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A Decoupled Approach to the Simulation of Converter Controlled Induction Machine Drive Dynamics

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Abstract

A unified , modular and decoupled approach for the simulation of converter fed induction machine systems is presented. The system under consideration could have semiconductor devices connected to the stator or the rotor of the induction machine for the purpose of controlling its performance. The machine model, however is invariant to these aspects. The model spans the circuit and equation domains of description thus allowing the advantages of both these domains of descriptions to be utilised. A hybrid switch model to describe the semiconductor device action is also proposed which improves the conditioning of the system matrix. The results obtained using this machine and switch model for a VSI fed induction machine (stator fed, rotor shorted) are compared with those from laboratory experiment to establish the validity and accuracy of the approach. Results for a slip energy recovery system are also presented and compared with those of earlier workers to establish the performance of the models and algorithms in the doubly-fed mode of operation of induction machine systems.

I. Introduction

Induction machines have been the workhorses of the industry for many years now. With the advancement of power electronic technologies, possibilities of control are becoming increasingly sophisticated. The induction machine can be fed from the stator only (singly-fed operation) or through the rotor also (doubly-fed operation). While in the former case the power electronic control circuitry almost always is on the stator side, the latter contains the controls on the rotor side. Providing controls on both sides however, may be a thing for the future. A general schematic diagram of a doubly-fed induction machine is shown in Fig. 1.

Simulation of induction machinery without any switching control circuitry is a relatively simple task. One needs to concentrate only on providing the right electromechanical description of the machine. The description is usually given in a reference frame different from the natural frame of description to avoid time varying matrices. Presence of switching circuitry complicates matters as the system now goes into different modes of operation depending on the states of the power electronic switches (ON/OFF). Further, to determine the states of the switches, one needs to know the motor terminal variables in the natural reference frame.

Several methods have been reported in the literature for digital simulation of such systems. One of these is to assume the waveforms of voltages or currents provided by the power electronic converter (as a function of time) and concentrate attention on the machine only. Ward et al.[1] follow this approach for a stator supplied, inverter fed machine running at constant speed. Enjeti and Lindsay[2] follow this method for a complete transient study of a pulse width modulated (PWM) inverter fed induction machine. The stator voltage waveform is assumed in these cases. Subrahmanyam et al.[3], make assumptions on the rotor current waveforms to study a doubly-fed machine system. In this type of approach, no information is generated about the behaviour of the switching circuit and is hence it is limited in use and application.

In an attempt to include the converter behaviour, the second method considers various modes of operation that the system can enter into as a result of device switching. System equations are then formulated for each of these modes and the algorithm for simulation then selects the appropriate set of equations depending

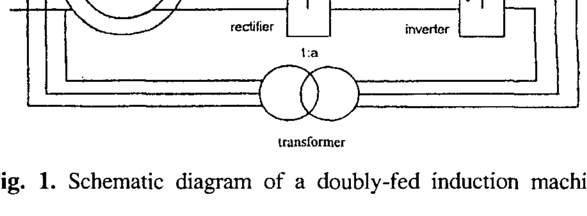

Fig. 1. Schematic diagram of a doubly-fed induction machine.

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on device state changes. An example of this approach can be found in the analysis of an AC voltage controller fed induction machine drive by Nath and Berg[4] (singly-fed), and a slip energy recovery system by Akpinar and Pillay[5] (doubly-fed). This approach requires considerable labour to formulate the different sets of equations. It also requires a good apriori knowledge of the working of the system to determine the actual modes of operation out of the several switch state combinations. This would be very difficult or almost impossible for a new topology.

In a third approach that is found in the literature, researchers have attempted to formulate equivalent circuits for the machine. Lavers and Cheung[6] propose an equivalent for the stator controlled machine (singly-fed). Ghani[7] developed an equivalent circuit with controlled sources for use with SCEPTRE. The equivalent circuit of[7] was demonstrated only for a singly-fed case, while the doubly-fed operation is mentioned as a possibility. Equivalent circuit based approaches lack the flexibility of description of the machine that equation based approaches (the first and second methods, for example) provide. The system matrix size increases by a large number, 23 for the case of[6], when the modified nodal (MN) method of system analysis is used. However, equivalent circuit based methods give a way of handling mode changes automatically.

