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지진에 의하여 댐에 작용하는 동수압의 고전 이론에 대한 재고

Notes on Incompressible Theory of Hydrodynamic Pressureon Dams during Earthquakes

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Abstract

Classical theory of hydrodynamic pressure on dams during earthquakes is revisited and examined with linear transient theory. Because the ground motion during earthquakes is not only transient but also consists of random horizontal and vertical motions, it is proper to treat hydrodynamic pressure on dams with transient theory for random, transient earthquake motions. The present study finds that surface waves are negligible and that the present theory agrees well with the classical theory if the ground motion is horizontal and harmonic with a high frequency.

요 지

지진 발생시 댐에 야기되는 동수압에 대하여 고전이론에서는 이를 정상상태로 취급하였는데 반하여 본 논문에서는 지진운동을 일시적인 현상으로 취급하여 비정상 선형 이론을 가지고 동수압을 검증하였다. 더욱이 지진운동은 불규칙한 수평 및 수직운동으로 구성되어있으며, 이 운동은 일시적인 현상으로 곧 사라짐으로 정상운동이 아니다. 따라서 지진운동에 의한 동수압은 비정상적으로 취급해야하며 불규칙한 지진운동을 그대로 입력하여 동수압을 구해야한다. 고전 이론에서는 지진운동을 수평적이며 단일 Sine 함수로 구성된 정상운동으로 가정하였으며 또한 물을 비압축성 물질로 가정하였으며 지진시 야기되는 수면파는 적다고 가정하여 무시하였다. 이러한 가정들을 모두 본 이론에 적용하였을 시 계산되는 동수압은 고전 이론에서 제시한 결과와 잘 일치함을 보여준다.

keywords : dam, earthquake, hydrodynamic pressure, transient theory

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1. Introduction

The classical theory for the hydrodynamic pressure on dams during earthquakes is based on the following assumptions : 1) A seismic motion is a steady state harmonic motion with a high frequency and surface waves induced during earthquakes are small and negligible, 2) the viscosity of water is negligible and the water is incompressible, and 3) the dam is rigid (Chopra, 1967). According to spectral analysis, an earthquake is shortlived, random, and composed of harmonics with frequencies distributed over a wide range. The classical theory cannot directly handle random ground motions of earthquakes. Instead, the complex frequency response together with the Fourier integral and use of an impulse response together with the convolution integral are employed to treat the random motions. But such procedures to treat random earthquake motions are not necessary in the present theory because it can directly handle such random earthquake motions. Surface waves would be present along the free surface during and after earthquakes because the dams not only disturb the water in the reservoir but the earthquake motion of the reservoir bottom also disturbs the water. Hence surface waves are also included in the present study. We introduce a time-dependent Green function in order to treat the problem with transient theory (Stoker, 1957; Chung, 1982).

Applying Green's theorem, the hydrodynamic pressure is conveniently formulated in terms of random earthquake accelerations. If the random earthquake motion is replaced by a horizontal harmonic motion with a high frequency, the present theory agrees well with the classical theory even though the present theory is a transient theory different from classical steady state theory. The computed hydrodynamic pressure on the vertical face of dams for horizontal earthquake motions is illustrated in the Table 1.

2. Transient Velocity Potential

The dam is considered as rigid and the water is assumed to be incompressible and inviscid. Further, it is assumed that the reservoir extends to infinity in the upstream direction with a constant depth h and that the motion of the dam during earthquakes is the same as the ground motion at the site. The water in the reservoir is excited by earthquake motions and the motion of the water is described by the velocity potential ϕ such that $u = \phi_x$ and $v = \phi_y$ where u and v are the horizontal and vertical velocities of the water, respectively.

The velocity potential depends not only on the coordinates (x, y) but also on t. When the earthquake starts, the water is initially at rest. According to linear transient theory, the potential ϕ satisfies the following equations (Stoker, 1957; Chung, 1982):

$$\phi_{xx} + \phi_{yy} = 0 \text{ ,}$$

$$\text{for } 0 \leq x < \infty \text{ , } \quad -h \leq y \leq 0 \qquad (1)$$

$$\phi_{tt} + g\phi_y = 0 \text{ , } \quad \text{at } y = 0, \qquad (2)$$

$$\phi_n = v\,(t) \quad \text{along rigid boundaries of water} \qquad (3)$$

$$\phi = \phi_t = 0 \text{ , } \qquad \text{at } t = 0 \qquad (4)$$

where $v(t)$ is the ground velocity component normal to the rigid boundaries into water. The rigid boundaries are the vertical face of the dam and reservoir bottom in Fig. 1.

