Active / Reactive Compound Compensation in Distribution System

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Abstract

In this paper the use of compensation based on a combination of active plus reactive power at distribution model system is proposed. The basic voltage-power relationships for the linearized case on an infinite bus are used and the compensation angle is defined based on the voltage magnitude response to small power injection. Compensation is supplied at several locations, and the system is subjected to varying fault scenarios, with its response observed under different system conditions. A number of control issues for a storage-based, active / reactive power compensator as a bus voltage regulator are explored to compare the effectiveness of active / reactive against reactive-only compensation.

I. Introduction

The correction of voltage fluctuation problems at distribution system is important because there are many dynamic loads and line faults. Over the past decades, the applications of reactive power-based compensating devices, such as Static Var Compensators, have become fairly well understood. The application of SVCs to compensate the voltage drop at the distribution system level has received considerable attention in recent years[1-4]. Recent articles have discussed the dynamic simulation and the transient behavior analysis of SVCs in distribution system. These contain the simulation study for the correction of voltage dips due to large motor startups with SVCs and the interaction between the AVC (Adaptive Var Compensator) and the distribution system[5-9].

Previously, most of the compensations in distribution system have been performed by reactive compensation like static var compensations. However, in this paper the use of compensation based on a combination of active plus reactive power at distribution model system is prescribed.

At first the basic voltage-power relationships for the linearized case on an infinite bus is induced. Based on this relation the compensation angle, which represents the condition at which maximum voltage magnitude compensation is achieved for a fixed amount of current, is induced. Simulation was done in 24-bus distribution model system under a number of control issues. System response is observed by some case studies; section II support, transformer out at bus 660, short fault in section II and transmission voltage fluctuations.

Finally the task of regulating bus voltage magnitude at a target bus, and the problem of regaining a degree of freedom by allowing the compensator output voltage (or current similarly) to change are considered.

II. Linearized Equation Model

Consider a single load bus connected to an infinite bus through a complex impedance. Designate V_L as the load bus voltage magnitude, V_i as the infinite bus voltage magnitude, δ as the phase angle between them (referenced to the infinite bus), Z as the interconnecting impedance amplitude, and θ_Z as the impedance angle.

Eq. (1) and (2) give the active and reactive power consumed at the load bus as a function of the above parameters[2].

$$P = \frac{V_L V_i}{Z}\cos(\delta + \theta_Z) + \frac{V_L^2}{Z}\cos\theta_Z \tag{1}$$

$$Q = \frac{V_L V_i}{Z}\sin(\delta + \theta_Z) + \frac{V_L^2}{Z}\sin\theta_Z \tag{2}$$

In considering the active power equation, the maximum power is transferred at a bus angle of $\delta = -\theta_Z$ when V_L, V_i and Z are given. Furthermore, for $\theta_Z < 90°$, active power is transferred even when $\delta = 0$.

Eq. (1) and (2) return bus power as a function of the load bus voltage magnitude and phase angle. It is equally possible to express bus voltage magnitude and phase angle as functions of the bus active and reactive power. As it turns out, even for the

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simple two-bus case, the resulting 4th order solution is difficult to write in close form, and is most expediently solved using a power flow.

For the two bus example, consider the functions f and g in Eq. (3) and (4), which define load bus voltage magnitude and phase angle in terms of the bus powers P and Q, and the remaining system parameters, designated $\underline{x}$.

$$V_L = f(P, Q, \underline{x}) \tag{3}$$

$$\delta = g(P, Q, \underline{x}) \tag{4}$$

If a small incremental power, $\Delta S = \Delta P + j\Delta Q =$ constant is applied at the load bus, the resulting changes to first order in bus voltage magnitude and phase angle can be written as in Eq. (5) and (6).

