# Concatenated Orthogonal / PN Spreading Scheme for a Multitone CDMA System

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# **Abstract**

In this paper, we propose a new communication scheme combining both MT modulation and DS-CDMA with a concatenated orthogonal / PN spreading sequence. This scheme incorporates the advantages of both DS-CDMA with a concatenated sequence, and an MT modulation technique to combat the effect of a multipath fading channel. It is shown that the proposed system outperforms the MT-CDMA system with a conventional PN sequence.

## I. Introduction

Direct sequence code division multiple access (DS-CDMA) is currently of great interest for third generation wireless personal communication systems. It provides good immunity to interference and fading. On the other hand, research on multitone (MT) modulation, also known as orthogonal frequency division multiplexing (OFDM), has been performed for wireless communications [1].

Exploiting the advantages of both DS-CDMA and MT modulation technique, multicarrier CDMA systems have been proposed [2, 3]. Recently Vandendorpe [4] has analysed the performance of an MT-CDMA system with a conventional PN sequence, in which the advantage of an MT spread spectrum transmission has been shown.

In this paper, we propose a new communication scheme combining both MT modulation and DS-CDMA with a concatenated orthogonal / PN spreading scheme. This proposed system incorporates the advantages of both DS-CDMA with a concatenated orthogonal / PN spreading scheme, and MT modulation technique to combat the effect of a multipath fading channel. The system performance is investigated for maximal ratio combiner (MRC). The performance of the receiver is analyzed in terms of bit error rate (BER) under a multiuser and multipath Rayleigh / Rician fading channels.

# II. System Models

We consider the synchronous transmission of an MT-CDMA

system where the benefit of concatenated sequence can be exploited.

The block diagram of the proposed MT-CDMA system is shown in Fig. 1. At the transmitter, the kth user's binary data stream  $b_k(t)$  of rate M/T is first converted into M parallel binary data sub-streams  $(b_{k,1}(t), b_{k,2}(t), \cdots, b_{k,M}(t))$  with duration T. Assuming BPSK modulation, the pth binary data sub-stream  $b_{k,p}(t)$  modulates a carrier frequency  $f_p = f_0 + (p-1)/T$  where  $f_0$  is the lowest frequency of the carriers.

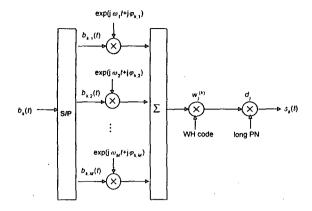
The spectra of the different carriers are shown in Fig. 2. The carrier frequencies are equally separated by the sub-stream data rate 1/T. The spectra of the different carriers mutually overlap, resulting in an optimum spectrum efficiency. The carrier frequencies are mutually orthogonal in the interval [0, T]. The multicarrier signal is obtained by the addition of the different carriers and then is spread by multiplying it with the spreading sequence  $a_k(t)$  associated with the kth user.

This spreading sequence used is a concatenated orthogonal / PN sequence, in which orthogonal sequences are concatenated with a long PN sequence to remedy their unsatisfactory and inhomogeneous behavior in a multipath environment [5]. The concatenated spreading sequence waveform  $a_k(t)$  can be written as

$$a_{k}(t) = \sum_{j=-\infty}^{\infty} a_{j}^{(k)} P_{T_{c}}(t-jT_{c}) = \sum_{j=-\infty}^{\infty} d_{j} w_{j}^{(k)} P_{T_{c}}(t-jT_{c})$$
 (1)

where  $a_j^{(k)} \in \{-1, 1\}$  is the concatenated spreading sequence for the kth user,  $d_j \in \{-1, 1\}$  represents the PN m-sequence, and  $w_j^{(k)} \in \{-1, 1\}$  is the orthogonal Walsh-Hadamard (WH) code for the kth user. The long PN sequence has a period  $P_c$  which is much greater than a period  $N_c$  of the WH codes.  $P_{T_c}(t)$  is the chip waveform which is a rectangular pulse with chip duration  $T_c = T/N_c$ .

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#### (a) Transmitter

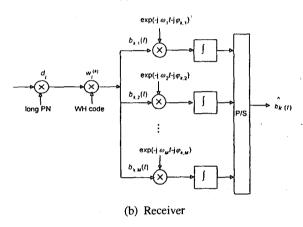


Fig. 1. Block diagram of the proposed system.

