A New Approach to Reduced-Order Modeling of Multi-Module Converters

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Abstract

This paper presents a new approach to obtaining a reduced-order model for multi-module converters. The proposed approach can be used to derive the reduced-order model for a wide class of multi-module converters including pulse-width-modulated (PWM) converters, soft-switched PWM converters, and resonant converters. The reduced-order model has the structure of a conventional single-module converter while preserving the dynamics of the original multi-module converter. Derivation procedures and the use of the reduced-order model is demonstrated using a three-module boost converter.

I. Introduction

To deliver the high current efficiently, power supplies for high-current applications employ several converter modules in parallel. The resulting multi-module converters offer improved reliability and built-in redundancy[1]. While benefits and dc characteristics of multi-module converters are well known, ac dynamics and control design procedures are rarely understood. This is primary due to the lack of a modeling method, which systematically characterizes the dynamics of multi-module converters, and in turn offers a simplified model that makes the design problem tractable. It is the purpose of this paper to present a general method for obtaining a simplified model which considerably eases the analysis and design of multi-module converters.

An earlier work[1] showed that a multi-module buck converter with a common output capacitor can be reduced to a simplified model. The approach used in[1] was based on the state-space analysis which needs to include dynamics of all reactive components of the power stage. This approach was successfully adapted to a buck converter with separate output capacitors[2] and a boost converter[3]. However, this approach readily becomes intractable when the number of reactive components of power stage increases, and is applicable only to PWM converters.

This paper presents a new approach to deriving a simplified model for multi-module converters. The proposed approach can be applied to a wide class of multi-module converters including PWM converters, soft-switched PWM converters, and resonant converters. The approach is based on standard circuit analysis techniques, and does not require any complex analytical manipulations. Once the derivation steps are fully understood, the simplified model can be obtained, in many cases, by an inspection.

The proposed approach results in a simplified model, hereafter referred to as the reduced-order model, which has the structure of a conventional single-module converter while preserving the dynamics of the original multi-module converter. Thus the approach allows one to analyze and design a multi-module converter using conventional techniques originally intended for single-module converters.

Complete details about derivation steps for the reduced-order model are demonstrated using a three-module boost converter. The use of the reduced-order model for the control design and dynamic analysis of the multi-module converter is presented. The equivalence between the original multi-module converter and the reduced-order model is verified by both frequency- and time-domain simulations. The robustness of the reduced-order model against statistical mismatches of individual converter modules is demonstrated using Monte Carlo analyses.

II. Reduced-Order Modeling

Figure 1 shows the schematic diagram of a three-module boost converter. Each module contains a boost power stage, a pulse width modulation (PWM) block, and a current sensing network (CSN) for current-mode control. Three identical modules are combined with a common voltage feedback circuit, forming a complete three-module converter.

Figure 2 shows the small-signal model of Fig. 1, obtained by replacing an active-passive switch pair with the PWM switch

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Manuscript received March 3, 1997; accepted July 14, 1997.

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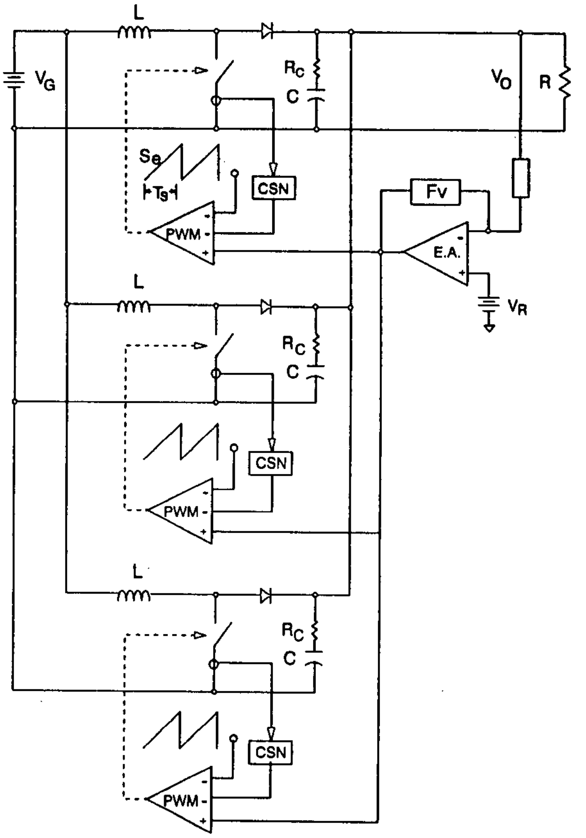