More recent approaches attempt to include the machine equations directly into the system equations without using equivalent circuits. Liu and Chang[8], attempt a tensor oriented approach and use two different models for the singly-fed and doubly-fed systems. Sudha et al.[9], report a similar method for the singly-fed case using MN formulation. Though the system matrix size does not increase by big numbers in these methods, lack of flexibility of description still persists as in the third approach since the machine description is embedded into the system equations.

Thus, the above approaches used for the simulation of induction machine system transients therefore, suffer from one or more of the following drawbacks :

1. Neglect of power electronic circuit transients[1-3]
2. Enormous manual labour[4, 5]
3. Prior knowledge of system operating modes is required[4, 5]
4. Lack of flexibility in describing the induction machine [6-9]
5. Different models for singly-fed and doubly-fed systems[8]
6. Inclusion of the rotor angle in the machine description leading to a computationally inefficient approach[4, 7]

It is the purpose of this paper to present an efficient method to model power electronic controlled induction machine systems that overcomes the drawbacks described above. The machine model proposed consists of two parts. One is a simple series RLE circuit. The other part is the detailed electromechanical description of the machine as equations based on standard dq-model. System equations are formulated using the circuit part of the machine description together with a topological description of the circuitry connected to the stator and rotor sides of the machine. The electromechanical description of the machine determines how the equivalent circuit reacts to the stimulus from the external circuitry. The proposed algorithm for simulation establishes this interaction to derive the transient response of the system. Use of an RLE equivalent circuit to represent the induction machine is not new[10, 11], but the representation has been used only for the steady state with an assumed waveform for E. In this paper, the RLE representation has been used to decouple the machine equations from the system equations to study the transient response also for any arbitrarily configured stator/rotor side power electronic circuitry.

Section II briefly describes the modified nodal method of system analysis which is used in this work for the formulation of system equations. A hybrid switch model that has been used to model the power electronic devices is also discussed in this section. Section III describes the machine representation while section IV discusses solution procedure. In section V simulation results are compared with those of laboratory experiment and with those of previous research workers to validate this approach. Finally, conclusions highlighting the advantages of this approach are presented in section VI.

II. Modified Nodal Formulation and Switch Modelling

The modified nodal method of formulating system equations was introduced in[12], to overcome the deficiencies of the nodal approach, and is now quite a popular approach for circuit simulation. In this, apart from the node voltages, the voltage source and inductor currents are also treated as system variables. The element current-voltage relations of the voltage sources and inductors are included to complete the equation set. The inductors and capacitors of the circuit are converted to their associate resistive forms by discretizing their dynamic relations by a suitable implicit integration algorithm. These associate resistive equivalents are used to formulate the system equations. The problem then reduces to one of solving simultaneously a set of linear equations of the form $\mathbf{Ax} = \mathbf{b}$ for every time step[13].

Binary valued resistors have been popular as macromodels for power electronic devices when the focus is on system simulation. Computational advantages that arise due to the fact that the resistor is a non-dynamic element, have also been pointed out [13]. While the value of the resistor used to model the device in the ON state can be determined from the static characteristics of the device found in manufacturer's data sheets, no clear rules exist for choosing the value of resistor in the OFF state. The choice of this value has therefore been based on numerical considerations, with higher values approximating more closely the ideal nature of the switch but resulting in an ill-conditioned system matrix. To model the device in its OFF state, it would be wiser to borrow Murakami's[14] idea of using a zero valued current source. Thus in this work, we have used a hybrid model for the device – a mixture of ideal (OFF state) and non-ideal (ON state) switches. The advantages of this approach will be spelt out presently.

