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# OUTPUT FEEDBACK SLEWING CONTROL OF FLEXIBLE SPACECRAFT BY LYAPUNOV STABILITY THEORY

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## ABSTRACT

Slewing maneuver and vibration suppression control of flexible spacecraft model by Lyapunov stability theory are considered. The specific model considered in this paper consists of a rigid hub with an elastic appendage attached to the central hub and tip mass. Attitude control to point and stabilize single axis using reaction wheel type device is tested. To control all flexible modes is so critical to designing an active control law. We therefore considered an direct output feedback control design by using Lyapunov stability theory. It is shown that the ouput feedback control law design with proposed configuration gives satisfactory results in slewing performance and vibration suppression control.

## 1. INTRODUCTION

During the past few decades, the family of problems which arise in maneuver and vibration suppression of lrare flexible spacecraft has received significant attention in the research literatures by Balas (1982), Linkins (1986), Junkins & Turnar (1986), and Meirovitch & Quinn (1987). Since flexible spacecrafts are mechanically flexible systems, they are most rigorously described by a *hybrid* (ordinary/partial) systems of nonlinear ordinary and integro-partial differential equations. Due to the inherent difficulties, there have been a large number of research issues associated with control of these spacecrafts. Dynamical modeling and control design problems have been subjects of intensive research. The main difficulty of flexible space structures control arises from the fact that the flexible structures are inherently infinite dimensional systems. Meirovitch (1990) extended the classical Lagrange's equations for hybrid systems using the extended Hamilton's principle. Although Meirovitch found the correct forms for the hybrid system, his equaitons embodied a differential operator that must be developed through integration by parts for each specific applicaiton. Also, the boundary condition operator in Meirovitch's developments must be found by integration by parts for each specific integration. We were motivated by Meirovitch's developments to establish, at least for significant classes of systems, explicit governing equaitons that make allowance for space structures. Since the equations of motion of infinite dimensional systems are usually described by Partial Differential Equations (PDEs), some kinds of approximation are necessary to develop finite dimensional systems for conventional control law design. The Finite Element Method (FEM) is one of the most popular methods of spatial discretization, especially for Large Flexible Space Structures (LFSS) (Kim *et al.* 1992, Reddy 1993).

In particular, we consider here large-angle rotational maneuvers with simultaneous vibration suppression. The motion is described by a system of hybrid coordinates, using a combination of discrete coordinates for translations and rotations of rigid bodies, and distributed coordinates for the deformations of elastic bodies. The specific model considered in this paper (see Figure 1) consists of a rigid hub with two identical elastic appendages attached symmetrically to the central hub and tip masses (Junkins & Turner 1986, Meirovitch & Quinn 1987, Junkins *et al.* 1991, Bang 1992, Kwon & Bang 1997). We consider only the case of a single-axis maneuver with flexible members restricted to displacements in the plane normal to the axis of rotation.

Modern control techniques impose another important issue on estimation of all degrees of freedom (DOF) with a limited number of sensors. The number of sensors and actuators are usually less than the DOF of approximated systems. Since the vibrational motion of the structure induces phase error at different locations of the structure, the sensor and actuator placement should take the phase difference into account. The best strategy is to place the sensor and the actuator at the same location called a collocated sensor/actuator system. Therefore, in this point of view, the Lyapunov function to be used is designed to have its global minimum at the target final state of the pointing maneuver, and considering a flexible spacecraft model. The commonly used total energy; i.e., the sum of both kinetic and potential energy and error energy between the target state and any intermediate neighboring state provide the main ingredients of the Lyapunov function (Bang 1992), but we have to augment the elastic potential energy of the appendage because the hub angle (rigid body coordinate) is a cyclic coordinate. Hance the output feedback control law technique by using Lyapunov stability theory to be designed later on must be based upon these control objectives.

## 2. EQUATION OF MOTION AND PROBLEM STATEMENT

Let us first consider only the hub with one appendage and tip mass for convenience, and assemble the two appendages later (Bang 1992, Kim *et al.* 1992). The coordinates we use are shown in Figure 1. The total transverse velocity (of a mass element on the appendage) is

$$v(x,t) \;=\; \dot{w}(x,t) \;+\; (x+r)\dot{\theta}(t) \tag{1}$$

where $x$ is the auxiliary variable measured from the outer radius $r$ of the hub along the undeformed appendage axis, $\theta(t)$ is the hub angle, $w(x,t)$ is the deflection measured from the $x$ axis, and overdots denote derivatives with respect to time $t$. In Eq. (1), it is evident that we have neglected radial deformations and the nonlinear radial velocity correction required to rigorously enforce zero elongation of the deformed appendage.

