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개정 Prandtl 이론을 이용한 유사 농도 분포식

A Sediment Concentration Distribution Based on a Revised Prandtl Mixing Theory

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Abstract

Modifications of Prandtl's mixing length theory were used to obtain a power velocity distribution in which the coefficient and exponent are variable over a range from 1/4 to 1/7. A simple suspended-sediment concentration distribution was developed which can be associated with this modified velocity distribution. Using nominal values of $\beta=1.0$, $\kappa=0.4$ and visual accumulation tube values of fall velocity, the comparison between theory and field measurements by the USGS on the Rio Grande is fair. Doubling the value of the exponent results in a good comparison. Further research is needed to be able to better choose β, κ, and fall velocity values, but such research will not be able to account for the effects of large-scale turbulence and secondary flows. In a pragmatic sense, a special set of fairly detailed measurements can establish coefficients and exponents for any gaging site.

요 지

멱 유속 분포를 구하기 위해 개정 Prandtl 혼합 거리 이론이 이용되었으며, 여기에서 사용된 지수값의 범위는 1/4~1/7이었다. 이 개정된 유속분포를 이용하여 간단한 부유사 농도 분포식을 개발하였다. 미국 지질조사국이 리오그란데강에서 실측한 자료와 명목값인 $\beta=1.0$, $\kappa=0.4$, 그리고 가시관에 의해 얻어진 침강속도를 이용한 개정 농도식 계산결과와 실측치와의 비교는 양호한 편이었으며, 지수에 임의로 두배를 해주었을 경우에는 좋은 결과를 보였다. 적당한 β, κ, 그리고 침강속도를 선택하기 위해 더 많은 연구가 필요하지만 이러한 연구가 대규모 난류와 이차류 영향에 대한 설명을 할 수는 없을 것이다. 하지만 실용적인 측면에서 보면 어느 관측지점에 대한 매우 자세하고 특별하게 측정된 자료는 그 지점에 맞는 지수나 계수를 찾아낼 수도 있다.

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1. Prandtl's Mixing Length Theory and Logarithmic Velocity Distribution.

Prandtl's notion of turbulence was, in essence, that small masses of fluid are exchanged without losing much, if any, of their momentum (velocity). When mass from a higher-velocity region passes a point, the instantaneous velocity at the point becomes greater than the temporal mean velocity; when mass is from a lower-velocity region the instantaneous velocity is less than the mean velocity ($u=\bar{u}+u'$ and $u=\bar{u}-u'$, respectively). Prandtl's (1926) original mixing length theory and the logarithmic velocity distribution begin with the equation for shear:

$$\tau=-\rho\overline{u'v'} \tag{1}$$

in which ρ is the mass density of the fluid and the bar over the product denotes mean value.

The concepts used to relate the turbulent fluctuation u' and v' to the mean velocity gradient $d\bar{u}/dy$ are:

$$u \propto \iota\frac{d\bar{u}}{dy} \tag{2}$$

$$v' \propto u' \tag{3}$$

$$\iota=\kappa y \tag{4}$$

$$\frac{\tau}{\rho}=\kappa^2 y^2\left(\frac{d\bar{u}}{dy}\right)^2 \tag{5}$$

where y is distance from the boundary, ι is the mixing length (the effective distance the mass of fluid off density ρ moves laterally in turbulent flow), and κ absorbs all the proportionality coefficients.

Prandtl then assumed the internal shear was equal to the boundary shear:

$$\tau=\tau_0 \tag{6}$$

The differential equation becomes:

$$\frac{d\bar{u}}{\sqrt{\tau_0/\rho}}=\frac{1}{\kappa}\frac{dy}{y} \tag{7}$$

which can be integrated as:

$$\frac{\bar{u}}{\sqrt{\tau_0/\rho}}=\frac{1}{\kappa}\ \text{In}\ y+\text{constant} \tag{8}$$

which agrees very well with Nikuradse's measurements in pipes.

2. Rouse's Concentration Distribution

Rouse (1937) used the logarithmic velocity distribution, O'Brien's (1933) diffusion equation for suspended sediment, mixing coefficients for momentum (velocity) and suspended sediment, and assumed that the two mixing coefficients were proportional.

$$C_W=-\in_s\frac{dC}{dy} \tag{9}$$

$$\tau=\rho\in_m\frac{d\bar{u}}{dy} \tag{10}$$

$$\tau=\left(1-\frac{y}{D}\right)\tau_0 \tag{11}$$

$$\in_s=\beta\in_m \tag{12}$$

Combining and integrating, one gets:

$$\frac{C}{C_a}=\left(\frac{D-y}{y}\cdot\frac{a}{D-a}\right)^{\frac{w}{\beta\kappa\sqrt{\tau_0/\rho}}} \tag{13}$$

where the a is the value of y such that $C(y=a)$

$= Ca$; C = sediment concentration at level y; C_a = reference sediment concentration at level a; D = flow depth; w = fall velocity; y = height above the channel bed; β = coefficient of proportionality between $\in_s$ and $\in_m$, β = 1.0 unless otherwise noted; κ = the "universal" mixing length coefficient (not necessarily Prandtl's κ, and probably variable, not constant); τ_0 = tractive force at stream bed; ρ = density of water; $\in_s$ = mixing coefficient for sediment; and $\in_m$ = mixing coefficient for momentum.