Power electronic devices can be included in the MN formulation easily by introducing their currents as variables and including their element V-I relations to complete the equation set. Using the hybrid model, the relations for devices during the ON state become

$$-V_1^{(n+1)} + V_2^{(n+1)} + i^{(n+1)}R = 0 \tag{1}$$

where V_1 and V_2 refer to the voltages at the terminals of the device with the current direction being from terminal 1 to terminal 2, i being the device current. The superscript (n+1) refers to the (n+1)th computational instant. R is the value of the resistor used to model the device in the ON state. During the OFF state, the device equation becomes

$$i^{(n+1)}=0 \tag{2}$$

The left hand sides of equations (1) or (2) are included in matrix A, while the right hand sides are included in matrix B. The advantages in this approach are as follows :

If equation (1) had been used to model the device in the OFF state, the value of R would have been high. This is avoided by use of (2). Apart from eliminating the problem of choosing a suitable value for the OFF state resistor (which could be circuit dependent), this has been found to reduce the condition number of the system matrix (by three orders of magnitude compared to a case where the device is modelled by an OFF state resistance of 1MΩ).

The right hand side of the equation used to model the device is zero irrespective of the state of the device. This enables the speedup procedures of[13] to be used.

The static models for several devices in the ON state could be enhanced by adding a constant voltage source in series with the resistor. This gives a good picture of the ON state losses of the device. This detail, if required, can be included very simply by making the right hand side of the ON state equation to be the negative of this voltage source (magnitude can be determined from data sheets).

The leakage currents of devices remain practically constant with applied voltage in the OFF state. This can also be included easily by altering only the right hand side of the switch equation. This entry is then equal to the negative of the leakage current value.

Thus this hybrid approach, in conjunction with the introduction of the device current as variable, permits inclusion of greater detail in modelling of the switch without affecting the system matrix. The inclusion of the details in points (3) and / or (4) above, would ofcourse mean that simulation would be slower.

III. Induction Machine Model

In this section the induction machine model used is described. Unlike the approaches used earlier, this model spans two domains of description. It consists of an equivalent circuit in part (circuit domain) while the other part consists of the electromechanical description of the machine (equation domain). This set of equations can be in any frame of reference suitable for the particular case under study. It may or may not include the effects of flux saturation or variation of rotor parameters with speed. We used the standard stator attached dq-model so that the number of describing equations will be only four.

A pictorial representation of the machine model used is shown in Fig. 2, connected to some circuitry on the stator as well as on the rotor. Both sides may contain switches in general, configured in any arbitrary fashion. The resistors used in the circuit are the resistances per phase of the induction machine and the inductors are leakage inductances per phase, both values being given in the natural frame of reference (stator described in a reference frame attached to the stator and rotor in a reference frame attached to the rotor). $E_{1\text{-}6}$ are fictitious EMF sources, and

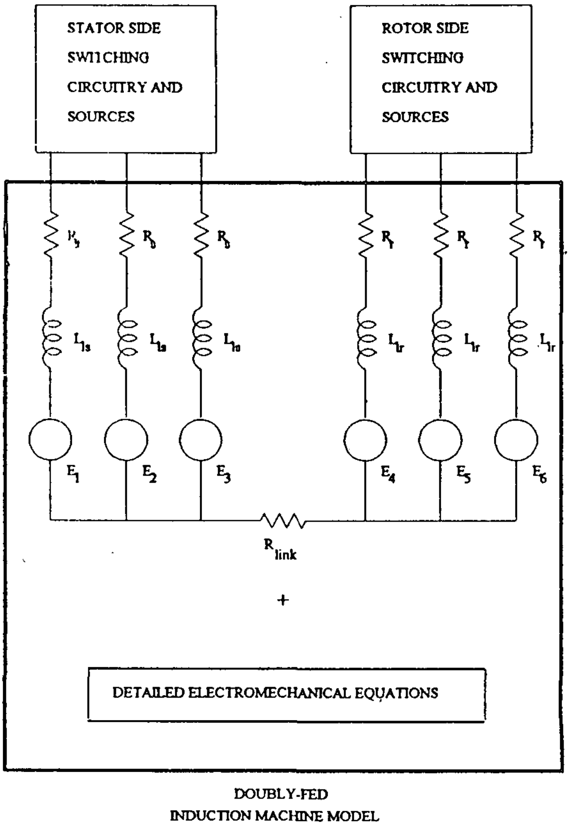

Fig. 2. Mixed domain model for the doubly-fed induction machine.

their values have to be determined during simulation. No assumption is made with respect to the nature of these EMF sources. The resistance Rlink used in the model is to aid the formulation of circuit equations and its value can be chosen arbitrarily. The detailed equations describing the machine are also a part of the model, as described earlier. When it is desired to include the effects of varying machine electrical parameters which are present in the equivalent circuit, say the rotor resistance with speed for example, it is convenient to think of it as having a constant part and a variable part. The equivalent circuit can then comprise of only the constant parts, while the voltage drop across the variable parts can be conveniently included in the fictitious voltage source. The actual varying parameters can be a part of the machine equation description. Thus the equivalent circuit can be made invariant to varying machine parameters.