If the solution ϕ to Eqs. (1) through (4) is found, the hydrodynamic pressure along the

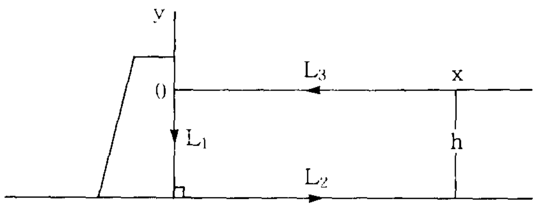

Fig. 1. Schematic Diagram

vertical face of the dam is given by

$$p(0,y;t) = -\rho\,\phi_t(0,y;t) \tag{5}$$

Surface waves induced by earthquakes are given by

$$Y(x;t) = -\frac{1}{g}\phi_t(x,0,t) \tag{6}$$

Let us introduce the timedependent Green function (Stoker, 1957; Chung, 1982):

$$G(x,y;t\mid\xi,\eta;\tau) = -\frac{1}{2\pi}\Bigg[\log\sqrt{\frac{(x-\xi)^2+(y-\eta)^2}{(x-\xi)^2+(y+\eta)^2}} -2\int_0^\infty \frac{e^{-kh}\sinh ky \sinh k\eta \cos k(x-\xi)}{k\cosh kh}dk\Bigg] + \frac{1}{\pi}\int_0^\infty \frac{\cosh k(y+h)\cosh k(\eta+h)\,[1-\cos\nu(t-\tau)]\cos k(x-\xi)}{k\cosh^2 kh \tanh kh}dk \tag{7}$$

where $\nu=\sqrt{kg \tanh kh}$. The Green function satisfies $G_{tt}+G_y=0$ at $y=0$, $G_y=0$ at $y=-h$ and $G=G_t=0$ at $y=0$ and $t=\tau$. Since $\phi(x,y;t)$ is harmonic in the domain of the water, $\phi_t(x,y;t)$ is also harmonic in the domain of the water.

3. Hydrodynamic Pressure

We apply Green's theorem to the Green function and ϕ_t. Then integrals along the free surface vanish. The integrals involving G_ξ along the face of the dam and G_h along the bottom of the reservoir also vanish (Stoker, 1957; Chung, 1982). The hydrodynamic pressure along the face of the dam is given by

$$p(0,y;t) = 2\rho v'_1(t)\int_{-h}^{\infty} G(0,y;t\mid 0,\eta;t)d\eta + 2\rho\int_{-h}^{0}d\eta\int_0^t G_t(0,y;t\mid 0,\eta;\tau)v'_1(\tau)d\tau -2\rho v'_2(t)\int_0^\infty G(0,y;t\mid\xi,-h;t)d\xi - 2\rho\int_0^\infty d\xi\int_0^t G_t(0,y;t\mid\xi,-h;\tau)v'_2(\tau)d\tau \tag{8}$$

where $v'_1(t)=dv_1(t)/dt$ and $v'_2(t)=dv_2(t)/dt$. $v'_1(t)$ is the horizontal earthquake accelerlation at the dam and $v'_2(t)$ is the vertical earthquake acceleration of the reservoir bottom. The hydrodynamic pressure in Eq. (8) is expressed in terms of those horizontal and vertical accelerations. Surface waves in Eq. (6) is similarly given by

$$Y(x;t) = \frac{2}{g}\int_{-h}^{0}d\eta\int_0^t G_t(x,0;t\mid 0,\eta;\tau)v'_1(\tau)d\tau - \frac{2}{g}\int_{-h}^{0}d\eta\int_0^t G_{\xi t}(x,0;t\mid 0,\eta;t)\phi_\tau(0,\eta;\tau)d\tau - \frac{2}{g}\int_0^\infty d\xi\int_0^t G_t(x,0;t\mid\xi,-h;\tau)v'_2(\tau)d\tau \tag{9}$$

Let us consider horizontal earthquakes for which we let $v'_2(\tau)=0$ in Eq. (9). We introduce the following dimensionless variables for convenience:

$$(x',y',\xi',\eta') = h^{-1}(x,y,\xi,\eta),\quad k'=kh \tag{10}$$

Eq. (8) is written only for horizontal earthquakes as

$$p(0,y;t) = \rho h v'_1(t)\sigma(y') + 2\rho h\int_{-1}^{0}d\eta'\int_0^t G(0,y';t\mid 0,\eta';\tau)v'_1(\tau)d\tau \tag{11}$$

where

$$\sigma(y') = \frac{1}{\pi}[(1+y')\log(1+y') - 2y'\log|y'| - (1-y')\log(1-y')] + \frac{2}{\pi}\int_0^\infty \frac{e^{-k'}\sinh k'y'(\cosh k'-1)dk'}{k'^2\cosh k'} \tag{12}$$

$\sigma(y')$ is evaluated by the Laguerre quadrature formula. Computed values of $\sigma(y')$ are given in the following Table 1.

