

# Nonlinear State Feedback for Minimum Phase in Nuclear Steam Generator Level Dynamics

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## Abstract

The steam generator level is susceptible to the nonminimum phase in dynamics due to the thermal reverse effects known as "shrink and swell" in a pressurized water reactor. A state feedback assisted control concept is presented for the change of dynamic performance to the minimum phase. The concept incorporates a nonlinear digital observer as a part of the control system. The observer is devised to estimate the state variables that provide the true indication of water inventory by compensating for shrink and swell effects. The concept is validated with implementation into the steam generator simulation model.

## I. Introduction

Water level control is complicated by the thermal reverse effects known as "shrink and swell" phenomena in a steam generator of the pressurized water reactor. Due to the destabilizing vapor content in the tube bundle region, the water level measured in the downcomer temporarily reacts in a reverse manner to water inventory change. These effects are accentuated during low power operations. Increased feedwater flow adds mass to the steam generator, which would be expected to increase the measured downcomer water level, does increase it at high power. But at low power, the cold feedwater addition can cause a decrease in the vapor content of tube bundle, a shift in the liquid from downcomer to tube bundle, and a temporary decrease in level (shrink). Similarly, a decrease in feedwater flow can cause a temporary increase in water level (swell). The reverse effects incur the unstable zero in the open loop water level dynamics that shows nonminimum phase characteristics. Thus, the closed loop control system may be unstable with a high gain value. Limitations in the gain value make automatic control unsatisfactory. In order to compensate for the reverse effects, the flow mismatch between steam and feedwater is used as a feedforward signal, which provides the only true indication of water inventory change. However, instrumentation uncertainties at low power exclude the use of steam and feedwater measurements. Current control systems require the operator to take manual operation at low power. Shrink and swell effects are still confusing for the operator, thus

great care needs to be exerted to avoid a reactor trip due to the failure to control the water level. Experience has shown that both automatic and manual control give a reactor trip rate that is too high.

Some possible solutions to these problems have been precluded by the limited capabilities of analog controller. The digital controller makes it possible to accommodate advanced complicated algorithms[1]. Recent literature has demonstrated successful progress in steam generator level control through implementation of digital controllers. A common approach in those papers is to design the control parameters, such as gains, setpoints, etc., to vary as a function of power. These functions are obtained from off-line tuning studies carried out at various power levels[2, 3]. It may achieve stable automatic control practically, but still be subject to the constraints due to nonminimum phase. The other approach is to accommodate an observer into the controller to estimate state variables, thus to mitigate shrink and swell effects. It requires adaptive processes for on-line prediction. Most of observer designs[4, 5, 6] proposed adopt the linear level dynamics model that provides transparency of control for implementation of modern control theory. Model imperfection, however, is a bottleneck of application to the highly nonlinear actual system. In the meanwhile, the observer presented in this study is based on the nonlinear model that allows on-line implementation with minor adaptive process. Recently, model-free approaches are also studies using fuzzy logics[7, 8] or neural networks[9, 10].

## II. Problem Description

A simplified linear model is adopted to describe the steam generator control problem[11]. This model gives an approximate

representation of the true nonlinear steam generator downcomer level dynamics. The linear model is given as a Laplace transfer function with the output downcomer water level and the input feedwater flow rate. That is:

$$G(s) = \frac{L_w(s)}{W_{fw}(s)} = \frac{G_1}{s} - \frac{G_2}{1 + \tau_2 s} + \frac{G_3 s}{\tau^{-2} + 4 \pi^2 T^{-2} + 2\tau^{-1}s + s^2} \quad (1)$$

- $L_w$  : steam generator water level
- $W_{fw}$  : feedwater flow rate
- $G_1$  : mass capacity constant
- $G_2$  : reverse dynamics constant
- $\tau_2$  : reverse dynamics time constant
- $G_3, T, \tau$  : mechanical oscillation dynamics constants

where 's' is a Laplace variable.

The first term gives the mass storage effect of the steam generator, designated "mass capacity term". The second term provides for shrink and swell effects, designated "reverse dynamics term". And the third term represents the mechanical oscillation of the water, designated "mechanical oscillation term". Numerical values for the parameters in Eq. 1 are obtained through system identification from transient level responses by the nonlinear model to step feedwater flow variations and are given in Table 1[12].

