonably approximate several well known distributions.

Combinations of exponential and gamma distributions have been shown the capability of approximating a wide variety of distributions[10]. A distribution consisting of the weighted sum of gamma distributions is, therefore, proposed for modeling of capacity outage. This $2n$-parameter distribution is given by equation (32).

$$\begin{aligned} f_{2n}(x) &= \sum_{i=1}^{n} \omega_i f_2(x \mid \rho_i, \alpha) \\ &= \frac{1}{\Gamma(\alpha)} \sum_{i=1}^{n} \omega_i \rho_i e^{-\rho_i x} (\rho_i x)^{\alpha-1} \end{aligned} \quad (32)$$

where

$f_2(x \mid \rho_i, \alpha) =$ two parameter gamma distribution with parameters ρ_i and α,

$\rho_i =$ scale parameter of the $i-$th gamma distribution,

$\alpha =$ shape parameter assumed the same for all the n distributions,

and the weighting factors ω_i :

$$\sum_{i=1}^{n} \omega_i = 1, \quad 0 \le \omega_i \le 1 \quad (33)$$

Parameters of multi-parameter distribution $f_{2n}(x)$ can be determined by the moments of the distributions from generating unit parameters. The $2n$ parameters of $f_{2n}(x)$ are $\alpha, \rho_1, \cdots, \rho_n$ and $\omega_1, \cdots, \omega_{n-1}$ since ω_n can be obtained from $\omega_1, \cdots, \omega_{n-1}$ by using equation (33). These parameters are obtained by using the moment matching technique. If all the moments of two distributions are equal, then the two distributions are identical. In practice, reasonable approximation can be obtained by matching a few moments.

The $2n$ non-linear equations are obtained by matching the first $2n$ moments of $f_{2n}(x)$ with those of the capacity outage. However, this set of non-linear equations is very sensitive to the initial values, and without knowing the initial values near exact solutions, it cannot be solved by general Newton-Raphson type methods. In this paper, the Complex method is used to obtain $2n$ parameters, which is a kind of Simplex method extended to the non-linear cases with the initial values selected randomly[11].

Once $f_{2n}(x)$ is known, then the cumulative probability of capacity outage required in generation capacity studies is easily obtained as follows;

$$P_g(X) = \sum_{i=1}^{n} [\omega_i e^{-\rho_i X} \sum_{j=1}^{\alpha} \frac{(\rho_i X)^{j-1}}{(j-1)!}] \quad (34)$$

Since it has been observed in this paper that increasing the number of expansion terms does not contribute to the accuracy at more than 6 parameters used, the number of parameters (or moments) instead of expansion terms is increased in order to improve the accuracy.

Ⅳ. Case Studies

To test the accuracy of the proposed multi-parameter method, IEEE-RTS is adopted. This system consists of 32 generating units ranging from 12 to 400 MW capacities. The *FOR*s are in the range of 0.01 to 0.12. The generation capacity is 3405 MW with the peak load of 2850 MW. The unit data for this system is shown in reference [8]. This appears to be a reasonable system to test the accuracy of the proposed method and compared with other methods since it has a mixture of small and large units. Several sets of studies (the Gram-Charlier, Legendre series, and multi-parameter gamma distributions) were conducted by varying the *FOR*s and expansion terms. First, the cumulative probabilities of capacity outage are obtained by recursive method and the other continuous distribution methods, and then, using these results, *LOLE*s for power generation system are calculated and compared for each case.

1. Cumulative Probability of Capacity Outage

Figures 2, 3 and 4 show the cumulative probabilities of the IEEE-RTS system obtained by the method of Gram-Charlier expansion with the distribution term only and 1 expansion term included, and 6 and 10 parameter gamma distributions, compared with the results from the recursive method in cases of normal *FOR* and *FOR*s divided by 2 and 4, respectively.

It can be seen from Figure 2 that when the *FOR*s are normal, as the number of expansion terms is increased, Gram-Charlier curve appears to be fitted to the recursive. In the case of *FOR*s divided by 2 and 4, however, it can be seen from Figures 3 and 4 that the employment of more expansion terms causes fluctuations and numerical instability. Particularly, when *FOR*s are divided by 4, the exact discrete results of the cumulative probability do not form a smooth curve and it is harder to fit a continuous distribution to it. On the contrary, it can be seen that 6 and 10 parameter gamma distributions are well on the track of recursive curves for all cases of *FOR*s.