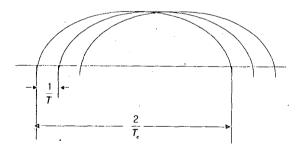


Fig. 2. The spectra of the different carriers.

The transmitted signal of the kth user can be expressed as

$$s_k(t) = \sqrt{2P} \sum_{p=1}^{M} \text{Re}\{b_{k,p}(t)a_k(t) \exp(j2\pi f_p t + j\varphi_{k,p})\}$$
 (2)

where P is the power of one carrier and  $\varphi_{k,p}$  is the pth carrier phase for the kth user. The total signal transmitted to K users is

$$s(t) = \sum_{k=1}^{K} s_k(t)$$
 (3)

The channel model for both Rician and Rayleigh multipath fading is considered as [6]:

$$h(t) = \sum_{n=1}^{L} \alpha_n \delta(t - \tau_n)$$
 (4)

where L is the number of resolvable multipath,  $\delta(t)$  is the unit impulse function,  $\tau_n$  is the time delay of the nth path, and  $\alpha_n$  is the complex gain coefficient of the nth path with

$$\alpha_1 = \beta + \gamma_1 A_1 \exp(i\theta_1)$$

$$\alpha_n = \gamma_n A_n \exp(i\theta_n), \qquad 2 \le n \le L.$$
(5)

 $\gamma_n A_n$  and  $\theta_n$  are the attenuation and the phase shift of the nth path, respectively. For Rician fading, the faded component is assumed to be insignificant compared to the non-faded specular component and therefore,  $\gamma_1 = 0$ .  $A_n$  is a Rayleigh distributed random variable and  $\gamma_n$  is a constant for the path strength. Without loss of generality,  $\beta$  and  $E[A_n^2]$  are normalized to be 1 so that  $\gamma_n^2$  can be seen as the relative power of the nth path to that of the first path. For Rayleigh fading,  $\beta = 0$  and  $\gamma_1 = 1$  since there is no specular path.  $A_n$ ,  $\theta_n$ , and  $\tau_n$  are mutually independent random variables that are assumed to be constant over the duration of a data bit.  $\theta_n$  is uniformly distributed in  $[0, 2\pi)$ . All of the resolvable paths are assumed to be pairwise separated by a time difference greater or equal to  $T_c$ , i.e.  $\tau_n = nT_c + \tau_n$  $(1 \le n \le L)$  where  $\tau_n$  is assumed to be uniformly distributed in [0, T<sub>c</sub>). The maximum delay spread is assumed to be quite less than T so that intersymbol interference can be ignored.

The received signal at a mobile is

$$r(t) = \sum_{n=1}^{K} \sum_{n=1}^{L} \text{Re}\{\alpha_n s'_k (t - \tau_n)\} + n(t)$$
 (6)

where  $s'_k(t)$  is the complex representation of  $s_k(t)$  and n(t) is additive white Gaussian noise (AWGN) with two-sided spectral density  $N_0/2$ . After the sub-stream data bits are decided at a rate 1/T, they are reconverted as a serial bit stream.

# III. Performance Analysis

An MRC is used under the assumption that the complex gain coefficient and the time delay of each path are perfectly estimated. The decision variable associated with the detection of the zeroth symbol of the ith user and qth carrier is given by

$$Z_{i,q}^{0} = \sum_{l=1}^{B} \int_{\tau_{l}}^{\tau_{l}+T} \operatorname{Re}\{\alpha_{l}^{*} r'(t)\} \Psi_{i,q}(t-\tau_{l}) dt$$
 (7)

where  $B(\le L)$  is the number of branches of the combiner, \* denotes complex conjugate, r'(t) is the complex representation of

$$r(t)$$
, and  $\Psi_{i,q}(t) = a_i(t) \cos(2\pi f_q t + \varphi_{i,q})$ .