Fig. 1. Three-module boost converter: F_V is the voltage feedback compensation. T_S represents the switching period, and S_e is the slope of the ramp function. $V_g = 24\,V$, $V_o = 48\,V$, $L = 15\mu H$, $C = 133\mu F$, $R_c = 0.06\Omega$, $R = 1\Omega$, $T_s = 20\times 10^{-6}$ *Sec*.

model[4], and replacing the PWM and CSN blocks with their small-signal models[5]. The parameters for the PWM switch model and definitions of the gain blocks are given in Table I. From the small-signal model of Fig. 2, three criteria for the closed-loop performance can be identified:

- loop gain measured at "A", which carries the information about the absolute and relative stability of the converter,
- audio-susceptibility $A_u \equiv \hat{v}_o / \hat{v}_g$, which represents the input-to-output noise transmission characteristics, and
- output impedance $Z_o \equiv \hat{v}_o / \hat{i}_o$, which characterizes the output voltage perturbation produced by a varying load.

Figure 2 contains all information about the small-signal dynamics of the converter. Due to the complexity of the model, however, it is impractical to use Fig. 2 for the analysis and design purpose. Thus it is necessary to develop a simplified model, which can be used for the analysis and control design of the complicated multi-module converter.

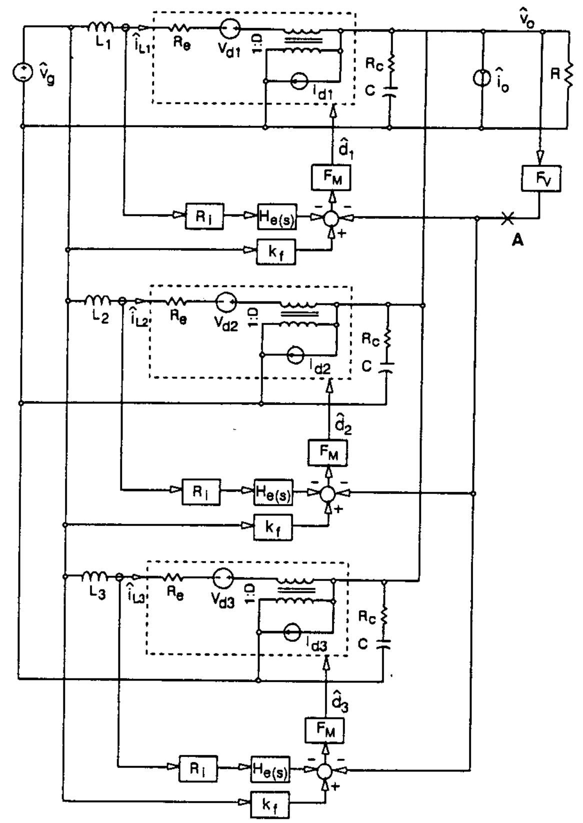


Fig. 2. Small-signal model of the three-module converter: F_M represents the modulator gain of the PWM block, and R_i is the dc gain of CSN. $H_e(s)$ represents the sampling gain of the CSN [5], and K_f is the feedforward gain created by the CSN [5]. Refer to Table I for parameters for the PWM switch.

Table 1. Parameter for Small-Signal Model of Fig. 2.