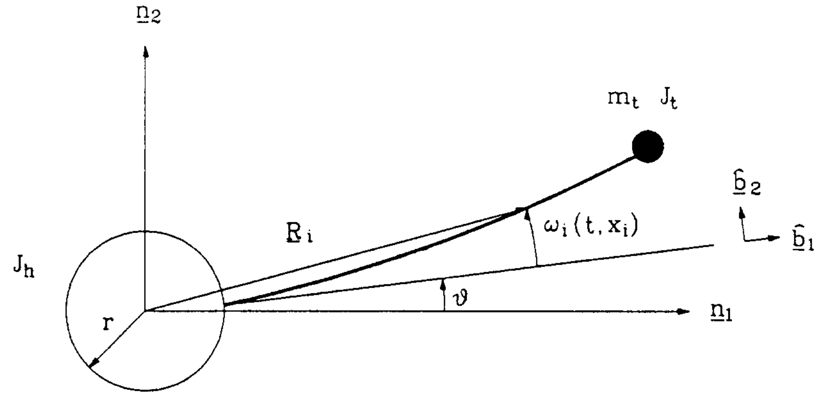


Figure 1. Deformation of a flexible appendage with rigid body motion.

Assuming Euler-Bernoulli beam theory and small deformation, we can express the total kinetic and potential energy as follow (Meirovitch 1990):

$$T = \frac{1}{2}J_h\dot{\theta}^2 + \frac{1}{2}\int_0^L \rho v^2 dx + \frac{1}{2}m_t v^2(L) + \frac{1}{2}J_t\left(\frac{\partial v}{\partial x}\bigg|_L\right)^2 \tag{2}$$

and

$$V = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{3}$$

where $J_h$ is the rotary inertia of hub, $J_t$ is the rotary inertia of the tip mass, $\rho$ is the mass density of the appendage, $m_t$ is mass of tip mass, $L$ is the length of the appendage, and $EI$ is the appendage flexural rigidity. The virtual work done by the external torque $u_1$ is given by

$$\delta W_{nc} = u_1 \delta\theta(t) \tag{4}$$

Substitution of Eqs. (1)-(4) into *extended Hamilton's principle* as follows;

$$\int_{t_1}^{t_2} \delta(T - V)dt + \int_{t_1}^{t_2} \delta W_{nc} dt = 0 \tag{5}$$

The displacement $w(x,t)$ can be discretized using a FEM expansion. The Hermite-cubic polynomial shape functions that satisfy the conditions for admissibility and that are defined over the finite element (Reddy 1993, Kwon & Bang 1997).

Substitution of Eq. (6) into Eq. (5) and integration over the spatial domains leads to the global mass, stiffness, and forcing matrices. The assembled set of matrix differential equations is as follow:

$$\begin{bmatrix} J_h + M_{\theta\theta} & M_{\theta\nu} \\ M_{\nu\theta} & M_{\nu\nu} \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{\underline{\nu}} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_{\nu\nu} \end{bmatrix} \begin{Bmatrix} \theta \\ \underline{\nu} \end{Bmatrix} = \begin{bmatrix} \mathbf{f} \\ 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ 0 \end{Bmatrix} \tag{6}$$

By using Eq. (6), the equations of motion of the hub with an appendage and tip mass can be assembled. Note that boundary conditions are already imposed in the above matrices. The assembled set of differential equations of motion can be expressed as follows:

$$M\ddot{\mathbf{q}} + K\mathbf{q} = F\mathbf{u} \tag{7}$$

with configuration and control vectors $\mathbf{q} = [\theta; \underline{\nu}_1^T]^T$ and $\mathbf{u} = [u_1; 0]^T$, respectively. In this study, the control actuation is assumed to be applied to the center hub using a reaction wheel type device. The matrices $M$ and $K$ are symmetric and positive definte, so obviously $M_{\nu\theta} = M_{\theta\nu}^T$, and the dimension of the configuration vector is $2N + 1$, when we discretize the appendage using $N$ elements.

## 3. FEEDBACK CONTROL LAW DESIGN

For control applications, the system dynamics are usually modeled in the first order state space differential equation. Let us introduce the $2n \times 1$ dimensional state vector

$$\mathbf{x} = \left[q^T \ \dot{q}^T\right]^T \tag{8}$$

Eq. (7) can be written as the first-order system

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{9}$$

$$\mathbf{y} = C\mathbf{x} \equiv \theta \tag{10}$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix}, \quad C = [1, 0, \cdots, 0]$$

As a special case, we assume a collocated sensor/actuator pair for the rotating beam. The actuator is located at the hub producing torque about the vertical axis and the sensor is also located at the hub measuring the angular displacement and/or angular velocity of the hub. With the collocated sensor/actuator set, the control law design is relatively simple.

