A Study on the Introduction of Fuzzy Theory to the Adjustment of Time Variant Parameter of Storage Function Method

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ABSTRACT : The Parameters of the storage function model (SFM) are taken as constants, while they have different values during every rainfall period and the duration of the runoff. Therefore, the results of the SFM generally show remarkably large errors. In this study, the modified storage function model (MSFM), in which the time variant parameters are introduced, is proposed to improve the SFM which is a conceptual rainfall-runoff model. The fuzzy reasoning method is applied as a real-time control one of the time variant parameters of the proposed model. The applicability of the MSFM was examined in the Bochung river, at a tributary of the Geum River, Korea. The pattern of the predicted runoff hydrograph and the peak discharge by the MSFM with fuzzy control are very similar to the measured values, compared with the results produced by the SFM.

1. Introduction

It is very hard to model and reproduce hydrological phenomena accurately, because their characteristics are, in general, nonlinear and stochastic as well as deterministic widely scattered, time variant and dynamic. Moreover, the measured data generally have uncertainties caused by missing data, measuring errors and variation in space and time, so it is not easy to obtain the accurate values of the parameters for the hydrological model(Singh, 1988). In order to overcome simplicity of the model, the uncertainties of input data and parameters of the model, and improve the prediction accuracy of rainfall runoff, fuzzy reasoning is introduced into the flood forecasting model.

A fuzzy control of time variant parameters can be considered to improve the storage function model (SFM) used in forecasting floods, operating dams and keeping the remaining fundamental model structure unchanged. Because the parameters of the rainfall-runoff models are treated as

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constants or defined simply, though they have, in reality, the time-variant parameters, the accuracy of the predicted output is not usually so good.

Therefore, it is necessary to raise the accuracy, introducing fuzzy reasoning as a control of the time-variant parameters.

The pioneering research of Mamdani(1977) on the fuzzy control was motivated by Zadeh(1965)'s investigation on the linguistic approach and the system analysis based on the theory of fuzzy sets. Recently, Fujita et al. (1992, 1995) reported the application of fuzzy reasoning to utilize rainfall data for forecasting the runoff without using the conventional rainfall-runoff model.

This study is aimed at modifying the SFM, following its main structure, in order to improve the accuracy of the predicted results and reduce much dependence on the experienced hydrological engineers' judgment.

The modified storage function model (MSFM) employing time variant parameters is proposed. Fuzzy reasoning is used to control time variant parameters. In the MSFM for basin routing, K (parameter of storage function) and f_1(variable runoff coefficient) are treated as time variant parameters and controlled by fuzzy reasoning. To examine the validity of the proposed MSFM with fuzzy control, the rainfall and discharge data of the Bochung stream, a tributary of the Geum River in Korea (one of the representative basins of the IHP) are used.

2. Modified Storage Function Model (MSFM)

The governing equations of the SFM are expressed as follows (Kimura, 1962) :

$$S_l = K O_l^P \tag{1}$$

$$\frac{1}{3.6} f r_{ave} A - O_l = \frac{dS_l}{dt} \tag{2}$$

where K and P are parameters of storage function, f is the inflow factor, r_{ave} is the rainfall intensity (mm/hr), A is the watershed area (km^2), S_l is the hypothetical storge volume over a watershed (m^3) considering the time lag T_l between the rainfall excess and the flood runoff discharge, $O_l = O(t+T_l)$ is the time lagged direct runoff discharge of the watershed (CMS).

However, Eqs. (3) and (4), transformed in terms of the water depth per unit area, are practically used instead of Eqs. (1) and (2) in the actual computation. Then the runoff discharge is finally computed by Eq. (5).

$$s_l = K q_l^P \tag{3}$$

$$r_{ave} - q_l = \frac{ds_l}{dt} \tag{4}$$

$$O=\frac{A}{3.6}\left[f_1 q_l+(1-f_1)\, q_{sa,l}\right]+O_i \tag{5}$$

where s_l is the hypothetical storage depth of rain water over the watershed (mm), q_l is the runoff discharge depth from the runoff area (mm/hr), $q_{sa,l}$ is the runoff depth from the infiltration area, O is the runoff discharge (CMS), O_i is the base discharge (CMS), and f_1 is the primary runoff coefficient.

Fig. 1 is a conceptual sketch of the SFM, showing the temporal change of the runoff area and the runoff coefficient or their change with the increase of the rainfall amount. As the rainfall amounts from the beginning of rain to that of runoff is nearly negligible, compared with the saturation rainfall (Hydrologic research society of Japanese Ministry of Construction, 1971).