Thus, the decision variable is given by

$$Z_{i,q}^{0} = \sqrt{P/2} T\{b_{i,q}^{0} \sum_{l=1}^{B} |\alpha_{l}|^{2} + I_{u} + I_{c} + I_{m}\} + n_{0}$$
(8)

where  $b_{i,q}^0$  is the zeroth symbol of the *i*th user and *q*th carrier, and  $n_0$  is a Gaussian random variable with zero mean and variance  $(N_0 T/4) \sum_{i=1}^{B} |\alpha_i|^2$ .  $I_n$  is the interference due to the other users on the same path and the same carrier, given by

$$I_{u} = \frac{1}{T} \sum_{l=1}^{B} |\alpha_{l}|^{2} \sum_{\substack{k=1\\ \neq i}}^{K} \cos(\varphi_{k,q} - \varphi_{i,q}) b_{k,q}^{0} \int_{\tau_{i}}^{\tau_{i} + T} a_{k}(t - \tau_{l}) a_{i}(t - \tau_{l}) dt.$$
 (9)

 $I_c$  is the interference due to the other users on the same path and the other carriers, given by

$$I_{c} = \frac{1}{T} \sum_{l=1}^{B} |\alpha_{l}|^{2} \sum_{k=1}^{K} \sum_{\substack{p=1\\ \neq i}}^{M} b_{k,q}^{0}$$

$$\times \int_{\tau_{c}}^{\tau_{l}+T} \alpha_{k}(t-\tau_{l}) \alpha_{s}(t-\tau_{l}) \cos[(\omega_{p}-\omega_{q})t+\boldsymbol{\Phi}_{k,p,l}-\boldsymbol{\Phi}_{i,q,l}]dt$$
(10)

where  $\Phi_{k,p,l} = -\omega_p \tau_l + \theta_l + \varphi_{k,p}$ ,  $I_m$  is the interference due to the remaining paths on all the carriers from all the users, given by

$$I_{m} = \frac{1}{T} \sum_{l=1}^{B} \sum_{k=1}^{K} \sum_{p=1}^{M} \sum_{\substack{n=1 \ p \neq l}}^{L} |\alpha_{l} \alpha_{n}| \times \int_{\tau_{l}}^{\tau_{l}+T} b_{k,p}(t-\tau_{n}) a_{k}(t-\tau_{n}) a_{i}(t-\tau_{l})$$

$$\cos \left[ (\omega_{p} - \omega_{a})t + \Phi_{k,p,n} - \Phi_{i,a,l} \right] dt.$$
(11)

Here, the Gaussian approximation is used for the total interference [6]. First we obtain the variances of the interferences in Rayleigh fading channel. It is noticeable that for the concatenated sequence,  $I_u = 0$ . For the nonconcatenated sequence,

$$Var(I_u) = \frac{K-1}{2N_c} (\sum_{l=1}^{B} |\alpha_l|^2)^2.$$
 (12)

After some manipulation, one can obtain that for both the concatenated and nonconcatenated sequences,

$$Var(I_c) = (\sum_{l=1}^{B} |\alpha_l|^2)^2 \sum_{\substack{p=1\\p\neq q}}^{M} \frac{(K-1)N_c}{4\pi^2(p-q)^2} [1 - \cos\frac{2\pi(p-q)}{N_c}].$$
 (13)

The interference  $I_m$  has the correlated terms when the interference is generated from  $n \le B$ , the same user, and the same carrier. Using the relation  $\sum_{l=1}^{B} \sum_{n=0}^{B} f(l,n) = \sum_{l=1}^{B-1} \sum_{n=l+1}^{B} \{f(l,n) + f(n,l)\}, I_{m,1}$  which has the correlated terms in  $I_m$  is given by

$$I_{m,1} = \frac{1}{T} \sum_{l=1}^{B-1} \sum_{n=l+1}^{B} |\alpha_{l}\alpha_{n}| \cos(\boldsymbol{\Phi}_{i,q,n} - \boldsymbol{\Phi}_{i,q,l}) ((b_{i,q}^{-1} + b_{i,q}^{+1}) \times R(k', \tau_{n} - \tau_{l}) + b_{i,q}^{0} [\widehat{R}(k', \tau_{n} - \tau_{l}) + R(k', T - \tau_{n} + \tau_{l})]$$
(14)