**Table 1.** Linear Water Level Dynamics Model Parameters.

Power %	5	15	30	50	100
$G_1$	0.058	0.058	0.058	0.058	0.058
$G_2$	9.63	4.46	1.83	1.05	0.47
$\tau_2$	48.4	21.5	4.5	3.6	3.4
$\tau$	41.9	26.3	43.4	34.8	28.6
$T$	119.6	60.5	17.7	14.2	11.7
$G_3$	0.181	0.215	0.310	0.215	0.015

In Eq. 1, the level dynamics due to the mechanical oscillations can be separately considered because it does not incur stability problems. Then, the open loop transfer function  $G(s)$  has a zero,  $G_1/(G_2 - G_1 \tau_2)$ , at the right hand side of the Laplace domain, designated "unstable zero", that represents nonminimum phase in dynamics. The unstable zero lowers the gain for the closed loop system to preserve stability, with scarifying performance. As the power level decreases, the zero moves to the right, stability condition being more limited.

The conventional controller is of a three element proportional - plus - integral (PI) type for the steam generator level. The control

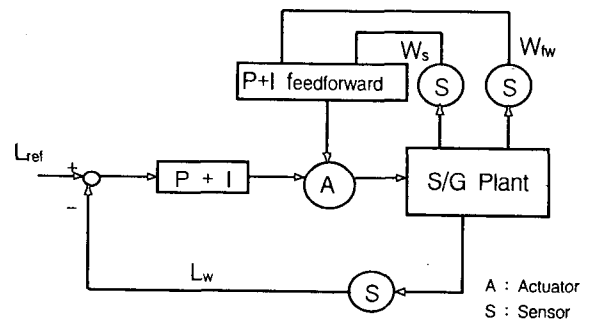
action is described by the following equation :

$$\frac{dW_{fw}}{dt} = K_{pw} \left( \frac{de_w}{dt} + \frac{1}{T_{iw}} e_w \right) + K_p \left( \frac{de_l}{dt} + \frac{1}{T_i} e_l \right) \quad (2)$$

where  $t$  = time ,  $K_{pw}$  = proportional flow gain,  $e_w$  = flow error,  $T_{iw}$  = flow integral time ,  $K_p$  = proportional level gain,  $T_i$  = level integral time, and  $e_l$  = level error.

The schematic for this controller is given in Fig. 1. At low power operation, flow error signal is not used because flow measurements are too erratic, thus a single element PI controller is executed. It can be simulated with Eq. 2 by setting  $K_{pw}$  to zero. A transfer function is obtained for the closed loop level control system based on the linear model with single element PI as follows:

$$M_{cl}(s) = \frac{N(s)}{D(s)} = \frac{K_p \left( 1 + \frac{1}{T_i s} \right) G(s)}{1 + K_p \left( 1 + \frac{1}{T_i s} \right) G(s)} \quad (3)$$



**Fig. 1.** Conventional controller.

- $W_s$  : steam flow rate
- $W_{fw}$  : feedwater flow rate
- $L_w$  : water level
- $L_{ref}$  : reference water level

The closed loop transfer function  $M_{cl}(s)$  has two zeros and three poles. The closed-loop zero and pole configuration determines the performance of the controlled system. The root-locus plot enables us to find the closed loop poles in the s-plane with the variation of proportional gain. At 5% of rated power, the root-locus plot is given in Fig. 2. It demonstrates the problems of the nonminimum phase system. As seen in the plot, the closed loop poles move to the right half s-plane as the gain increases. The stability requirements impose the following constraint on the gain  $K_p$  with setting  $T_i$  that makes the value of the bracket to be positive.

$$K_p < \frac{1}{G_2 - G_1 \tau_2} \left[ 1 - \frac{\tau_2}{T_i - (G_2 - G_1 \tau_2)/G_1} \right] \text{ if } G_2 - G_1 \tau_2 > 0 \quad (4)$$

Otherwise, the gain is free of the constraint.

For the high gain ( $K=500$ ), the corresponding poles lie in the

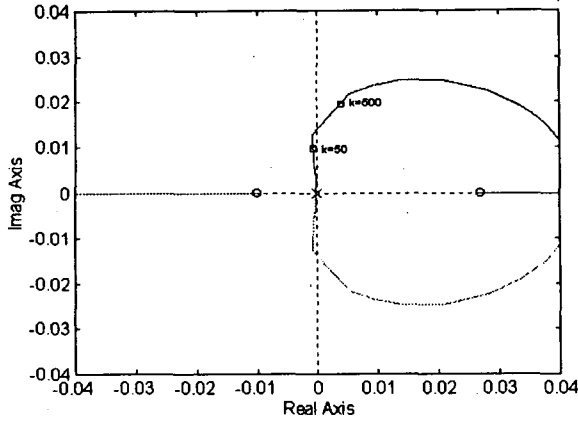


Fig. 2. Root locus plot for nonminimum phase.

right half s-plane and the system is expected to be unstable. If poles be placed in the left half s-plane with the low gain ( $K=50$ ), the transient behavior may be stable but very sluggish.

### III. New Control Concept

A new control concept is presented to solve the problems encountered by the conventional controller during low power operation. A big challenge is to achieve good performance both in stability and in transient responses without using flow measurements.