$$\Delta V_L = \frac{\partial f}{\partial P}\Big|_{P,Q,\underline{x}} \Delta P + \frac{\partial f}{\partial Q}\Big|_{P,Q,\underline{x}} \Delta Q \tag{5}$$

$$\Delta \delta = \frac{\partial g}{\partial P}\Big|_{P,Q,\underline{x}} \Delta P + \frac{\partial g}{\partial Q}\Big|_{P,Q,\underline{x}} \Delta Q \tag{6}$$

As a next step, consider the injection of a finite amount of Volt-Amperes at the load bus, subject to the constraint $\Delta S^2 = \Delta P^2 + \Delta Q^2$. Using this relation and Eq. (5), the ratio between ΔP and ΔQ that produces the greatest change in ΔV_L for a fixed ΔS could be solved. Taking the arc tangent of this ratio, it can be written the expression for the Compensation Angle, Φ_{comp}, as in Eq. (7).

$$\Phi_{comp} = \tan^{-1}\left[\frac{\dfrac{\partial f}{\partial Q}}{\dfrac{\partial f}{\partial P}}\right] \tag{7}$$

For a finite magnitude of compensating Volt-Amperes, this angle refers to the ratio between active and reactive components which produces the maximum change in bus voltage amplitude for the linearized case. As in most utility applications current is approximately constant, the Volt-Amperes are roughly proportional to current magnitude. From this perspective, Φ_{comp} represents the condition at which maximum voltage magnitude compensation is achieved for a fixed amount of current.

III. Compensated System Analysis

3.1 Model System

Fig. 1 shows the one-line diagram for the 12 KV, 23-bus distribution model system. The network is divided into three sections, with sections I and III fed from the transmission system (modeled as an infinite bus) through a pair of transformers. Section II is connected to sections I and III through switchable lines at busses 673 and 676. Table 1 gives the line and transformer data for the bus interconnections. The pu impedance values shown are given for the individual line base MVA at 12 KV operation. Bus loading coefficients and capacitive compensation data as shown in Table 2.

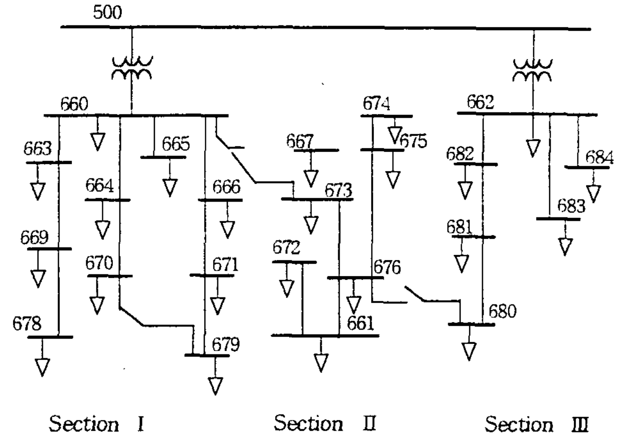

Fig. 1. 23-bus distribution model system.

Table 1. Data for distribution model system.

From	To	KV	MVA	Km	R(pu)	X(pu)	B(pu)
660	663	12	16	1.6	0.090	0.200	0.000
660	664	12	16	3.2	0.180	0.390	0.000
660	665	12	4	0.8	0.170	0.230	0.000
660	666	12	16	0.8	0.045	0.100	0.000
660	673	12	16	4.0	0.224	0.500	0.000
661	672	12	8	4.8	0.540	1.070	0.000
661	676	12	21	3.2	0.135	0.292	0.000
662	682	12	16	1.6	0.090	0.195	0.000
662	683	12	8	8.0	0.900	1.950	0.000
662	684	12	16	6.4	0.360	0.780	0.000
663	669	12	16	9.6	0.270	0.590	0.000
664	670	12	8	5.6	0.360	0.780	0.000
666	671	12	8	5.6	0.600	0.810	0.000
667	673	12	20	1.6	0.020	0.012	0.008
669	678	12	4	0.8	0.679	0.920	0.000
670	679	12	4	3.2	0.510	0.690	0.000
671	679	12	4	2.4	0.679	0.920	0.000
673	676	12	16	3.2	0.090	0.195	0.000
674	675	12	10	0.8	0.010	0.006	0.004
675	676	12	16	3.2	0.180	0.390	0.000
676	680	12	8	1.6	0.169	0.229	0.000
680	681	12	8	1.6	0.170	0.230	0.000
681	682	12	16	1.6	0.090	0.195	0.000
500	660	12/69	80	-	0.006	0.100	0.000
500	662	12/69	80	-	0.006	0.100	0.000