IV. Formulation of Equations and Solution

System equations are formulated using the MN method. The dynamic elements of the circuit are discretized using Gear's second order algorithm. Using this, the equation for the inductor becomes

$$-V_1^{n+1} + V_2^{n+1} + (3/2)*L/(dt)\ i^{n+1} = (4*i^n - i^{n-1})*L/(2*dt) \qquad (3)$$

where V_1 and V_2 refer to the voltages at the terminals 1 and 2 of the inductor, i is the current through the inductor from terminal 1 to 2 and superscripts refer to the computational instants, with (n+1) being the present instant. dt is the time step at the (n+1)th instant, and L is the value of the inductor. The capacitor equation becomes

$$V_c^{n+1} = 4/3 * V_c^n - 1/3 * V_c^{n-1} + 2/(3C) * i_c^{n+1} *dt \qquad (4)$$

where V_c refers to the voltage across the capacitor, and C is the value of the capacitor and i refers to the current through the capacitor.

With these discretizations, the equations for the system comprising the externally connected circuitry and the equivalent circuit part of the machine model can be formulated using the MN method. The equation oriented description of the machine is segregated as a subroutine. The algorithm described below establishes proper interaction between the modified nodal formulation and the subroutine containing the machine equations to determine the transient response of the system.

The algorithm used is described as a flow chart in Fig. 3, and is self explanatory. The machine description provided in the subroutine can be in any frame of reference. The stator attached reference quadrature axes (d, q) frame of description does not involve the rotor angle and hence the machine description is constant. However, the computation of currents and voltages by the modified nodal formulation is in the natural reference frame, which is essential for determination of the switching circuit performance. Therefore, the currents and voltages must be transformed to the appropriate frame of description of the machine in the subroutine and then the variables computed must be transformed back before being given back to the simulator. This computation requires the value of the rotor angle which is also computed as part of the solution. However the matrices involving the rotor angle which are used in the transformations need not be inverted.

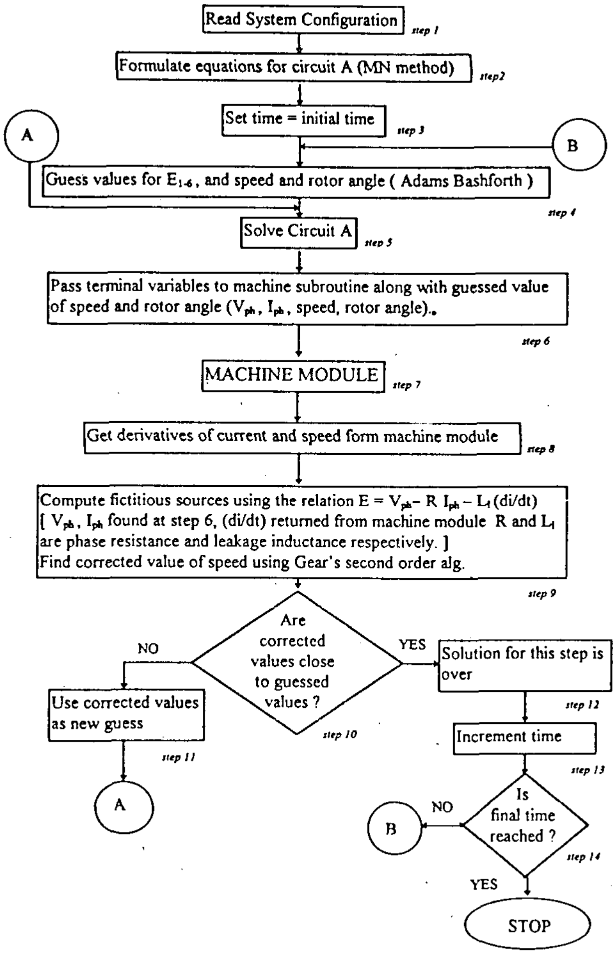

Fig. 3. Flowchart for the algorithm used for determination of transients in doubly fed induction machine systems.