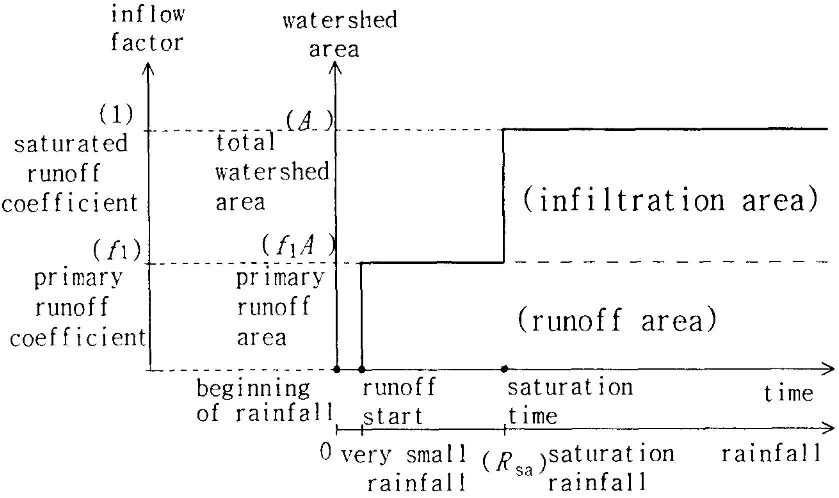

Fig. 1. Variation of runoff coefficient during flood period

At present, parameters of the SFM are determined by empirically or analytically using the hydrological observed data. Therefore, a lot of efforts are required to obtain numerical parameters, and sometimes they depends on the hydrologically much experienced engineers' udgment.

Reviewing the SFM, the following problems, in addition to uncertainty of data and parameters, are pointed out:

① The rainfall loss of a basin is not effectively represented.

② The variation of the runoff mechanism related to the antecedent precipitation is not considered.

③ The runoff depths from runoff and the infiltration area, whose properties are nonlinear, are superposed.

④ Variation of the lag time, depending on the storage volume of the basin, is not considered.

⑤ Variation of storage parameters is not considered.

⑥ The different runoff pattern related to rainfall distribution is not considered.

Problems ①, ② and ③ seem to occur because of using the concept of the primary runoff coefficient and the saturation rainfall. In order to minimize the above-mentioned problems, it is necessary to introduce the time-variant runoff coefficients.

Though the flow velocity is thought to be depend on the storage depth of the basin, the lag time is used as a constant in the SFM. So it can not avoid problems ④ and ⑤. For that reason, K, P and T_l should be handled as time-variant parameters.

In this study, K, P and T_l are determined by Brent method(Press et al., 1986) and linear regression, to reduce their uncertainties and determine them in the objective method. f_1 and R_{sa} are also obtained by two dimensional Brent method.

The MSFM is proposed as follows (Lee, 1996) :

The continuity equation, Eq. (4), is modified to Eq. (6). In other words, an inflow coefficient f_a which is computed by Eq. (7) using the measured rainfall-runoff data is introduced in Eq. (6).

$$f_a r_{ave} - q_{l'} = \frac{ds_{l'}}{dt} \tag{6}$$

$$f_a = \int_{t_1}^{t_2} O_l \, dt \Big/ \int_{t_1}^{t_2} I \, dt \tag{7}$$

where t_1 and t_2 are the times when the runoff discharges have the same values in rising and falling limb of the hydrograph, respectively, and I is the inflow discharge, which is replaced by $\frac{1}{3.6} r_{ave} A$ in the basin routing. The discharge is usually selected between 10~20% of the peak discharge. And, $q_{l'}$ is a specific discharge depth of the catchment area and $s_{l'}$ is the storage depth per unit area of it, but, as it is, in practice, no meaning to distinguish between $q_{l'}$ and $q_{l'}$ or s_l and s_l, hereafter q_l and s_l are used without a prime.

Making no distinction between the runoffs from the runoff and infiltration areas in Eq. (5), the equation calculating the unified runoff discharge over the watershed is proposed as follows :

$$O = \frac{A}{3.6} f_2 q_l + O_i \tag{8}$$

where f_2 is a variant runoff coefficient which is a time-dependent function (Lee et al., 1994).

In order to find a solution of the problems ④ and ⑤, it can be considered to treat K, P and T_l as time-variant parameters. But, when all the parameters are treated to be time- dependent, it is expected that the problems involved become very complex, and the interference effects between them are not clearly analyzed and a lot of computing time to control them is needed. So, it is necessary to select a proper one among three parameters as a time-variant variable.