3.2 Compensator Modeling in the Power Flow

In this section the multiple compensation scenarios, injecting both reactive-only and active/reactive power at a number of

Table 2. Bus loading coefficients and capacitive compensation data.

Bus	Section	K_P (MVA)	K_Q (MVAR)	Q_C (MVAR)
660	I	1.00	0.50	
663	I	0.85	0.40	-0.61
664	I	0.30	0.14	-0.67
664	I	1.00	0.50	
665	I	0.40	0.15	
666	I	0.15	0.08	-0.17
669	I	0.25	0.10	-0.33
670	I	0.05	0.01	
671	I	0.30	0.15	-0.41
678	I	0.06	0.03	
679	I	0.04	0.02	
661	II	0.25	0.10	
667	II	0.40	0.15	
672	II	0.08	0.03	
673	II	0.50	0.25	-0.53
674	II	0.12	0.06	
675	II	0.10	0.03	
676	II	0.20	0.08	-0.17
662	III	0.00	0.60	
680	III	0.20	0.10	
681	III	0.70	0.28	
682	III	0.40	0.10	
683	III	0.20	-.02	
684	III	0.40	0.10	

K_P, K_Q : active, reactive load power coefficient

Q_C : capacitor coefficient

different locations and under a number of switch states, are explored. The compensator is modeled as an additional bus attached to the network at some location through a transformer. The connecting transformer has 10% reactance and an X/R ratio of 10 on a base of 8 MVA.

Fig. 2 shows the compensator connected to a network bus through the transformer. In the figure the network bus voltage is labeled Vterm while the compensator bus voltage is labeled Vint. This nomenclature is analogous to a synchronous machine, with its terminal and internal voltages. Using this analogy, synchronous condensers have the ability to regulate Vterm by varying Vint are observed. Similarly, power electronic-based, reactive-only compensators such as STATCONs and SVCs can regulate Vterm directly, in the STATCON case by adjusting the energy in the storage element (and therefore Vint) in such a way so as to keep Vterm at the setpoint and injected active power at zero.

Consider a power electronic-based compensator where active power is supplied from a storage element, such as a battery-driven VSI(Voltage Source Inverter), and note that for normal step operation, the bus voltage will be at a fixed relation to the battery voltage. To operate in this mode it becomes impossible to both regulate Vterm as well as the amount of power injected by the compensator to the network. Since the purpose of this paper in some part is to compare the effectiveness of active/reactive compensation against reactive-only compensation, and since desiring this to be an apples-to-apples comparison, it will be assumed that active/reactive compensator has the ability (over at least a minimal range) to vary Vint. This method could be used to examine the case of in section 3.4.

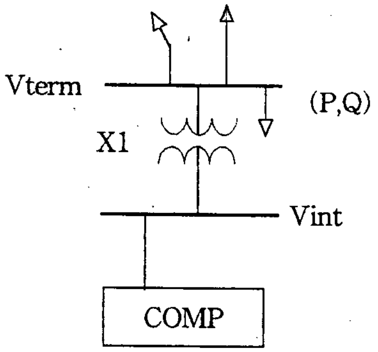

Fig. 2. Compensator connection to load bus via transformer.