	Parameters
PWM switch	$R_e = R_c DD'$ $v_{dk} = (V_o + I_{Lk}(D-D')R_c)\ \hat{d}_k (k = 1, 2, 3)$ $i_{dk} = I_{Lk}\ \hat{d}_k$ D: duty cycle $D' = 1 - D$ I_{LK}: inductor current of k^{th} module
Modulator gain of PWM	$F_M = \frac{1}{(S_n + S_e)T_s}$ S_n : on-time slope of sensed switch current S_e : slope of ramp function
Dc gain of CSN	R_i : constant
Sampling gain of CSN	$H_e(s) = 1 + \frac{s}{\omega_n Q_n} + \frac{s^2}{\omega_n^2}$ $Q_n = -\frac{2}{\pi}$, $\omega_n = \frac{\pi}{T_s}$
Feedforward gain of CSN	$k_f = -\frac{T_s R_i}{2L}$

The small-signal model of Fig. 2 is successively simplified to yield a reduced-order model. The reduced-order model has the structure of a conventional single-module model, while retaining

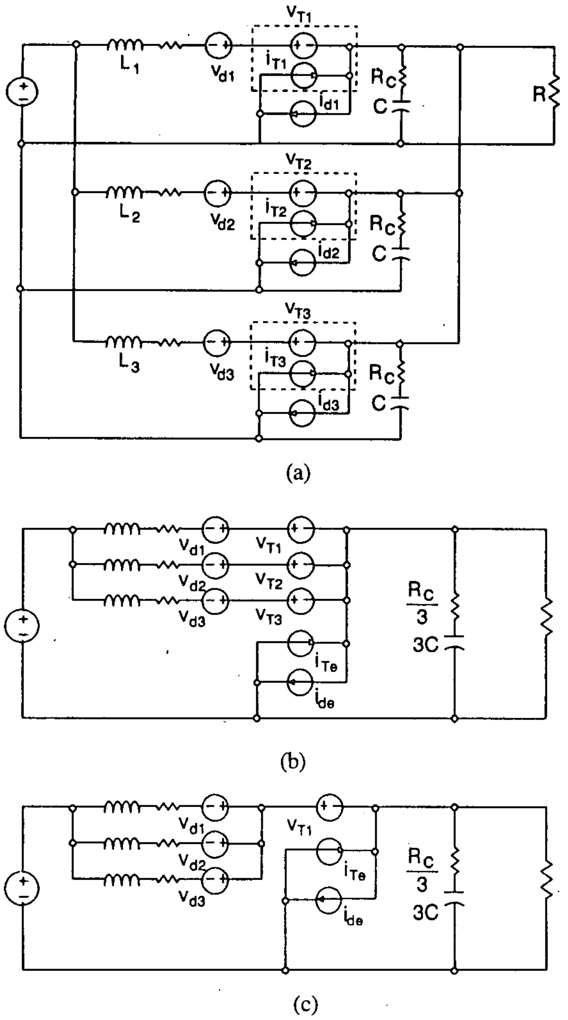


Fig. 3. Simplification of power stage model: (a) Modified power stage model: $i_{Tk}=-D\,\hat{i}_{Lk}(k=1,2,3)$, $v_{Tk}=D\,\hat{v}_o$; (b) Simplified power stage model: $i_{Te}=i_{T1}+i_{T2}+i_{T3}$, $i_{de}=i_{d1}+i_{d2}+i_{d3}=I_{L1}\,\hat{d}_1+I_{L2}\,\hat{d}_2+I_{L3}\,\hat{d}_3$; (c) Further simplification by eliminating v_{T2} and v_{T3}.

the closed-loop performance of the original three-module converter. Thus the reduced-order model allows the analysis and design of a multi-module converter to be same as for a single-module converter.

Figure 3(a) shows the power stage model of the converter, where the ideal transformer is replaced with a pair of current and voltage sources, v_{Tk} and i_{Tk} $(k=1,2,3)$. This model simplifies to Fig. 3(b) by combining the output capacitors and the current sources, i_{Tk} and i_{dk} $(k=1,2,3)$. Three identical voltage sources in Fig. 3(b), v_{T1}, v_{T2}, and v_{T3}, are combined, and subsequently v_{T2} and v_{T3} are removed to yield Fig. 3(c). Figure 4(a) is the simplified model for the entire three-module converter, obtained by adapting Fig. 3(c) and replacing v_{T1} and v_{Te} with an ideal transformer.