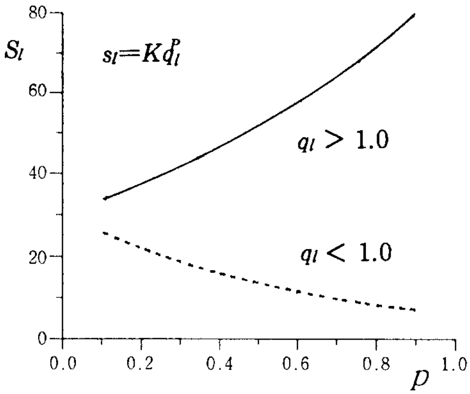

Fig. 2. Variation of storage depth with P

P is an exponent of an exponential function in Eq. (3). Therefore, storage depth s_1 increases with P when the runoff depth q_1 is larger than 1.0, on the other hand, s_l decreases with P when q_l is smaller than 1.0, as shown in Fig. 2. Therefore, it is not easy to control it effectively, because a variation of the storage depth have a serious effect on the model.

Next, the increase of T_l results in the decrease of the ordinate of the hydrograph on the rising limb and increases on the falling limb, as shown in Fig. 3. Furthermore, when T_l is treated as a time-variant, another problem rises on the interpolation of rainfall because the change of T_l make the computing time step adjust again. And the horizontal movement of the hydrograph by ΔT_l has a similar effect on its vertical change by ΔK. Therefore, K is selected as the time-variant variable among the parameters of the storage function.

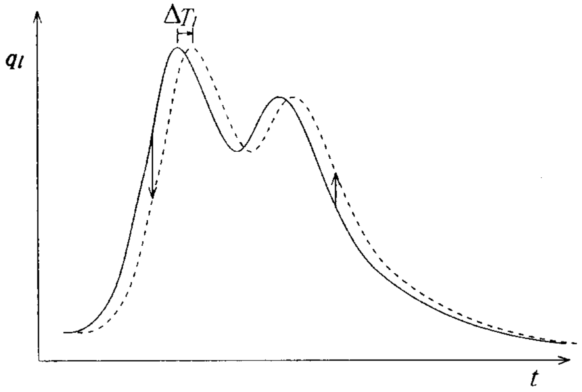

Fig. 3. The Variation of Runoff Depth with Lag Time T_l

Some features of the MSFM are summarized as follows:

① Eqs. (6) and (8) are proposed as governing equations.

② K is a time-variant parameter.

③ The variant runoff coefficient f_2 is used, instead of the primary runoff coefficient and the saturation rainfall.

④ The time-variant parameters f_2 and K are controlled by fuzzy reasoning.

⑤ If the simularted results obtained using the fuzzy control are in accord with the measurements, the problem ⑥ is not significant.

The results of the SFM often show large errors when the representative parameters of the watershed are used. So, we can not expect good results practically without the intuitional udgment of long-experienced engineers(Korea Water Resources Corporation, 1993). Therefore, introducing fuzzy reasoning into the MSFM, we can expect to take the place of knowledge and experience of hydrological engineers.

3. Fuzzy Reasoning

A fuzzy set F in a universe of discourse U is characterized by a membership function μ which takes values in the interval $[0,1]$ namely, $\mu : U \rightarrow [0,1]$. The existing methods have a limitation on representing hydrological complexities and a variety of variation characteristics of the parameters involved. So, there is a necessity for introducing of fuzzy reasoning, which gives much closer reasoning to human thinking and natural language than the traditional reasoning (see ref. (Lee, 1990, Zimmermann, 1991, Kosko, 1992) for the further study). Mamdani's reasoning method(1977), which first succeeded in introduction of the theory of fuzzy sets (Zadeh, 1965) into the engineering application, is adopted in this study.

The primary fuzzy sets usually have a meaning, such as NB (Negative Big), NM (Negative Medium), NS (Negative Small), ZO (Zero), PS (Positive Small), PM (Positive Medium), and PB (Positive Big). A typical example is shown in Fig. 4, depicting two fuzzy partitions in universe $[-1,+1]$ and $[-6,+6]$. A membership functions having the forms of the triangle-shaped and trapezoid-shaped functions are used here.