where  $R(k',\tau) = \int_0^\tau a_i(t+k'T_c-\tau)a_i(t+k'T_c)dt$  and  $\widehat{R}(k',\tau) = \int_{\tau}^T a_i(t+k'T_c-\tau)a_i(t+k'T_c)dt$ . After some not so straight forward deviation, one can obtain that for both the concatenated and nonconcatenated sequences,

$$\operatorname{Var}(I_{m}) = (A_{a1} + A_{a2})K \left\{ \frac{1}{3N_{c}} + \sum_{\substack{p=1\\ \neq q}}^{M} \frac{N_{c}}{4\pi^{2}(p-q)^{2}} \right\} \times \left[1 - \frac{N_{c}}{2\pi(p-q)} \sin \frac{2\pi(p-q)}{N_{c}} \right] + \frac{A_{1}}{6N_{c}}$$
(15)

where  $A_{al} = \sum_{l=1}^{B} |\alpha_{l}|^{2} \sum_{\substack{n=1 \ j \neq l}}^{B} |\alpha_{n}|^{2}$  and  $A_{a2} = \sum_{l=1}^{B} |\alpha_{l}|^{2} \sum_{\substack{n=B+1 \ j \neq l}}^{L} \gamma_{n}^{2}$ . Note that the last term in (15) is the additional interference power due to the correlation term in (11). Using the approximation  $\sum_{l=1}^{B} |\alpha_{l}|^{4} \cong (\sum_{l=1}^{B} |\alpha_{l}|^{2})^{2}/B, \quad A_{al} \cong \frac{B-1}{B} (\sum_{l=1}^{B} |\alpha_{l}|^{2})^{2}.$ 

In Rician fading channel the variances of the interferences can be obtained by a similar manner. The signal power is  $(P/2) T^2 (\sum_{l=1}^{B} |\alpha_l|^2)^2$ . For the concatenated sequence, the signal to noise ratio (SNR) can be obtained as

$$SNR_{1} = \frac{\left(\sum_{l=1}^{B} |\alpha_{l}|^{2}\right)^{2}}{\frac{N_{o}}{2E_{s}} \sum_{l=1}^{B} |\alpha_{l}|^{2} + \text{Var}(I_{c}) + \text{Var}(I_{m})}$$
(16)

where  $E_s$  is the energy per data symbol of one carrier. For the nonconcatenated sequence, the SNR can be obtained as

$$SNR_{2} = \frac{\left(\sum_{l=1}^{B} |\alpha_{l}|^{2}\right)^{2}}{\frac{N_{o}}{2E_{s}} \sum_{l=1}^{B} |\alpha_{l}|^{2} + \text{Var}(I_{u}) + \text{Var}(I_{c}) + \text{Var}(I_{m})}.$$
 (17)

The BER is obtained by averaging over the pdf  $f_Y(y)$  of the random variable  $y = \sum_{i=1}^{B} |\alpha_i|^2$ . The BER is given by  $P_{ei} = \int_{o}^{\infty} Q(\sqrt{SNR_i(y)}) f_Y(y) dy$  where i = 1 is for the concatenated sequence and i = 2 is for the nonconcatenated sequence.

## IV. Numerical Result and Discussion

Fig. 3 shows the BER of MRC versus the ratio of averaged symbol energy over white noise for Rayleigh/Rician fading channels. The number of multipath is 3 and the number of branches of combiner is 3. The relative path strengths of Rayleigh fading channel are (0, -6, -8 dB) and Rice factor of Rician fading channel is 7dB. It is shown that the proposed system outperforms the MT-CDMA with the conventional PN m-sequence. It is noticeable that larger improvement can be achieved for Rician fading channel compared to Rayleigh fading channel since the interference  $I_n$  in the strong specular path is removed by the concatenated sequence. Thus the proposed system can be more

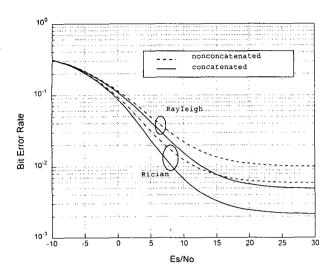


Fig. 3. Average BER of MRC versus  $E_s/N_0$  for Rayleigh / Rician fading channels. ( $N_c = 128$ , M = 4, K = 10)

favorable in a microcellular and an indoor picocellular environment.

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