3.3 Case Study

(A) Section II Support

In this case a compensator is attached to the network in section II at bus 661. This bus was shown to be susceptible to low voltage under load in the power flow studies. The compensator is configured to hold the bus 661 voltage at 1.0pu. Table 3 shows a comparison of reactive-only versus active/reactive compensation at minimum compensator MVA at varying load levels. For a given test, the actual demand power at the bus is found by multiplying the load coefficients by a global loading parameter k. For the full network with all line switches closed, values of k in range 0-1 correspond to a light load, while k values in the range 2-3 correspond to a heavy load, with multiple load busses near a lower limit of 0.9pu at k = 3. Value of k near 5 represent an extreme load, with most bus voltages down 20% or more, and several near the point of collapse.

At zero load in the network, with a transmission voltage of 1.02pu, the compensator has to absorb power. At this load level the compensator MVA output level is minimum at a Q/P ratio of approximately 65°, which is roughly the impedance angle of the system lines. The minimum MVA output is at 89% of the purely reactive level.

Table 3. Section II compensation at bus 661 for different load levels.

k	Q-only	Minimum MVA			
	Q_R(MVAR)	P_G(MW)	Q_G(MVAR)	S_{TOT}(MVA)	S/Q_R(%)
0	-0.83	-0.31	-0.68	0.74	89
1	0.42	0.17	0.33	0.37	88
3	3.55	1.50	2.48	2.90	82
5	8.31	3.30	4.95	5.95	72

As k increases, the Q/P ratio drops to an angle of 56° at k = 5. Similarly, the ratio of minimum MVA to reactive-only MVA drops somewhat, to a value of 72% at k = 5. These results indicate that even for high load levels, reactive compensation is on the whole more effective for supporting section Ⅱ voltages in an intact network.

(B) Transformer out at Bus 660

The transformer connecting the distribution system to the transmission system at bus 660 is disconnected. Two sets of tests are run. In the first set, the system is operated at a light load (k = 0.5). The effectiveness of compensation at bus 660 versus bus 661(location of compensator in previous case studies) is compared.

Table 4 shows the compensation levels for reactive-only(case 1) versus active/reactive (case 2) compensation at a load level of k = 0.5. At both bus locations the compensator regulates the load bus voltage to 1.0pu. At both positions, the minimum MVA point is located at a power factor angle of approximately 40°. When the compensator is placed at bus 660, however, the overall MVA amount to support the same level is approximately 30% less than when the compensator is located at bus 661.

Table 4. Comparison of compensator locations for bus 660 transformer out.

Com. Loc.	Power Source	Case 1			Case 2		
		P_G	Q_G	S_{TOT}	P_G	Q_G	S_{TOT}
660	Tx System	5.2	-0.16	5.20	4.60	-0.18	4.60
	Compensator	0.0	0.74	0.74	0.31	0.27	0.41
	Total	5.2	0.58	5.20	4.90	0.49	4.90
661	Tx System	5.2	-0.58	5.2	4.40	-0.48	4.40
	Compensator	0.0	1.00	1.0	0.49	0.41	0.64
	Total	5.2	0.42	5.2	4.90	-0.07	4.90

In the second simulation test, an active / reactive compensator is placed at bus 660 and the system is operated at load level k = 3. Note that at this load level, a reactive-only compensation solution does not exist. The effects of operating bus 660 at a voltage setpoint of 1.0pu versus 0.95pu are then compared with the minimum MVA point. The resulting power injections from the transmission system and compensator are summarized in Table 6.

Based on the sensitivity results with transformer feed at bus 660 open(Table 5), the compensator is operating at a power factor angle of approximately 33°, or a power factor of 0.84 at both voltage setpoints. This occurs in comparison to an overall power factor angle of 19° for the transmission system when both transformers are operating. It suggests that as the connection to the transmission system weakens, the compensator acts less like a compensator and more like a primary generator, in that the power factor angle tends to become what required by the load/network pair.

Table 5. Sensitivity results with transformer feed at bus 660 open.