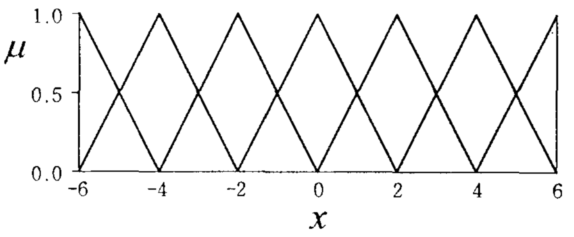

Fig. 4. Membership function of fuzzy variables

The fuzzy reasoning process requires the control rule. In order to establish the control rule, the input variables are divied into 7 fuzzy sets, consisting the elements of the control rule table. There is no common and theoretical method recommended for establishing the control rules.

Many control rules similar to Table 1 are necessary in the processes of fuzzy reasoning. For example, the meaning of the 1st row and the 4th column of Table 1 is that "If e is NB and Δe is ZO then ΔB is PB" is "If an error is very big on the negative and the change in error is approximately zero, then the adjusting value should be changed very big in positive". Table 1 is composed of 49 control rules as an example of preview.

The processes of introducing fuzzy reasoning into this study are as follows :

$$e(t) = O_c(t) - O_m(t) \tag{9}$$

$$\Delta e(t) = e(t) - e(t-1) \tag{10}$$

where $O_m(t)$ is the measured runoff discharge, O_c is the computed runoff discharge, $e(t)$ is the error between the measured and computed value, and $\Delta e(t)$ is the change in the error.

The impotant part of fuzzy reasoning is a reasoning process to determine the adjusting value ΔB from the error e, between the measured and computed values, and the change in error Δe.

Fuzzy reasoning is introduced into the reasoning process of the output variable ΔB from the input variables e and Δe, as shown in Fig. 5.

$$e, \Delta e \longrightarrow \Delta B \tag{11}$$

And then, ΔB is added to $B(t\text{-}1)$ in order to obtain the control parameter $B(t)$:

$$B(t) = B(t\text{-}1) + \Delta B \tag{12}$$

The input variables "e"s of K and f_2, which are used in Eqs. (3), (6) and (8), are computed from Eq. (13) and Eq. (14), respectively :

$$e_K = q_l - \frac{q_m}{f_2} \tag{13}$$

$$e_{f_2} = O - O_m \tag{14}$$

where q_m and O_m are the measured direct runoff depth and the runoff discharge, respectively.

Table 1. Example of the control rules

Δe e	NB	NM	NS	ZO	PS	PM	PB
NB	PB	PB	PB	PB	PM	PS	ZO
NM	PB	PB	PB	PM	PS	ZO	NS
NS	PB	PB	PM	PS	ZO	NS	NM
ZO	PB	PM	PS	ZO	NS	NM	NB
PS	PM	PS	ZO	NS	NM	NB	NB
PM	PS	ZO	NS	NM	NB	NB	NB
PB	ZO	NS	NM	NB	NB	NB	NB

A scale mapping is needed to obtain the input variables e and Δe, transformed by using scale factors. The scale factors are usually decided by the method of trial and error(Lee, 1995). The flowchart of introducing fuzzy control into the MSFM is shown in Fig. 6, including a scale mapping procedure.