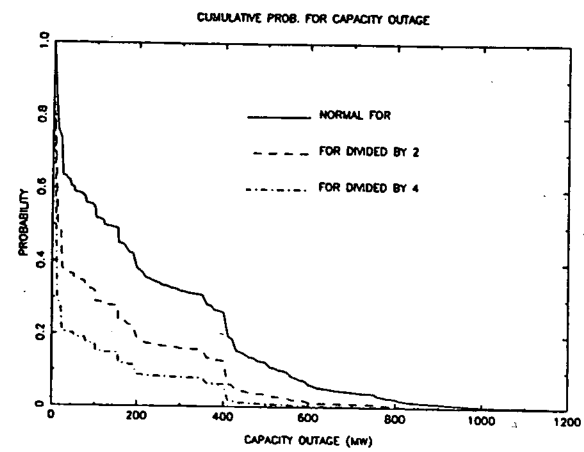

Fig. 1. Cumulative Probability of Capacity Outage by Recursive Method for IEEE-RTS.

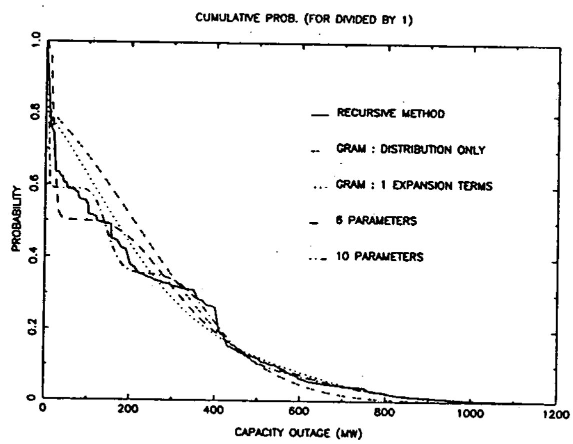

Fig. 2. Cumulative Probability using Gram-Charlier and Multi-parameter Gamma(Normal FOR).

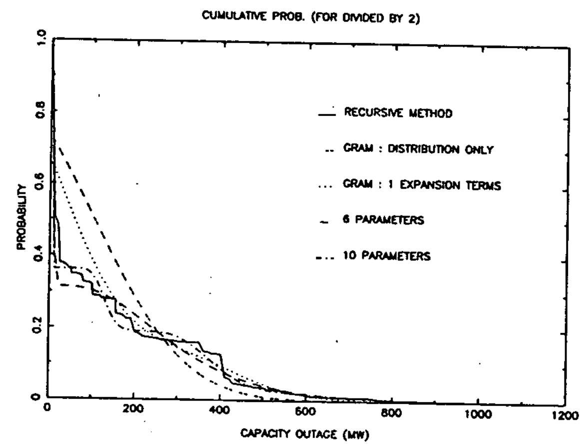

Fig. 3. Cumulative Probability using Gram-Charlier and Multi-parameter Gamma(FORs divided by 2).

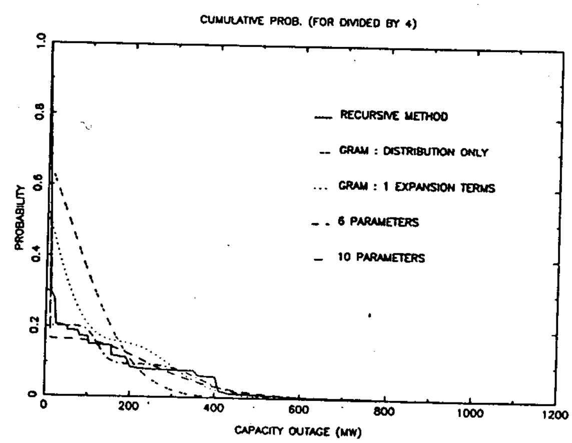

Fig. 4. Cumulative Probability using Gram-Charlier and Multi-parameter Gamma(FORs divided by 4).

2. *LOLE by Expansion Terms and Multi-parameter*

The comparison of *LOLE* by the Gram-Charlier expansion and Legendre series with the recursive method is shown in Table 1. For normal *FOR*s in the range from 0.01 to 0.1, the Gram-Charlier gives results close to the recursive method. However, when all *FOR*s are divided by 2 which makes their range quite small 0.005 to 0.05, the Gram-Charlier has large percentage deviations from the recursive method. For the *FOR*s divided by 4, ranging from 0.0025 to 0.025 which are unreasonably low, Table 1 doesn't show any meaningful results.

Table 1 also shows the *LOLE*s by the Legendre series for comparison. For the normal case of unmodified *FOR*s, the Legendre series shows a lot of variation but at some points does come close to the recursive method. There is no uniform convergence and it is hard to know where to stop. When the *FOR*s are divided by 2 or 4, the results show no relationship to the recursive values at all. Even if the normalized moments are

(1) Gram-Charlier Expansion
(2) Legendre Series

used as mentioned in the discussion of [7], the results still have quite large errors from the recursive method and show numerical instability when the number of terms used is increased. In this Tables 1, it can be seen that the result from this Legendre series are worse than those from the Gram-Charlier.

Table 2 shows the *LOLE*s by the 2, 3, and 6 parameter gamma distributions with their expansion terms. It can be seen from Table 2 that none of these distributions by themselves can give as accurate results as the multi-parameter distribution. With expansion terms added, the results are better but not as good as the distribution only with the number of parameters increased.