Since the closed-loop performance of the three-module converter is determined by the small-signal sources, $\hat{v}_g$, $\hat{i}_o$, and the injection signal at "A" for the loop gain measurement, the only considerations in deriving a reduced-order model are the system response due to these small-signal sources. In deriving a reduced-order model that preserves the closed-loop performance of the three-module system, $\hat{d}_1$, $\hat{d}_2$, and $\hat{d}_3$ can be considered identical due to the symmetry of the system. As a result, three dependent voltage sources, v_{d1}, v_{d2}, and, v_{d3} are the same, and the inductors of the converter modules can be combined to yield a simplified model of Fig. 4(b). In Fig 4(a), the output of the CSN of each module is given by:

$$V_{CSN}=\frac{R_i}{L_k}\int v_{Lk}dt \tag{1}$$

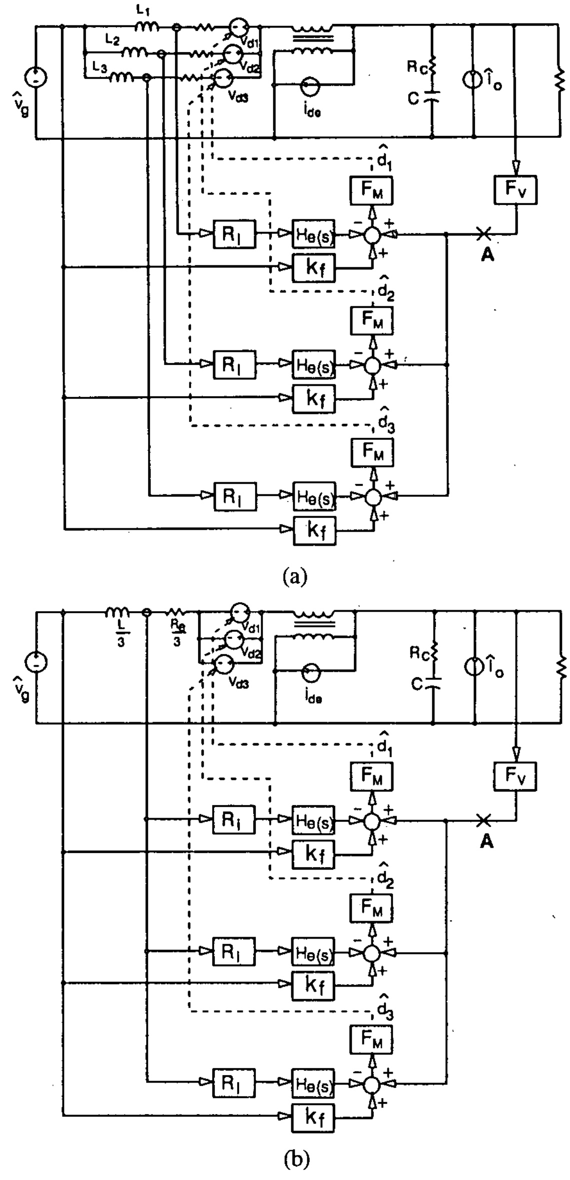


Fig. 4. Simplification of converter model: (a) Simplified model of converter: $i_{de}=I_{L1}\,\hat{d}_1+I_{L2}\,\hat{d}_2+I_{L3}\,\hat{d}_3$, $v_{dk}=(V_o+I_{Lk}(D-D')R_c)\,\hat{d}_k(k=1,2,3)$; (b) Further simplification using $\hat{d}_1=\hat{d}_2=\hat{d}_3$.

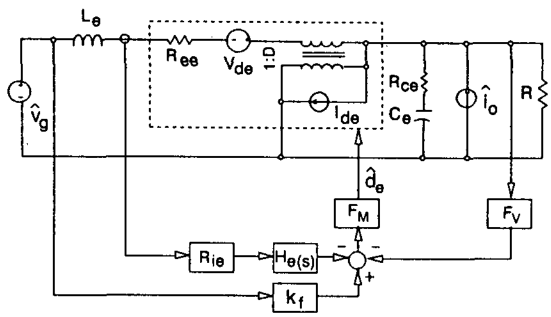


Fig. 5. Reduced-order small-signal model for Fig. 2: $L_e = L/3$, $C_e = 3C$, $R_{ce} = R_c/3$, $R_{ee} = R_{ce}DD'/3$, $V_{de} = (V_o + I_{Le}(D-D')R_{ce})\,\hat{d}_e$, $I_{de} = I_{Le}\,\hat{d}_e$, $I_{Le} = I_{L1} + I_{L2} + I_{L3}$, $R_{ie} = R_i/3$.