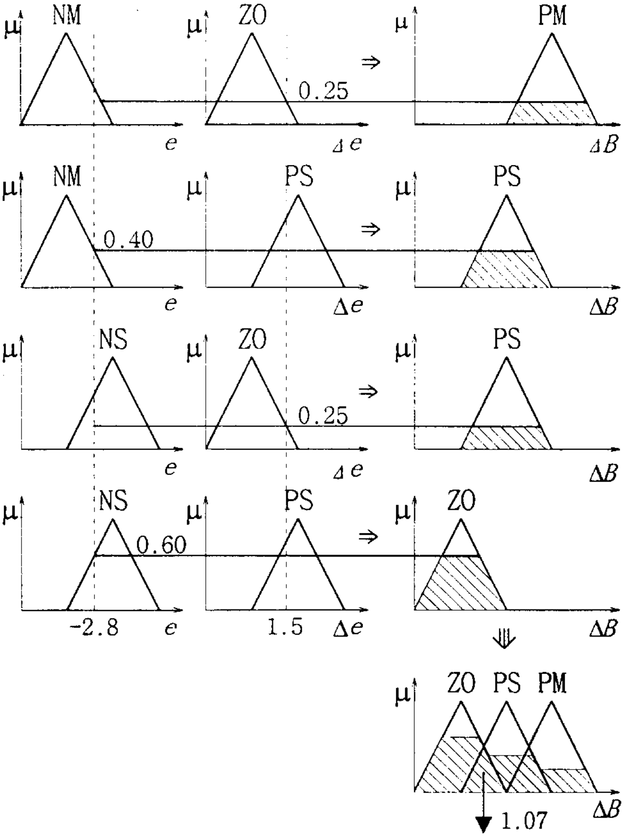

Fig. 5. Mamdani's fuzzy reasoning process

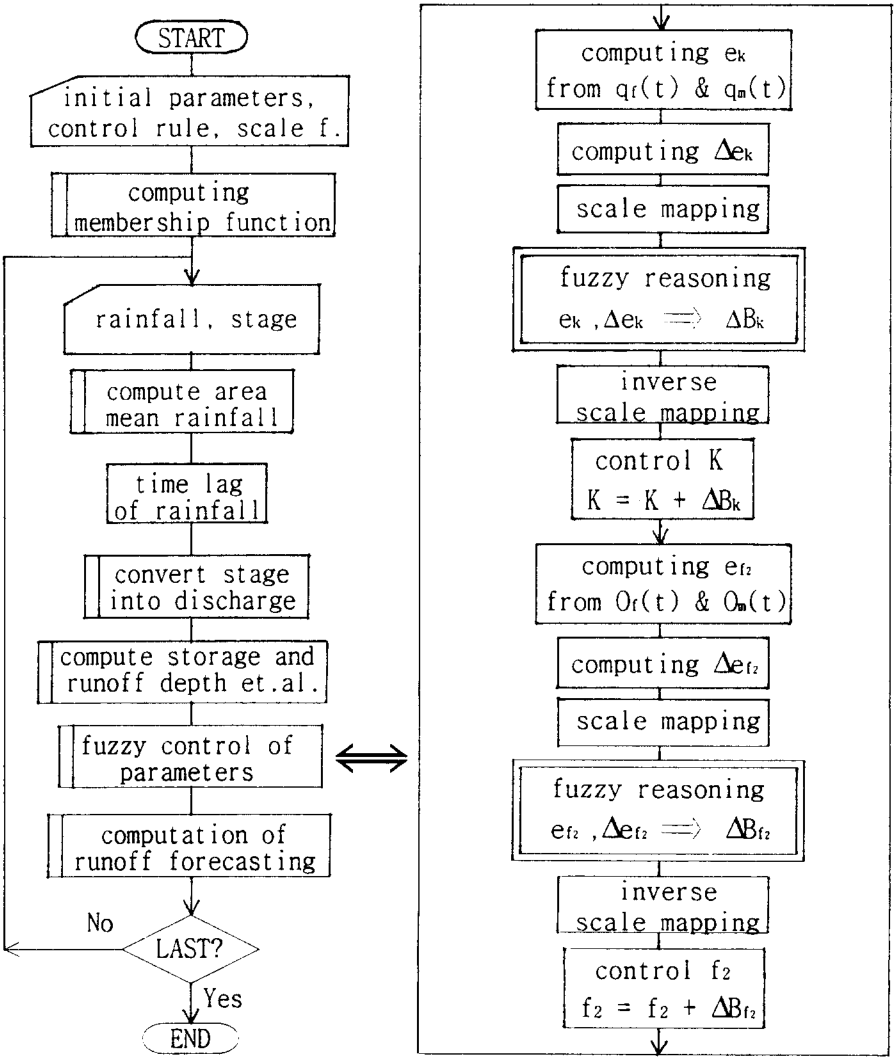

Fig. 6. Flowchart of introducing fuzzy control to the MSFM

4. Application of the MSFM and Discussion

The basin under study is the watershed(346.53 km^2) upstream (Gidae gaging station) of the Bochung stream, mentioned beforehand, which is one of the representative basins of IHP (International Hydrological Program), and rainfall-runoff events are 5 major storms occurred in 1992. Then, the optimum parameters of each rainfall event are computed by the optimization technique of Brent's method and the results are summarized in Table 2.

To examine the usability and the accuracy of the optimization method for parameters and fuzzy reasoning for the MSFM, the following four cases are performed :

Case (1) ; Flood-runoff simulation by the SFM, using the optimum constants of parameters for each storm.

Case (2) ; Flood-runoff simulation by the SFM, using the representative parameters of the basin.

Case (3) ; Flood forecasting with 4 hour lead time by the MSFM, using the representative parameters.

Case (4) ; Flood forecasting with 4 hour lead time by the MSFM, using the fuzzy control of the time-variant parameters.