From the linear level model in Eq. 1, the transient level response to a unit step increase in feedwater flow is:

$$\delta L_w(t) = G_1 t - G_2(1 - e^{-t/\tau_2}) \quad (5)$$

where,  $\delta L_w(t) = L_w(t) - L_w(0)$

To provide an idea about the proposed control concept, it is considered adding a compensation term  $G_2^c(1 - e^{-t/\tau_2})$ , thus a compensated level is obtained as follows:

$$\delta L_w^c(t) = G_1 t - (G_2 - G_2^c)(1 - e^{-t/\tau_2}) \quad (6)$$

When using the compensated level as a control variable, the gain  $K_p$  is now subject to the following modified constraint with setting  $T_i$  that makes the value of the bracket to be positive.

$$K_p < \frac{1}{(G_2 - G_2^c) - G_1 \tau_2} \left[ 1 - \frac{\tau_2}{T_i - ((G_2 - G_2^c) - G_1 \tau_2) / G_1} \right] \quad (7)$$

if  $(G_2 - G_2^c) - G_1 \tau_2 > 0$

Otherwise, the gain is free of the constraint.

By increasing compensation for the reverse dynamics term, we can therefore mitigate and eliminate stability constraint. It implies

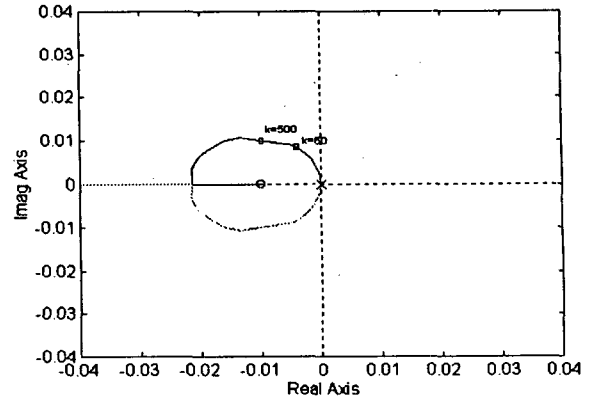
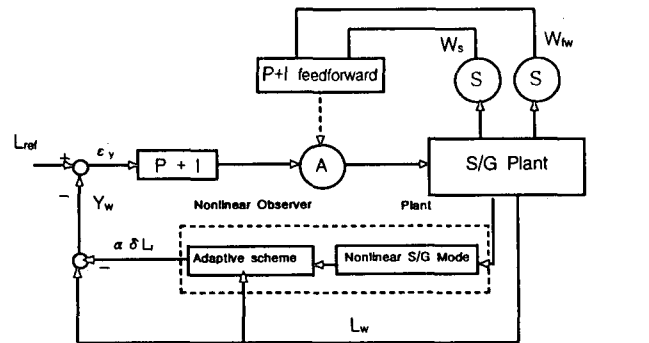


Fig. 3. Root locus plot for minimum phase.

that the compensated level dynamics has a minimum phase transfer function, for which Fig. 3 gives a root-locus plot of closed loop poles.

It was found that the tube bundle mass has a similar transient behavior to the reverse dynamics term[13]. Thus the tube bundle mass is used as a variable for state feedback to provide the compensation term. This state variable is not measurable but observable from the reliable outputs. A digital observer is incorporated into the controller to estimate the tube bundle mass. The schematic for the new controller is given in Fig. 4.



A : Actuator, S : Sensor

- $W_s$  : steam flow
- $W_{fw}$  : feedwater flow
- $\epsilon_y$  : compensated level error
- $L_w$  : water level
- $Y_w$  : compensated level
- $\delta L_w$  : reverse dynamics compensation term
- $\alpha$  : design tuning parameter

Fig. 4. New controller.

### IV. Digital Observer

The observer is based on the nonlinear steam generator level dynamics model that is of decoupled 5th order and real-time

executable in microprocessor.

The state-space equation is :

$$\underline{A} \, dx/dt = b(x, u, w, t) \tag{8}$$

where,

$$\text{State variables : } \underline{x} = [U_o, L_w, \alpha_r, \alpha_n, p]^T ;$$

- $U_o$  : steam generator inlet internal energy
- $L_w$  : steam generator water level
- $\alpha_r$  : steam generator riser inlet void fraction
- $\alpha_n$  : steam generator outlet riser void fraction
- $p$  : secondary pressure

$$\text{Input : } u = g(W_{fw}) ;$$

$W_{fw}$  : feedwater flow rate

$$\text{Disturbances : } w = h(W_s, T_{fw}, p_{pr}, W_{pr}, T_h)$$

- $W_s$  : steam flow rate
- $T_{fw}$  : feedwater temperature
- $p_{pr}$  : primary pressure
- $W_{pr}$  : primary flow rate
- $T_h$  : hot-leg temperature

The output equation is :

$$y = [0 \ 1 \ 0 \ 0 \ 0] \underline{x} \tag{9}$$

and  $\underline{A}$  is a 5x5 matrix with nonlinear elements.