Since the inductor current is divided by three in Fig. 4(b), R_i should also be divided by three to produce the same output voltage as (1). With this scaling of CSN gain, F_M and K_f remain the same in Figs. 4(a) and 4(b).

As the last step, the redundant voltage sources, v_{d2} and v_{d3}, together with their associated feedback controller, are removed, resulting in the reduced-order small-signal model of Fig. 5. The reduced-order model has the form of a conventional single-module converter, while retaining the closed-loop performance of the original three-module converter.

III. Use of the Reduced-Order Model

The reduced-order model considerably simplifies the analysis and design of the complex multi-module converter. Using the reduced-order model, any control scheme developed for a single-module converter can be directly adapted to the multi-module converter. The control design optimized using the reduced-order model offers a multi-module converter which matches its closed-loop performance by the reduced-order model. Also, multi-module converters operating either as subsystems of a large-scale power system or as stand-alone regulators can be replaced with their reduced-order models without compromising any analysis accuracy.

1. Control Loop Design

The reduced-order model of Fig. 5 allows control design of a multi-module converter to be same as for a conventional single-module converter:

1. Design R_{ie} using the reduced-order model. The switch current of the reduced-order model is the sum of switch currents of individual modules. Multiply the resulting gain by the number of modules to obtain R_i for the individual module. This CSN gain scaling compensates for the difference in switch current between the reduced-order model and the original three-module converter. For the given conditions, R_{ie} is determined to be *0.5* resulting in an R_i of *0.15*.
2. Select the slope of the external ramp for a desired modulator gain [5], using the reduced-order model. For this case, S_e is selected as *153600V/S* in order to critically damp the double pole, located at half the switching frequency due to the sampling effects of the current mode control [5].
3. Design the voltage compensation, F_V, using the reduced-order model. The familiar two-pole one-zero compensation is used for a good closed-loop performance:

$$F_V = \frac{\omega_I(1 + s/\omega_Z)}{s(1 + s/\omega_P)}.$$

The compensation pole, ω_P, is selected at *50000[r/s]* to cancel the esr zero of the power stage, the compensation zero, ω_Z, is select at *8944[r/s]* to obtain a fast response during transition periods, and finally the integrator gain, ω_I, is determined to be *6300* for a phase margin of *45°*.