Fig. 7 is the results of the rainfall events (July 16, Oct. 24 and Sep. 24, 1992) with Cases (1) to (4). As shown in Fig. 7, Case (1) represents a remarkably good result, which means that the optimization technique is useful. Case (2) shows the large discrepancy between the measured and computed values, especially at the peak discharge, so it is necessary to adjust the parameters. In Case (3), it is supposed that the representative constant parameters are not proper. On the other hand, Case (4) shows acceptable results, compared with Cases (1) to (3). It is thought that the superiority of the MSFM is ascribed to the control effect of the time-variant parameters by fuzzy reasoning.

Table 2. Optimum parameters of each rainfall event and representstive parameters at Gidae gaging station of the Bochung stream

Parameters / Rainfall event	T_l	K	P	R	f_1	R_{sa}	f
July 12, 1992	4.3336	17.438	0.41260	0.9720	0.10000	46.486	0.29060
July 16, 1992	2.5661	23.183	0.37247	0.9925	0.49440	42.412	0.67845
Aug. 14, 1992	5.7123	26.165	0.99807	0.9971	0.23236	52.776	0.23767
Aug. 24, 1992	3.8629	21.555	0.63467	0.9979	0.17617	46.656	0.38135
Sep. 24, 1992	5.7001	30.128	0.46661	0.9985	0.29854	44.387	0.56668
Representative Parameters	4.4350	23.694	0.57688	0.9916	0.26029	46.543	0.43095

For the fitness test of each case, the NRMSE (non-dimensional root mean square error) is used,

$$NRMSE = \frac{\sqrt{\frac{1}{n}\sum_{j=1}^{n}\{O_j - Q_j\}^2}}{Q_{max}} \tag{15}$$

where n is the number of data, O_j is the computed runoff discharge, Q_j is the measured discharge, and $Q_{\max}$ is the largest measured discharge. To compare the fitness of the hydrograph of each case with the measured one, the results are summarized in Table 3.

The peak discharge of the runoff has an important meaning in the point of the flood forecasting together with the shape of the runoff hydrograph. The comparison criterion of the peak discharge, E_{Qp}, is used (Yen, 1982) :

$$E_{Qp} = \frac{O_p - Q_p}{Q_p} \tag{16}$$

where E_{Qp} is a relative error of the peak discharge, O_p is the computed peak discharge, and Q_p is the measured peak discharge. And the results of E_{Qp} are also sumarized in Table 4.

Table 3. Comparison of NRMSE for each case

Application method Rainfall event	(1) Optimum P. with SFM	(2) Represen. P. with SFM	(3) Represen. P. with MSFM	(4) Fuzzy Contol with MSFM
July 12, 1992	0.1241	0.2377	0.0747	0.0612
July 16, 1992	0.0487	0.2000	0.1372	0.1288
Aug. 14, 1992	0.0611	0.1521	0.0473	0.0404
Aug. 24, 1992	0.0400	0.0883	0.0842	0.0709
Sep. 24, 1992	0.0298	0.0571	0.0891	0.0594
Mean	0.0607	0.1470	0.0865	0.0721

Table 4. Comparison of E_{Qp} for each case

Application method Rainfall event	(1) Optimum P. with SFM	(2) Represen. P. with SFM	(3) Represen. P. with MSFM	(4) Fuzzy Contol with MSFM
July 12, 1992	0.3988	0.6920	-0.0370	0.0181
July 16, 1992	0.1153	-0.3930	-0.1872	0.0405
Aug. 14, 1992	0.0156	0.3294	-0.0329	0.0064
Aug. 24, 1992	0.1577	0.3268	-0.1006	0.0216
Sep. 24, 1992	0.0884	0.1860	-0.2602	-0.0703
Mean	0.1552	0.3854	0.1236	0.0314

Fig. 8 shows the temporal variation of K and f_2 using the rainfall event on Sep. 24. In this figure, the scale of the runoff discharge are the same as Case (4) of Fig. 7(c). Following the error between the measured and the computated values, the time-variant parameters increase or decrease according to fuzzy reasoning, and after 80 hours, the control values of parameters become very small, as shown in Fig. 8.