We can make sure of this fact from Table 3, where it can be seen that the generalized multi-parameter distribution model provides *LOLE* index quite close to the recursive method for eight and more moments. Also there is no significant variation in

Table 1. Comparison of *LOLE* by Gram-Charlier and Legendre Series.

FOR	normal case		divided by 2		divided by 4	
Expansion Terms	(1)	(2)	(1)	(2)	(1)	(2)
1	1.81	641.78	0.02	384.94	4×10^{-5}	256.51
2	7.17	532.18	0.25	215.85	2×10^{-3}	56.41
3	9.32	-163.63	0.51	-276.50	6×10^{-3}	-318.46
4	9.06	-421.17	0.66	-527.18	0.01	-559.60
5	9.77	-133.84	1.01	-153.34	0.03	-151.32
6	9.41	-151.18	1.44	-127.02	0.09	-102.48
7	9.38	-59.92	1.59	-64.32	0.19	-59.40
8	9.33	-39.99	1.52	-44.37	0.33	-42.50
9	9.39	-20.54	1.32	-22.67	0.43	-17.51
10	9.55	-7.06	1.22	-0.47	0.34	11.19
11	9.50	0.89	1.32	4.40	-0.07	12.66
12	9.50	20.27	1.44	31.95	-0.53	43.66
13	9.42	14.27	1.34	18.44	-0.01	25.30
14	9.40	15.60	1.10	12.81	2.38	13.48
15	9.41	14.05	1.07	10.88	4.52	11.88
16	9.41	13.81	1.42	7.60	-0.96	6.86
17	9.44	13.42	1.77	7.35	-18.5	6.81
Recursive	9.39		1.35		0.24	

Table 2. Comparison of *LOLE* by gamma distributions with their expansion terms.

FOR		normal case	divided by 2	divided by 4
No. of parameters	No. of expansion terms			
2	1	28.36	9.14	3.57
	2	24.16	8.33	3.40
	3	18.00	6.92	3.07
	4	11.55	5.08	2.61
3	1	12.86	2.88	0.87
	2	12.86	2.88	0.87
	3	11.33	2.60	0.82
	4	9.72	2.13	0.71
6	1	9.35	1.27	0.19
	2	9.28	1.26	0.18
	3	9.26	1.27	0.18
	4	9.46	1.32	0.27
Recursive		9.39	1.35	0.24

accuracy after ten moments. The author's experience with this method indicates that use of 8 to 10 moments gives quite accurate results. It should also be noted that the solution exists for all even moments.

Following conclusions can be drawn from the results of these studies :

1. For the small values of *FOR*s, the irregularity in the exact discrete model increases, and therefore, it is harder to fit continuous distributions to the exact model.
2. The methods using a series expansion based on Normal distribution work well only when the system is large, homogeneous, and reasonable *FOR*s. However, the results obtained from these Normal type distributions and expansions are likely to be inaccurate for small systems, or systems with small forced outage rate units.
3. The first three or four terms of the expansion could be used to improve the accuracy of the solution. Addition of more terms, however, does not ensure higher accuracy of the calculated indices.
4. The most recently published paper using Legendre series still has the same problems, and when the *FOR*s are divided by 2 and 4, it completely breaks down.
5. The proposed generalized multi-parameter distribution model method is stable. Methods based on expansions may fluctuate widely with the addition of more terms. The proposed method is, however, steady and stable as the number of parameters is increased.
6. The proposed approach is conceptually attractive since the distribution is easily integrable and provides a simple equation for reliability. The parameters of this distribution can be easily calculated and modified if new units are added or removed.
7. Although the excellence of the proposed method has been shown from the case studies, not from the strict mathematical proof, the author's experience with other sample systems reveals the similar results as discussed above.
8. From the cases studied, the multi-parameter model appears to be a good alternative approach for generation capacity reliability evaluation.

Table 3. Comparison of *LOLE* by multi-parameter distribution without expansion terms.

FOR	normal case	divided by 2	divided by 4
No. of parameters			
6	9.35	1.27	0.19
8	9.51	1.33	0.22
10	9.43	1.31	0.22
12	9.44	1.33	0.22
14	9.44	1.33	0.22
16	9.43	1.33	0.22
18	9.44	1.33	0.22
Recursive	9.39	1.35	0.24

V. Conclusion

Generalized multi-parameter distribution model is compared in this paper with the Gram-Charlier and Legendre series. From the cases studied, the proposed multi-parameter model appears to be more accurate than any other methods with higher expansion terms. It seems because the use of more moments implies that more information of the system is utilized. It appears, moreover, that using higher moments in a higher parameter distribution provides better results than using more expansion terms.

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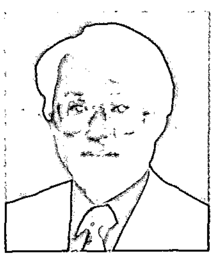

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