First, we select a candidate Lyapunov function as (Junkins *et al.* 1991, Bang 1992)

$$\begin{aligned} 2U = a_1 J_h \dot{\theta}^2 &+ a_2 \left[ \int_0^L \left[ \rho(x\dot{\theta} + \dot{\omega})^2 + EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \right] dx + m_t \left( x\dot{\theta} + \dot{\omega}(x,t) \right)^2 \right. \\ &+ \left. J_t(\dot{\theta} + \dot{\omega}'(x,t))^2 \right] + a_3(\theta - \theta_f)^2 \end{aligned} \tag{11}$$

where prime denotes derivative with respect to displacement $x$. The Lyapunov function is shown as the combination of each sub-structure's energy; hub, appendages and tip masses. Furthermore, $a_1$, $a_2$ and $a_3$ are positive weighting constants determining the relative importance of sub-structure's energy, and $\theta_f$ is a constant final desired angle. The Lyapunov function is positive definite with respect to the steady equilibrium point

$$(\theta, \dot{\theta}, w, \dot{w})_f = (\theta_f, 0, 0, 0) \tag{12}$$

Table 1. The configuration parameters (Bang 1992).

| Parameter | Symbol | Value |
|---|---|---|
| Hub radius | $r$ | $1\,ft$ |
| Rotary inertia of hub | $J_h$ | $8\,slug \cdot ft^2$ |
| Mass density of appendage | $\rho$ | $0.0271875\,slug/ft$ |
| Elastic modulus of appendage | $E$ | $0.1584 \times 10^{10}\,lb/ft^2$ |
| Moment of inertia of appendage | $I$ | $0.4709502797 \times 10^{-7}\,ft^4$ |
| Appendage length | $L$ | $4.0\,ft$ |
| Appendage thickness | $t$ | $0.01\,ft$ |
| Appendage height | $h$ | $0.49\,ft$ |
| Tip mass | $m_t$ | $0.156941\,slug$ |
| Rotary inertia of tip mass | $J_t$ | $0.0018\,slug \cdot ft^2$ |

The control torque at the center body should be designed in such way that the Lyapunov function decreases asymptotically toward the equilibrium point. For this purpose, we take the time derivative of the given Lyapunov function

$$\dot{U} = a_1\Big[u + g_1(\theta - \theta_f) + g_3(rS_0 - M_0)\Big]\dot{\theta} \tag{13}$$

where $g_1 = a_3/a_1 > 0$, $g_3 = (a_2 - a_1)/a_1 > -1$ are design parameters or feedback gains of the control law. $M_0$ and $S_0$ are the internal bending moment and shear force, respectively at the root of the appendage. Since our goal is to design a stabilizing control torque input, the most natural choice is to make the time derivative of the Lyapunov function negative in such a way that

$$u + g_1(\theta - \theta_f) + g_3(rS_0 - M_0) = -g_2\dot{\theta}, \qquad g_2 > 0 \tag{14}$$

Therefore,

$$u = -g_1(\theta - \theta_f) - g_2\dot{\theta} - g_3(rS_0 - M_0) \tag{15}$$

so that

$$\dot{U} = -a_1 g_2 \dot{\theta}^2 < 0 \tag{16}$$

As we can see, $\dot{U} < 0$ as long as $\dot{\theta} \neq 0$. At $\dot{\theta} = 0$, the Lyapunov function is equal to zero which does not mean that the system is at equilibrium condition due to other nonzero motions like angular position error and flexible motion (Bang 1992). In order to reach the steady equilibrium state, the Lyapunov function continues to decrease as dictated by Eq. (12).

## 4. APPLICATION AND SIMULATION

We have simulated and discussed the slewing maneuver and vibration suppression problem of the hub-appendage-tip configuration in the previous sections. The configuration parameters of spacecraft undergoing large rotations are as following Table 1. A reaction wheel type actuator is mounted on the hub and imparts torques about the vertical (symmetry) axis of the system. This actuator has sufficient bandwidth to simultaneously impart both the large amplitude, low frequency torques required for large angle maneuvers and the low amplitude, high frequency torques required for vibration control.