Determination of the exact instant of switching of power electronic devices is crucial to the accurate determination of the transient response. This aspect of computation has been omitted from the flowchart for the purposes of clarity. Fig. 4 shows the action the algorithm takes when it notices that the state of a device needs to be changed. The curve shown in the figure could be taken to be the current through a diode for the purposes of explanation. When the current becomes negative at a computational instant, that computation is rejected. The algorithm then tries to locate the exact time instant at which the device changes state by varying the time step of simulation from the previous

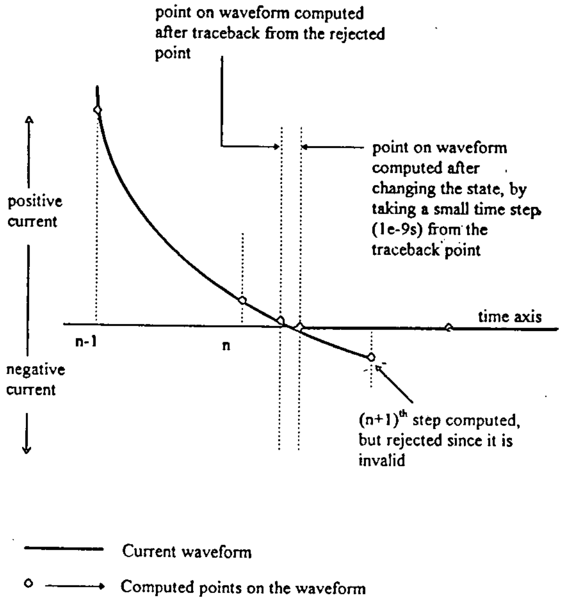

Fig. 4. Action taken by the algorithm when the current through a diode goes through zero.

instant. The 'exact switching instant' is then found within a specified tolerance interval (0.1ms, in this work) and simulation proceeds after changing the state of the device.

It can be seen from the flowchart of Fig. 3 that the waveforms of the fictitious voltage sources in each phase are computed during the solution. The RLE representation for the induction machine has been used before in many cases[10, 11], but they have been used as steady state representations. The algorithm outlined above permits a way to calculate the EMF sources in transient condition without any assumptions. In some converter configurations, the system goes into a mode of operation where the terminals of the motor are open circuited periodically. The presence of the EMF source in the equivalent circuit allows us to determine the motor terminal voltage in such cases accurately.

V. Results

The switch model, machine model and the algorithm have been tested on several configurations and found to give very goodresults. The results of two such studies are presented below.

1. Voltage source inverter feeding an induction machine

An experimental setup of a voltage source inverter fed induction machine (singly-fed case) as shown in Fig. 5 was run in the laboratory and the measured stator phase voltage and current waveforms are shown in Fig. 6. This is a very common technique used for speed control of induction motors. The setup was simulated using the techniques mentioned above. The rotor terminals of the model were shorted to achieve the singly-fed mode of operation. The results of the simulation are shown in Fig. 7. It can be seen that the results agree very well with those measured experimentally. The slight discrepancies in the smaller peaks of the current waveforms can be attributed to errors in determination of the machine parameters. The induction machine used for the experiment was a 3 KW, 380 V, 50 Hz, 4 pole, star connected machine with $R_s = 1.44\ \Omega$, $R_r = 1.97\ \Omega$, $L_s = L_r = 8$ mH, $L_m = 0.126$ H, and Moment of Inertia = 0.1 Kgm^2. The load torque used for the experiment was 1.0 Nm. All machine parameters are

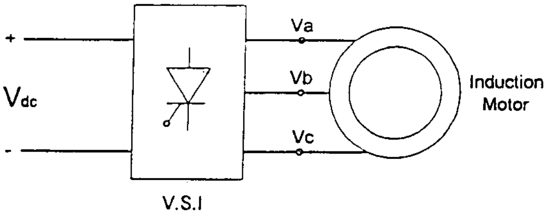

Fig. 5. Schematic diagram of a voltage source inverter fed induction machine.