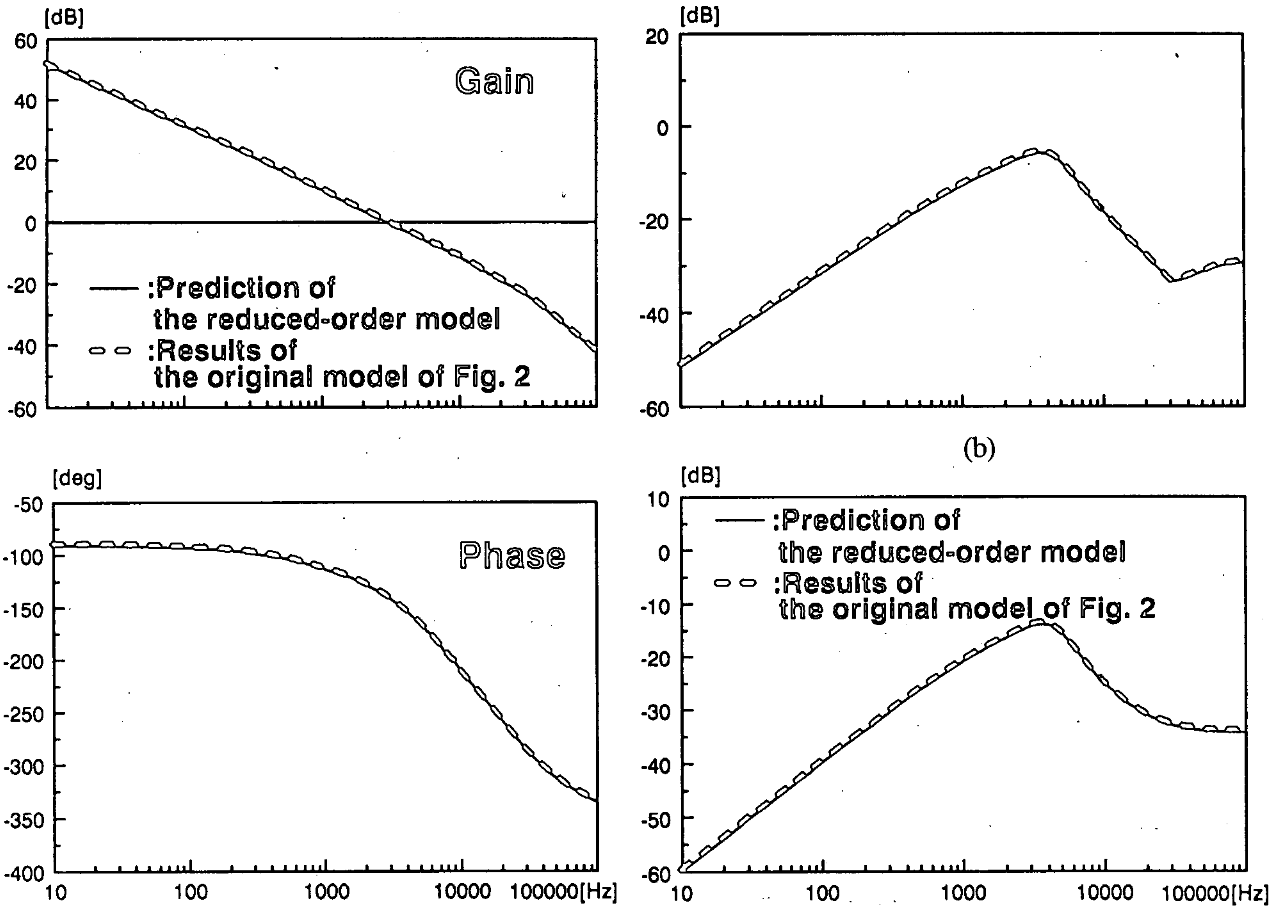
Figure 6 shows the performance of the three-module converter obtained using the original small-signal model of Fig. 2, in comparison with the prediction of the reduced-order model of Fig. 5. The exact agreement in the transfer functions verifies the modeling results and design procedures.

2. Time-Domain Simulation

Figure 7 shows the time-domain counterpart of the reduced-order small-signal model of Fig. 5. Figure 8 compares transient response of the output voltage obtained using the reduced-order model of Fig. 7 and the transient responses obtained using the original time-domain model for the three-module converter. For this analysis, a step change in the load current from *48 A* to *60 A* is assumed. Following two observations can be made from Fig. 8:

1. The prediction of the reduced-order model (upper figure in Fig. 8(a)) is identical to the response of the three-module converter (middle figure in Fig. 8(a)) whose switching signals of individual modules are synchronized.
2. Except the ripple component, the reduced-order model produces the same transient response as that of three-module converter (bottom figure in Fig. 8(a)) whose switching signals of individual modules are *120°* apart. Figure 8(b) shows the detailed comparison between the prediction of the reduced-order model (dashed line) and the response of the actual three-module converter (solid line).

Preceding analysis indicates that the time-domain reduced-order model of Fig. 7 can be used for the time-domain simulations of multi-module converters. The reduced-order model offers a significant save in the modeling effort and simulation time, while providing all necessary information to analyze the large-signal dynamics of the system.



(b)

(a)

(c)

Fig. 6. Small-signal performance of the three-module system: (a) Loop gain; (b) Audio-susceptibility; (c) Output impedance: The exact agreement in the transfer functions verifies the modeling results and design procedures.