Bus	k = 0		k = 0.5			
	Φ_{comp}	$\%\Delta V/\Delta S$	Angle	V_{MAG}	Φ_{comp}	$\%\Delta V/\Delta S$
660	42	14.1	-28	0.76	33	177
671	44	22.1	-30	0.77	31	219
678	50	37.1	-30	0.76	40	246
661	45	10.7	-19	0.79	39	95
673	42	11.0	-22	0.78	37	117
676	42	9.70	-1.9	0.79	39	93
662	86	0.12	-0.3	1.00	86	0.13
680	49	6.20	-12	0.86	48	38
683	65	26.3	-1.8	1.00	64	27

Table 6. Effects of voltage setpoint on compensation levels at bus 660 with transformer out.

Power Source	V_{660} = 1.0			V_{660} = 0.95		
	P_G	Q_G	S_{TOT}	P_G	Q_G	S_{TOT}
Tx System	13.0	4.3	13.7	12.7	4.7	13.5
Compensator	13.4	8.3	16.0	13.6	8.8	16.2
Total	26.4	13.0	29.4	26.3	13.5	29.6

Of interest in this second test is the slight increase in active and reactive power required from the compensator when the bus 660 voltage is allowed to drop to 0.95pu. A likely explanation is that as bus voltages sag in section Ⅱ, the currents required to meet the section I demand are higher, and thus a greater amount of active and reactive power is consumed in the lines.

(C) Short Fault in Section Ⅱ

For this test the compensator is again attached to bus 661, regulating to a voltage setpoint of 1.0pu. The system is operated with both transformers in, at a load level of k = 0.5. Bus 674 is effectively short-circuit to ground through an admittance of 1,000 pu. Table 7 shows the compensation results for two cases: (case 1) almost entirely reactive (a purely reactance compensation solution does not exist), and (case 2) active/reactive compensation. For case 1, with the transmission system providing most of the MW, the compensator has to provide a substantial number of MVAR, to account for the substantial reactive power lost in the lines while delivering current from the transmission system to the short circuit.

Table 7. Compensation results for faulted bus 674.

Power Source	V_{660} = 1.0			V_{660} = 0.95		
	P_G	Q_G	S_{TOT}	P_G	Q_G	S_{TOT}
Tx System	19.9	6.66	21.0	7.79	6.80	10.3
Compensator	1.00	32.0	32.0	8.60	19.4	21.2
Total	20.9	38.7	44.0	16.4	26.2	30.9

The minimum MVA point occurs when the compensator takes over the majority of the power delivery from the transmission system. The compensation angle of 66° for the optimal case corresponds very closely to the impedance angle the path from bus 676 to the short at bus 674. This indicates that the network behavior from a compensation viewpoint is dominated by the shorted line impedance.

(D) Transmission Voltage Fluctuations

In this section a different aspect of compensation, specifically how the size of the distribution transformer affects the amount of compensation required to offset a fluctuation in the transmission system voltage was examined. The compensator is placed at the secondary bus of the distribution transformer, in this case bus 660, for the purpose of regulating that bus voltage to 1.0pu. As these results thus far indicate this is a reactance-dominated location, only reactive compensation is considered.

Table 8 shows the amount of compensation required for transformer impedances of $X = 0.1$ pu with an X/R ratio of 15 on bases of 80MVA and 20MVA. The connection from the compensator to bus 660 is made through a transformer with $X = 0.1$pu, with an X/R ratio of 10 on a base of 100MVA (large). The results show quite clearly that a stronger connection (80MVA) to the transmission system requires a larger amount of compensation to offset variations in the transmission system voltage.

Table 8. Compensation levels required to hold 660 at 1.0pu for different distribution transformer sizes.