Table 1. Values of $\sigma(y')$

$-y'$	$\sigma(y')$	$-y'$	$\sigma(y')$
0.	0.		
0.05	0.13487	0.55	0.63659
0.10	0.22558	0.60	0.65963
0.15	0.29957	0.65	0.67958
0.20	0.36265	0.70	0.69659
0.25	0.41755	0.75	0.71079
0.30	0.46587	0.80	0.72228
0.35	0.50867	0.85	0.73115
0.40	0.54666	0.90	0.73743
0.45	0.58039	0.95	0.74112
0.50	0.61026	1.00	0.73662

If the earthquake motion is harmonic with a high frequency ω, we write $v'_1(t)$ as $v'_1(t) = a \sin \omega t$. Then the second term on the right side of Eq. (11) is of order ω^{-1} and is negligibly small. Eq. (11) becomes

$$p(0,x;t) = \rho h v'(t) \sigma(y') + O(\omega^{-1}) \quad (13)$$

The hydrodynamic pressure in Eq. (13) agrees well with those of Zangar (1953).

The earthquake motion is not harmonic, but it is highly oscillatory. In the case, the second term in Eq. (11) is small compared with the first term and could be ignored. Otherwise, the second term affects the hydrodynamic pressure. Surface waves induced by horizontal earthquakes are given by

$$Y(x;t) = \frac{2}{g} \int_{-h}^{0} d\eta \int_0^t G(x,0;t \mid 0,\eta;\tau) v'_1(\tau) d\tau \quad (14)$$

If the earthquake motion is harmonic, then

$$Y(x;t) = O(\omega^{-1}) \quad (15)$$

Hence surface waves are negligibly small.

Next, let us consider the hydrodynamic pressure due to vertical earthquakes alone. We let $v'_1(t) = v'_1(\tau) = 0$ in Eqs. (8) and (9). Then Eqs. (8) and (9) reduce to

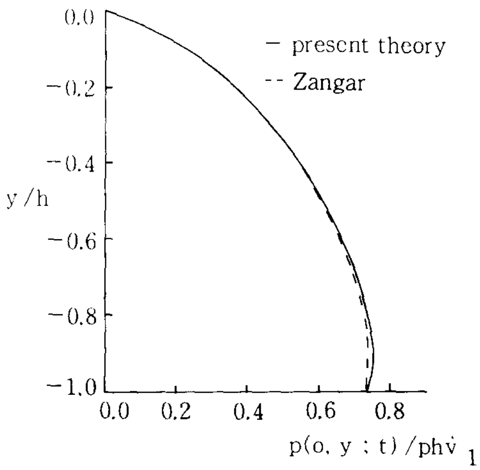

Fig. 2. Normalized Hydrodynamic Pressure on Dam

$$p(0,y;t) = -2\rho v'_2(t) \int_0^{\infty} G(0,y;t \mid \xi,-h;\tau) d\xi$$
$$-2\rho \int_0^{\infty} d\xi \int_0^t G_t(0,y;t \mid \xi,-h;\tau) v'_2(\tau) d\tau \quad (16)$$

$$Y(x;t) =$$
$$-\frac{2}{g} \int_{-h}^{0} d\eta \int_0^t G_{\xi t}(x,0;t \mid 0,\eta;\tau) \phi_t(0,\eta;\tau) d\tau$$
$$-\frac{2}{g} \int_0^{\infty} d\xi \int_0^t G_t(x,0;t \mid \xi,-h;\tau) v'_2(\tau) d\tau \quad (17)$$

If the vertical earthquake motion is harmonic with a high frequency ω_1, Eqs. (16) and (17) are similarly written as

$$p(0,y;t) = -\rho v'_2(t)y + O(\omega_1^{-1}) \quad (18)$$

$$Y(x;t) = O(\omega_1^{-1}) \quad (19)$$

We now return to Eq. (8). The vertical ground motion during earthquakes is not the same as the horizontal ground motion. If the horizontal and vertical ground motions are harmonic with frequencies ω and ω_1, respectively, Eq. (8) is written as

$$p(0,y;t) = \rho h v'_1(t)\sigma(y') - \rho v'_2(t)y + O(\omega^{-1}, \omega_1^{-1}) \quad (20)$$

Eqs. (8) and (20) indicate that vertical earthquakes affect the hydrodynamic pressure. But the vertical earthquakes are ignored in the classical theory. Similarly as before, surface waves are given by

$$Y(x;t) = O(\omega^{-1}, \ \omega_1^{-1}) \quad (21)$$

Therefore, surface waves are small if the earthquakes are highly oscillatory.

4. Concluding Remarks

Because earthquakes last about 30 seconds, transient theory is the exact theory to apply. Furthermore, the present theory can handle random earthquake motion directly. If the earthquake motion is horizontal and harmonic with a high frequency, the classical theory agrees well with the present theory. Surface waves are also found to be negligible.

Vertical earthquakes affect hydrodynamic pressure on dams as seen from Eqs. (8) and (20). If the vertical acceleration $v'_2(t)$ does not depend on the coordinates $(x,-h)$ along the bottom of the reservoir, the hydrodynamic pressure due to the vertical earthquakes is hydrostatic with respect to $v'_2(t)$. This is a significant result because vertical earthquakes have been ignored and have never been treated in detail before.

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