It is not surprising that the reduced-order model produces the exactly same performance as the original multi-module system. A multi-module converter is a non-minimal order system due to the symmetry in its structure. A non-minimal order system can be reduced to a minimal order system, the reduced-order model, keeping all input-to-output characteristics unchanged. The performance criteria of a converter, such as audio-susceptibility, output impedance, and transient response due to line and load variations, are the input-to-output characteristics. Thus the reduced-order model retains all dynamics necessary to design the controller and produces the same performance criteria as the multi-module system.

IV. Model Sensitivety

The reduced-order small-signal model of Fig. 5 is derived assuming all individual converter modules are identical. However, converter modules are usually non-identical due to finite tolerance of components. Thus it is necessary to investigate the sensitivity of the reduced-order model to the mismatch of power

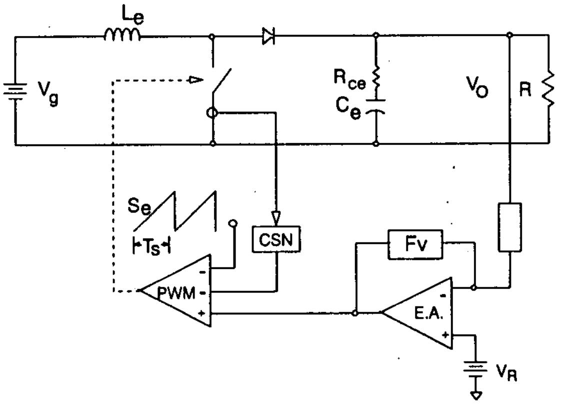
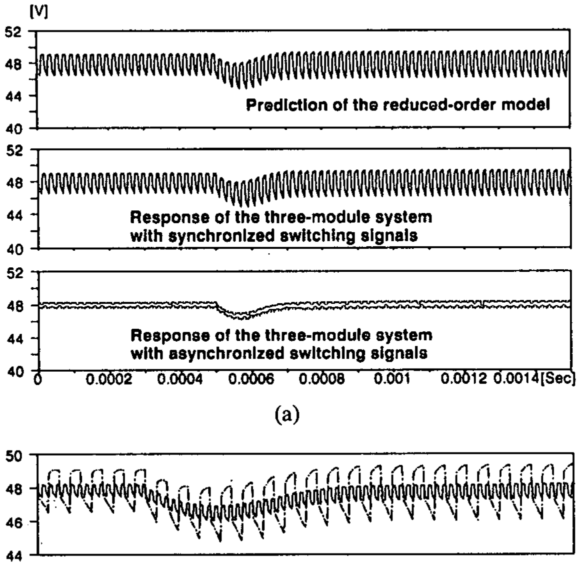


Fig. 7. Time-domain counterpart of reduced-order model: This model generates the same transient response as the original three-module converter. The dc gain of CSN is one-third of the CSN gain of the three-module converter; $L_e = L/3$, $C_e = 3C$, $R_{ce} = R_c/3$.



(a)

(b)