	Xfmr Base = 80MVA	Xfmr Base = 20MVA
V_{TX} (pu)	$Q_{G,COMP}$ (MVAR)	$Q_{G,COMP}$ (MVAR)
0.92	72.4	19.1
1.12	-89.5	-23.0

While up until now in this paper it has appeared advantageous to have a strong connection to the transmission system, a result that suggests a certain advantage exists in not making the transmission system connection too stiff. Specifically, in some cases it appears that a larger transformer impedance reduces the amount of current required to provide voltage regulation in the distribution area. Philosophically, it points to a tradeoff between the advantages of "local autonomy" versus the benefits of "centralized support".

(E) Summary of Case Study

The results of section Π compensation at bus 661 for different load levels show that even for high load levels, reactive compensation is on the whole more effective for supporting section Π voltages in an intact network. It suggests that as the connection to the transmission system weakens, the compensator acts less like a compensator and more like a primary generator, in that the power factor angle tends to become what required by load/network pair. The network behavior from the viewpoint of compensation is dominated by the shortline impedance. In case of transmission voltage fluctuations a stronger connection to the transmission system requires a larger amount of compensation to offset variations in the transmission system voltage.

3.4. Active Compensator Control

In the previous section a purely reactive to active/reactive compensation was compared, in the context of determining the amount of power was required for a given load to hold a distribution bus at a given voltage. For a number of circumstances, if the internal voltage (i.e., voltage at the inverter) could be allowed to vary, then it is possible to regulate the distribution bus voltage to setpoint with an overall lower MVA magnitude while still maintaining control over the active power injection.

This capacity to vary the voltage at the compensator itself comes naturally to synchronous machines, and reactive compensators such as STATCONS and SVCs. However, it appears harder to come by for compensators that deliver power from a fixed DC storage element. A VSI operating from a battery is needed to operate at an essentially fixed AC voltage. Similarly, a CSI driven by a SMES coil is also needed to operate at an AC current linearly proportional to the DC current.

In this paper an alternate approach, based on the observation that with a strong connection between the compensator and distribution busses(effectively through only a transformer leakage) will be looked, it only need a small amount of voltage control range to achieve our desired ends. In the following case study, it will be considered a stored voltage-based compensator that achieves voltage regulation through a modest amount of switching.

An implementation of a simple quasi-steady state controller for the compensator, which regulates the distribution bus voltage and active power output to command setpoints, is attempted. Fig. 3 shows the block diagram for the VSI and controller. The VSI has two state variable inputs: θ and δ, θ is the pulse width angle as described in the previous section, controlling the magnitude of the inverter output voltage **Vint**. δ controls the reference firing angle for the VSI. The controller takes advantage of the following relationships:

1. increases in the inverter output voltage **Vint** (and thus θ) will tend to cause increases in the regulated bus voltage **Vterm**.
2. increases in the phase angle δ will tend to increase the amount of active power transmitted by the inverter.

Errors in voltage magnitude and injected active power are measured, and input to a pair of PI controllers which deliver the operating angles to the inverter. Also shown in the diagram is a tap control. Through the use of static switches and multiple transformer taps, the effective regulation range for the invert vol-

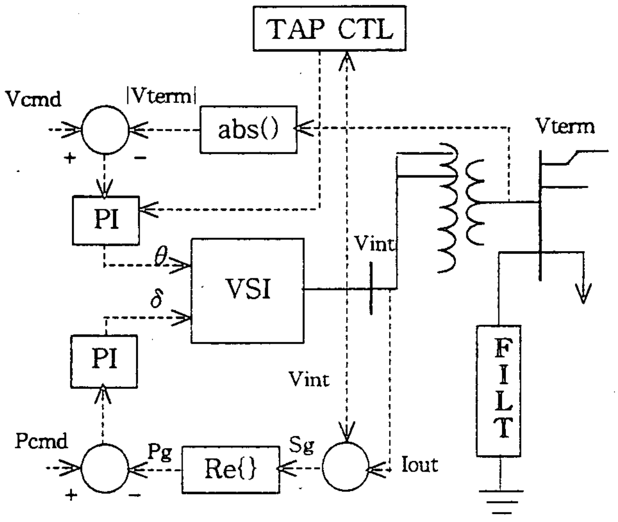

Fig. 3. Diagram of quasi-steady state (P, V) control at distribution bus.

tage can be increased for the same range of θ. The tap control block monitors the inverter output voltage, and switches as necessary (on a half-cycle basis) to the optimal transformer tap.