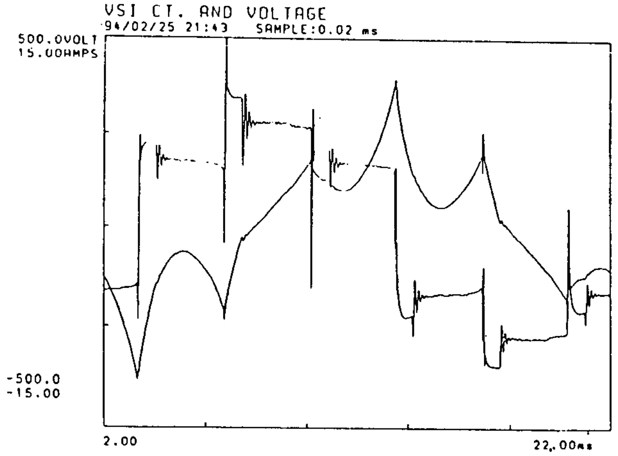

Fig. 6. Stator current and phase voltage waveforms for a VSI fed induction machine : Experimental results.

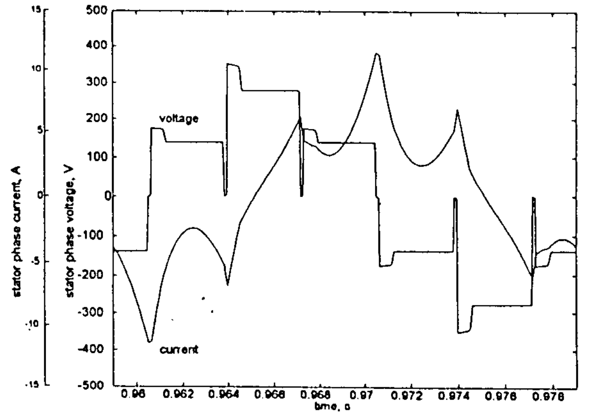

Fig. 7. Stator current and phase voltage waveforms for a VSI fed induction machine : Simulation results.

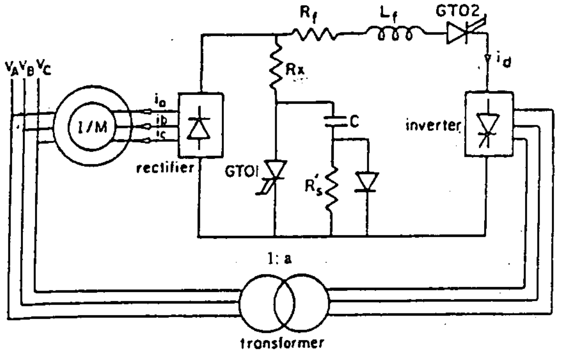

Fig. 8. Schematic diagram of a slip-energy recovery scheme of an induction motor.

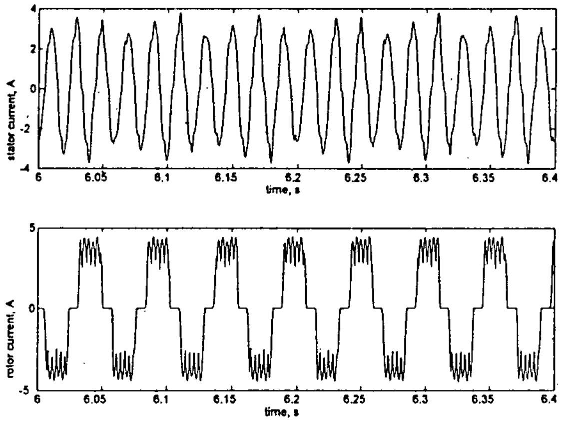

Fig. 9. Stator and rotor current waveforms of the slip energy recovery system.

referred to the stator.

2. Induction machine in a slip energy recovery scheme.

At very high powers, it becomes advantageous to recover the power that is wasted in the rotor of the induction machine.