The tube bundle mass is calculated by the above 5 state variables. A salient feature of the observer is to exclude the use of uncertain measurements for steam flow rate, and instead to accommodate the use of reliable measurements for secondary pressure as a disturbance. Thus, the state-space equation in the observer is reduced to be of 4th order as follows:

$$\underline{A}^* \, dx/dt = b^*(x, u, w, t) \tag{10}$$

where,

- State variables :  $\underline{x} = [U_o, L_w, \alpha_r, \alpha_n]^T ;$
- Input :  $u = g(W_{fw}) ;$
- Disturbances :  $w = h(p, T_{fw}, p_{pr}, W_{pr}, T_h)$

output equation :

$$y = [0 \ 1 \ 0 \ 0] \underline{x} \tag{11}$$

and  $\underline{A}^*$  is a 4x4 matrix with nonlinear elements.

The digital observer is incorporated into the new controller as follows :

$$\frac{dW_{fw}}{dt} = K_p \left( \frac{d\varepsilon_y}{dt} + \frac{1}{T_i} \varepsilon_y \right) \tag{12}$$

where

$$\varepsilon_y = L_{ref} - Y_w$$

$$Y_w = L_w - \alpha \delta L_r$$

The description for each term is given in Fig. 4. The term  $\delta L_r$ , designated a "reverse dynamics compensation term", is the compensation for a variation in the downcomer level due to the change of the tube bundle mass calculated by the digital observer. The amount of compensation can be adjusted by introducing a proportionality constant  $\alpha$  as a tuning design parameter. Also, to account for model imperfection, the nonlinear observer is also required to be adaptive, but less critical than the linear observers. A simple filter is used for tuning the parameters of the observer with an error signal between the estimated level and the measured one.

## V. Testing and Results

The proposed control concept is evaluated with implemented into the detailed steam generator simulation model that has been validated for model accuracy by the actual plant data. The proposed controller is employed during the water level transients. It achieves satisfactory performance for the cases that the conventional controller fails to control. Some of the results are given in Fig. 5 and Fig. 6. For the case of power transient from 5% to 10% of rated value as in Fig. 5, the water level settles down within a few minutes and the maximum overshoot is less than 10% of limit value. It is comparable to achievements by a good manual control. The other case in Fig. 6 represents the level set point change of 0.1 m at low power. The proposed controller also performs well. In both cases, the design values of  $\alpha$ ,  $T_i$ ,  $K_p$  are used as follows ;  $\alpha=1.5$ ,  $T_i=200(s)$  and  $K_p=428.9$ . In addition, the controller is tested for various gain values. Control performances are consistent and good. It implies the robustness of the proposed control concept.

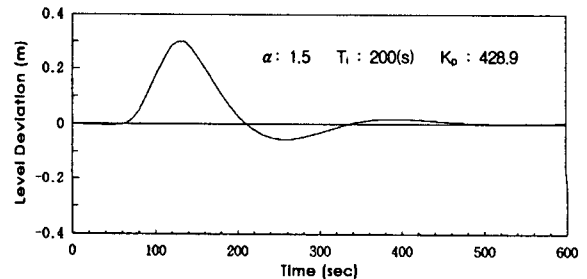


Fig. 5. S/G Level control simulation (power change).

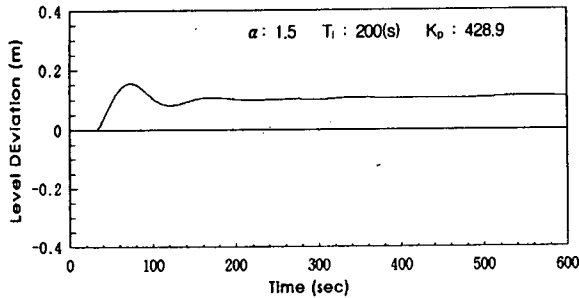


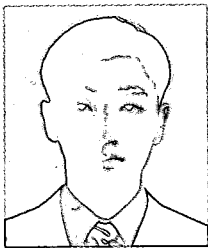
Fig. 6. S/G Level control simulation (level change).

## VI. Conclusions

A new control concept is proposed to solve the problems associate with the steam generator water level control during low power operation. The feedback of state variables related to shrink and swell effects changes water level dynamics to the minimum phase that achieves satisfactory automatic control performance. A salient features of the controller is the incorporation of the digital nonlinear observer to estimate the state variable. The controller is validated with implementation into the detailed steam generator simulation model. The results insist the new concept be judged to be feasible.

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