Fig. 4 and 5 show the same waveforms for an active power regulation test. In the first event the distribution bus drops an active power load from approximately 3.3 to 0.5pu. Transient errors are again observed on both voltage and power as the controller drops the inverter voltage and reactive output power. In the second event the power setpoint is reversed from 0.5 to -0.5pu. The controller compensates by raising the inverter output and therefore the amount of reactive power.

The above case studies show that a basic mechanism for regulating terminal voltage and active power output, which was the basis behind many of the power flow studies in the earlier sections, is fairly easily achieved. The test results show that the amount of leading or lagging reactive power required is naturally found by the controller. It is expected that other control modes would be required for a practical system, including fast-reacting modes which mitigate the transient errors seen in voltage and power as a result of step changes in load. It would also be interesting to implement an additional control mode which finds the optimal steady state Volt-Ampere output point, as indicated in the preceding studies.

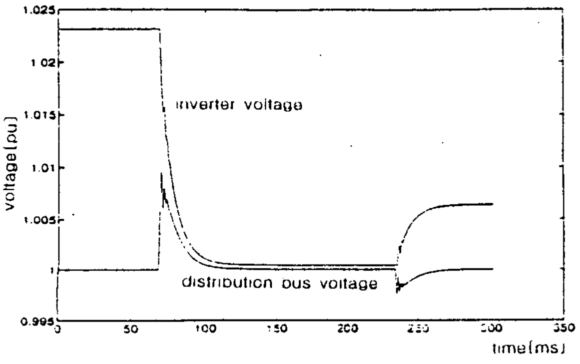

Fig. 4. VSI, distribution bus voltage magnitudes for power setpoint test.

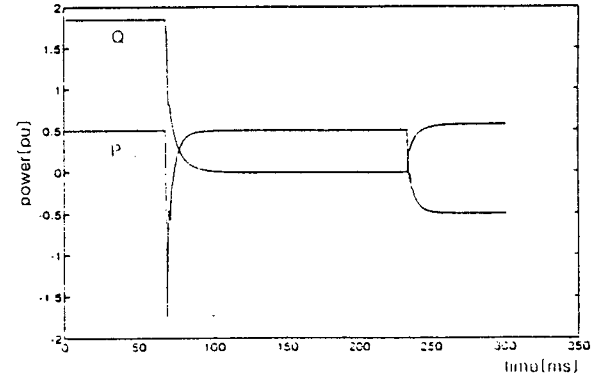

Fig. 5. VSI active and reactive power outputs for power setpoint test.

IV. Conclusions

An active/reactive compound compensation studies at the distribution system level has been described. For the given load coefficients, overall the uncompensated network is VAR-deficient in 23-bus model system. Due to the weaker connections between section Π and other sections, the system lacks the ability to stay up in entirety at all but light loads when one of the two transmission connections is lost. Depending on the load level, compensation angles ranges from 30 to 60° and active power is more effective than reactive power for voltage support in several fault scenarios.

The compensation angles which were in reasonable agreement with those predicted by linearized results and the net MVA savings in the range of 10 to 40%, depending on the circumstance, were observed. A number of cases during faults, where a steady-state solution was unobtainable without some active power compensation were encountered.

With the inverter voltage control capability it is fairly straightforward to regulate the target bus voltage magnitude while maintaining control over the active power output. The limiting case being that when the transmission line was tripped completely, in which case the role of the compensator changes to that of power supply, whatever power factor the load demands.

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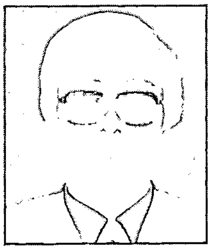

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