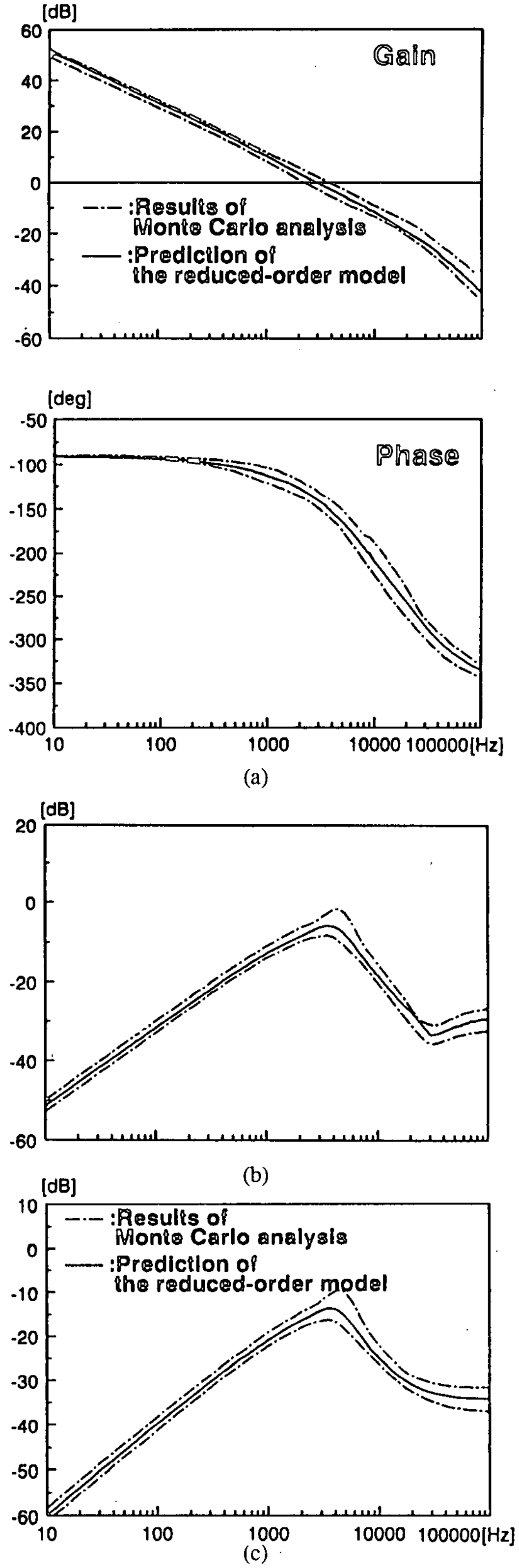
Fig. 8. Step load response: (a) Transient response of the output voltage: the prediction of the reduced-order model (upper figure), the response of the three-module converter with synchronized switching signals (middle figure), and the response of the three-module converter with asynchronized switching signals(bottom figure); (b) Detailed comparison: Prediction of the reduced-order model (dashed line) and actual response of the three-module converter (solid line).

stage parameters and operating conditions of converter modules.

Figure 9 shows the results of the statistical analysis of closed-loop transfer functions. The dashed lines are the upper and lower envelopes of the transfer functions, obtained 400 runs of Monte Carlo analysis, and the solid curve is the prediction of the reduced-order model. For Monte Carlo analysis, the original system model of Fig. 2 is used assuming $\pm 40\%$ variation for parasitics, and $\pm 20\%$ variation for power stage parameters and $\pm 10\%$ variation for control parameters. As shown in Fig. 9, the envelopes do not show any significant deviation from the prediction of the reduced-order model of Fig. 2. This analysis confirms that the reduced-order model is highly robust against the mismatched modules, and can be used for most practical applications.

V. Extention of Results

The procedures of obtaining the reduced-order model can be readily adapted to any converter that has a circuit representation for its small-signal model. For example, the reduced-order model of a multi-module zero-voltage-switched full-bridge pulse-width-modulated (ZVS-FB-PWM) converter [6, 7] can be directly deduced by applying the procedures discussed in II to the small-signal circuit model [8] of the individual ZVS-FB-PWM converter module. Similarly, the small-signal model of multi-module resonant



(a)

(b)

(c)

Fig. 9. Monte Carlo analysis (a) Loop gain; (b) Audio-susceptibility; (c) Output impedance: The upper and lower envelopes of the transfer functions (dashed line) do not show any significant deviation from the predictions (solid line) of the reduced-order model.

converters can be derived from the small-signal circuit model [9] of the individual converter modules. For many cases, the reduced-order model can be deduced by inspection without any derivation or calculation.

VI. Conclusions

A circuit oriented procedure for obtaining a reduced-order model for multi-module converters is developed. The derivation steps are straightforward and the results are general enough to be directly extended to a large class of multi-module converters. The resulting reduced-order model has the structure of a conventional single-module converter, while preserving both frequency- and time-domain performance of the original multi-module converter. Using this model, any control scheme developed for a single-module converter can be directly adapted to a multi-module converter. Also, multi-module converters operating either as subsystems of a large-scale power system or as stand-alone regulators can be replaced with reduced-order models, offering a considerable save in the modeling and simulation efforts without compromising any analysis accuracy.

Acknowledgement

This work was supported by the Kyungpook National University under 1996 General Research Funding Program.

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