A 45 deg ($\theta_f = 45deg$) slewing maneuver with vibration suppression is simulated. For the Lyapunove stability control law a constant gain feedback law in Eq. (15) with the results presented

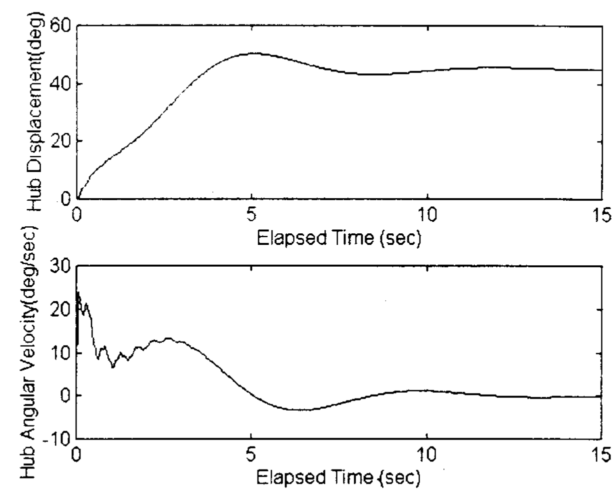


Figure 2. Simulation results of large angle maneuver and control.

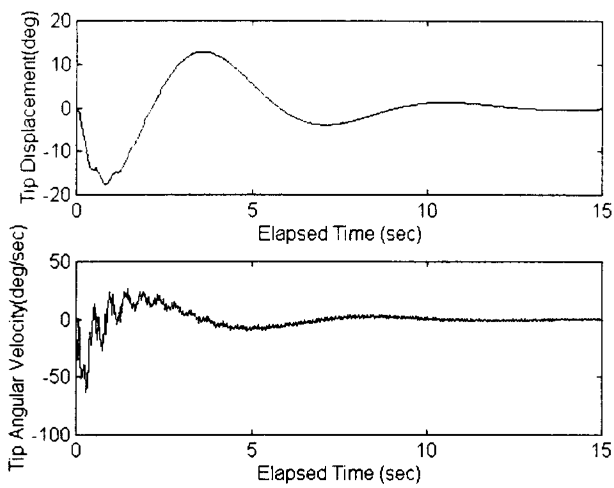


Figure 3. Simulation results of vibration suppresion control.

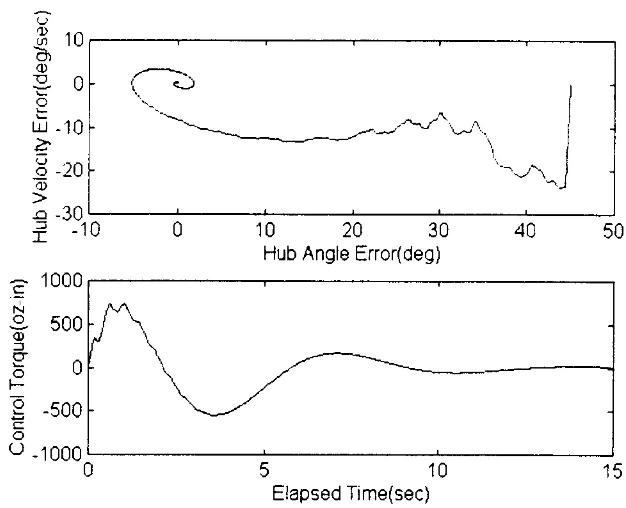


Figure 4. Hub error and control torque history by reaction wheel.

in Figures 2, 3, and 4, where the displacements and angular velocity of hub and tip mass are shown. This is due to the final target angle combined with the feedback gains

$$g_1 = 500 oz in./rad, \quad g_2 = 800 oz in./rad, \quad g_3 = 0 \, or - 0.5$$

The feedback gain ($g_3$) on the boundary force feedback is tested to investigate its effect on the closed-loop performance.

As we can see in Figure 2, the hub angle converges to the final angle 45° within 15 seconds of simulation time. Also, we observe large overshoot ($\sim 5deg$) in Figures 2 and 4, requiring large amount of torque instantaneously at the start of the maneuver. The significant structural vibration is settled around 11 sec, and the control action was terminated at 13 sec in Figure 3.

## 5. CONCLUSION

The results of this paper provide a basis for systematic constant feedback gain solution of large-angle single-axis flexible spacecraft rotational maneuvers, when distributed hybrid system is employed. Output state feedback control scheme by using Lyapunov stability theory is applied to obtain reaction wheel torquer control law for pointing and stabilizing single axis of a flexible spacecraft model. It is shown that the Lyapunov stability control law with proposed configuration gives satisfactory results in slewing performance and vibration control. Further study can be made with dynamically more complex space structures including more vibrational modes. It is hoped that the insight gained from consideration of this paper will prove beneficial in the large angle maneuver and vibration suppresion control technique for the practical applicaiton into actual spacecraft.

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