Akpinar and Pillay[5, 15], report theoretical and experimental investigations on one configuration used for slip energy recovery scheme shown in Fig. 8. Power from the rotor is fed back to the mains while the motor is fed from the stator. This is a doubly-fed operation.

Some results of the simulation of this scheme are shown here. Fig. 9 shows the stator and rotor current waveforms and Fig. 10 shows their harmonic spectrum. From the theory of induction machines, we know that harmonics in the rotor current waveform are reflected on to the stator side. The 95 Hz fifth harmonic on the rotor would get reflected as 64.2 Hz frequency on the stator which causes the stator current waveform to have an envelope. These phenomena are predicted well by the approach outlined in the earlier sections as shown by the waveforms of Fig. 10. The current waveforms themselves match well with the experimental investigations of[15]. The machine details for this investigation are as follows (from [15]) : 3.5 KW, 380 V, 50 Hz, 4 pole, star connected machine with rotor/stator turns ratio of 0.4. The parameters in the natural reference frame are $R_s = 0.90\ \Omega$, $L_s = 6.94$ mH, $M_s = -133.15$ mH, $R_r = 0.2\ \Omega$, $L_r = 1.1$ mH, $M_r = -21.3$ mH, $M_{sr} = 106.5$ mH, Moment of Inertia = 0.2 Kgm^2. The load torque used was 4.0 Nm.

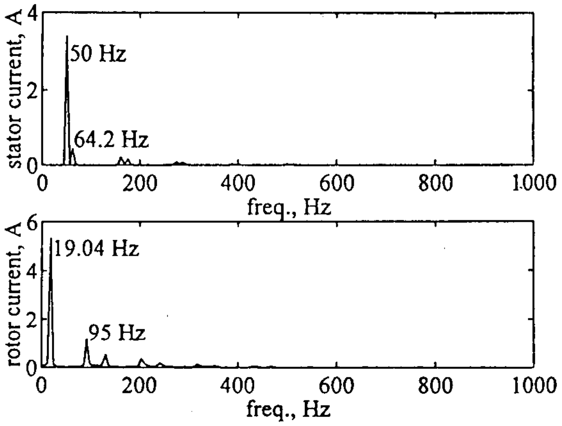

Fig. 10. Frequency spectra of stator and rotor current waveforms of the slip energy recovery system.

VI. Conclusions

A unified approach to the modelling and simulation of converter fed induction machine systems – both singly-fed and doubly-fed – has been presented. The approach is also modular as far as the machine description is concerned. Simulation results have been compared with experimental and previously published results to establish the validity and accuracy of this approach. The method has the following advantages :

(1) The machine model spans two domains of description viz., the topological circuit description and the physical equation domain. This gives the advantages of a circuit oriented approach for simulation where mode changes can be handled automatically, and an equation oriented approach which gives flexibility of modelling the machine.

(2) A series RLE circuit has been used to represent each phase of the machine. This enables us to determine the motor terminal voltages accurately even when the machine phases open.

(3) No assumptions are made regarding the nature of the EMF sources in the equivalent circuit. Their values are computed as the transient solution progresses.

(4) The hybrid switch model used gives better conditioning of the system matrix and a possibility to determine the ON state losses. At low switching frequencies the device losses can be approximated by the ON state losses.

(5) The machine description can be given in any reference frame. It can also include slip dependent parameters and effects

of saturation. The circuit part of the machine description can be made invariant to these changes, resulting in a system description that is independent of the machine model.

(6) Mode changes are handled at the equivalent circuit level and hence the machine equations provided need not reflect this aspect.

(7) The machine description presented could be used with methods other than MN approach as well. However, current trends indicate that this is the preferred approach for circuit simulation.

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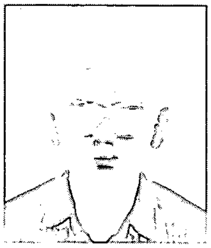

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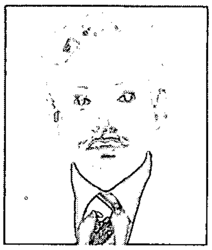

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