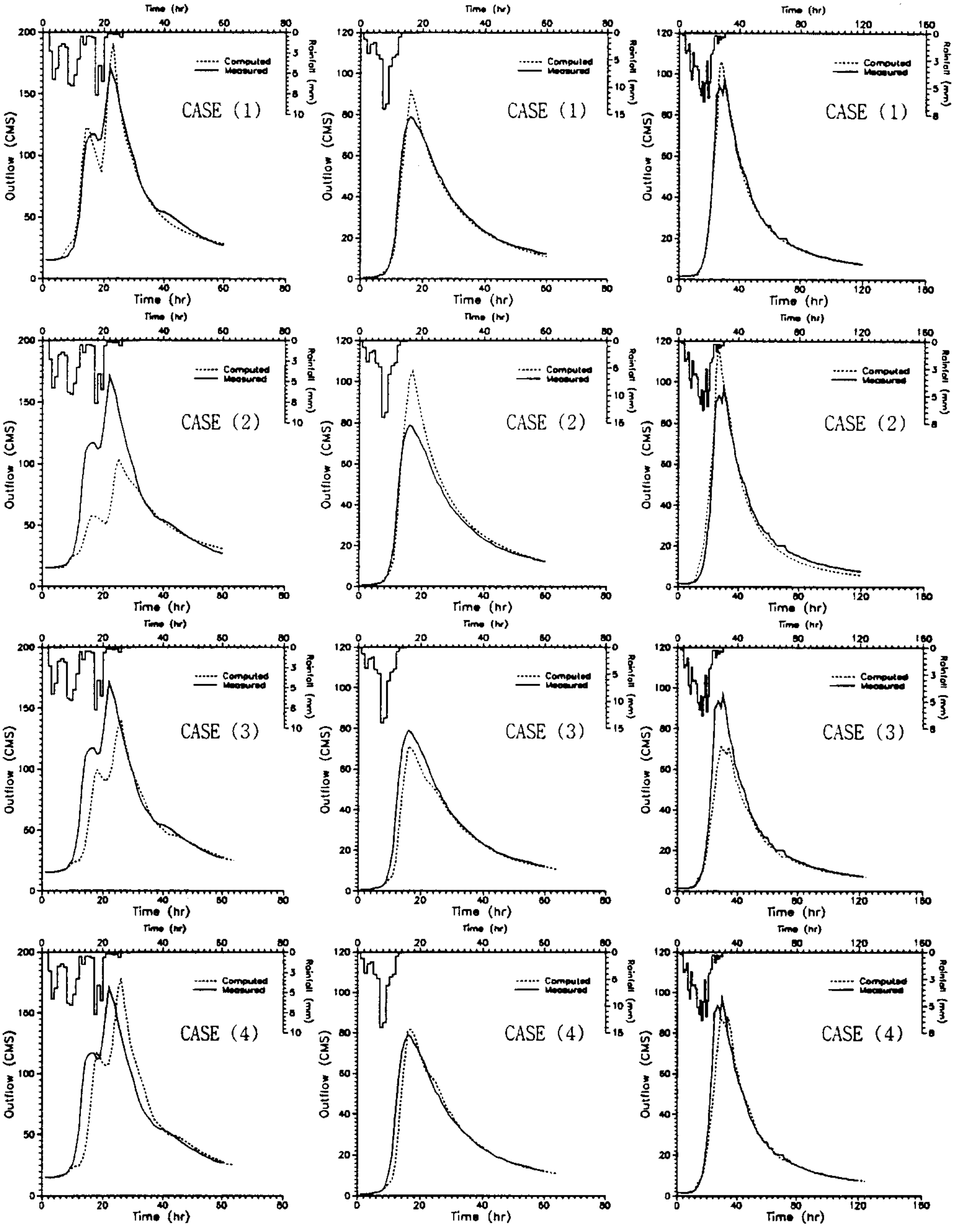

Fig. 7. Results of each rainfall event and case (1992, Bochung Stream)

(a) July 16, 1992 (b) Oct. 24, 1992 (c) Sep. 24, 1992

Case (1) : Optimum parameter with SFM, Case (2) : Representative parameter with SFM,

Case (3) : Representative parameter with MSFM, Case (4) : Fuzzy control with MSFM

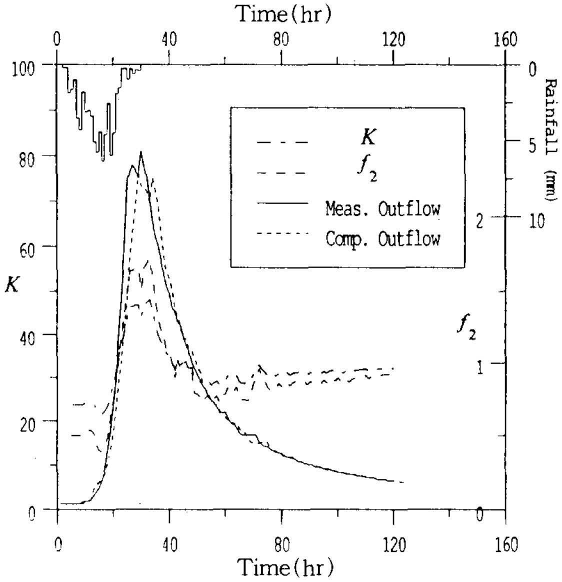

Fig. 8. Temporal variation of K and f_2, Case (4) of Sep. 24, 1992

5. Conclusion

Hydrological models, including the SFM, inevitably involve many error sources, because of the complex hydrological cycle processes and the uncertainties of both the observed data and parameters. Therefore, in this study, the MSFM with the fuzzy control is proposed to improve the predicted outflow hydrograph and the peak discharge. The feature of the MSFM is to treat the two parameters, K and f_2, of the storage function as a time-variant and to be able to use it practically without a lot of experience. The applicability of the MSFM is examined using 5 major storms, occurred in 1992, of the Bochung stream of the Geum River, Korea.

The concluding remarks are summarized as follows;

(1) The optimization method of the parameters reduces both the working time and efforts and improves the accuracy of the parameters.

(2) Reviewing the SFM, the MSFM is proposed by introducing the fuzzy control to the variant parameters.

(3) The MSFM shows generally a better agreement with the measured hydrograph for the basin under study than the SFM.

Therefore, we can use it as an efficient tool for the flood forecasting and the dam operation.

And the further study will be the extention to the development of fuzzy reasoning for good fitness to the hydrological data, the improvement of the control rules and the determination of the scale factor, etc.

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Reference

Fujita, M., Yoshitake, H. , and Lee, B. H. (1995). "Runoff forecasting using a radar hyetometer based on fuzzy reasoning." International Congress on Modelling and Simulation Proceedings, Vol. 3 : Water Resources and Ecology, Australia, pp. 351-356.

Fujita, M., and Zhu, M. -L. (1992). "An application of fuzzy set theory to runoff prediction." Proceedings of the Sixth IAHR International Symposium on Stochastic Hydraulics, Taipei, pp. 727-734.

Hydrologic research society of Japanese Ministry of Construction. (1971). Example handbook of runoff computation (Ⅱ), pp. 81-146. (in Japanese).

International Hydrological Program (IHP) Report. (1992). The Ministry of Construction, Korea. (in Korean).

Kimura, T. (1962). "Storage Function Method (II)-The Basic Structure of Storage Function Method." Civil Eng. Data, Vol. 4, No. 1, pp. 654-661. (in Japanese).

Kosko, B. (1992). Neural Networks and Fuzzy Systems. Prentice Hall International, N. J.

Korea Water Resources Corporation. (1993). "Rainfall-runoff model improvment for Namgang multi-purpose dam." WRRI-WR-93-7. (in Korean).

Lee, C. C. (1990). "Fuzzy logic in control systems: fuzzy logic controller-part I." IEEE Trans. Syst. Man Cybern. , Vol. 20, No. 2, pp. 404-418.

Lee, C. H. (1995). "A study on storage function model by fuzzy control of time variant parameters," Ph. D. thesis, Hangyang University, Seoul, Korea. (in Korean).

Lee, J. K., Lee, C. H., and Lee, J. I. (1994). "The applicability of fuzzy reasoning to flood runoff analysis." Proceedings of Korean Society of Civil Engineers, Vol. II, KSCE, pp. 279-282. (in Korean).

Lee, K. H., and Oh, K. R. (1991). Fuzzy Theory and Application. Hongreung Science Press. (in Korean)

Mamdani, E. H. (1977). "Application of fuzzy logic to approximate reasoning using linguistic synthesis." IEEE Transactions on Computers, Vol. C-26, No. 12, pp. 1182-1191.

Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T. (1986). Numerical recipes. Cambridge Press.

Singh, V. P. (1988). Hydrologic systems: Volume Ⅰ. Rainfall-runoff modeling. Prentice Hall.

Viessman, W. Jr., Knapp, J. W., Lewis, G. L., and Harbraugh, T. E. (1977). Introduction to hydrology. IEP-A Dun-Donnelley Publisher.

Yen, B. C. (1982). "Some mearsures for evaluation and comparison of simulation model." Urban stormwater hydraulics and hydrology. Water Resources Publications, pp. 341-349.

Zadeh, L. A. (1965). "Fuzzy Sets." Information and control, Vol. 8, pp. 338-353.

Zimmermann, H. -J. (1991). Fuzzy Set Theory and Its Applications, Kluwer Academic Publisher.