If a β value of about 1.5 is used, Rouse's equation agrees quite well with measurements. However, it is difficult to explain why sediment is mixed more effectively than momentum. Rouse's internal shear varies linearly in the vertical (as it should), whereas Prandtl's internal shear was constant. There are other details of Prandtl's concept that should also be examined further. However, the best reason to revisit Prandtl and Rouse is the difficulty in integrating the product of their equations to obtain the suspended sediment load.

$$q_s = \int_0^D \bar{u}\, C dy \quad (14)$$

3. Laursen's (1980) Revision of Prandtl's Assumptions

Several of Prandtl's statements or assumptions do not stand up well to critical scrutiny: (1) the instantaneous turbulence components are not proportional to the product of the mixing length and the mean velocity gradient, (2) the temporal averaged turbulent shear is not proportional to some measure of the product of the two temporal averaged turbulence components, and (3) the internal shear is not a constant equal to the boundary shear.

A few simple revisions are sufficient to make the derivation of the logarithmic velocity distribution more rigorous. Let

$$\sqrt{\overline{u'^2}} \propto l \frac{d\bar{u}}{dy} \quad (15)$$

$$\sqrt{\overline{u'^2}} \propto' \sqrt{\overline{v'^2}} \quad (16)$$

$$\frac{\overline{u'v'}}{\sqrt{\overline{u'^2}}\sqrt{\overline{v'^2}}} = R \quad (17)$$

where R is a correlation coefficient equal to:

$$R = \left(1 - \frac{y}{D}\right) \quad (18)$$

This assumption (Laursen, 1980) will result in the familiar logarithmic velocity distribution when the correct internal shear distribution, Eq. (11), and Prandtl's mixing length assumption, Eq. (4), are used.

4. Laursen's Concentration Distribution (Laursen, 1980)

If, now, the difference between the mixing coefficients for sediment and momentum is partly due to the correlation coefficient R, the relationship between the two is:

$$\in_s = \frac{\beta \in_m}{(1 - y/D)} \quad (19)$$

and the mixing coefficient for sediment becomes:

$$\in_s = \beta\kappa\sqrt{\tau_0/\rho}\, y \quad (20)$$

The differential form of the sediment diffusion equation is then:

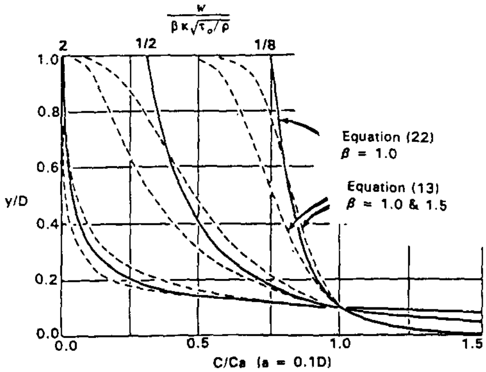

Fig. 1. Comparison of Rouse (Eq. (13)) and Laursen (Eq. (22)) Concentration Distributions ($\beta=1.0$, $\kappa=0.4$, a=0.1D; Laursen, 1980)

$$\frac{dC}{C}=-\frac{\mathrm{w}dy}{\beta\kappa\sqrt{\frac{\tau_0}{\rho}}\,y} \tag{21}$$

which integrates simply to:

$$\frac{C}{C_a}=\left(\frac{a}{y}\right)^{\frac{\mathrm{w}}{\beta\kappa\sqrt{\tau_0/\rho}}} \tag{22}$$

Note that β in this equation will not be the same as the β value in the Rouse equation. Both β values are found by forcing the theoretical equations to fit measured data. A comparison of the two equations is shown in Fig. 1.

5. Jung's (1989) First Revision of Laursen's Revision of Prandtl's Assumptions

Laursen's mathematically simpler sediment concentration distribution equation still requires integration involved in computation to find the suspended sediment load. Use of a power distrin for the velocity would make the integration much easier. However, there would be a logical inconsistency since the logarithmic velocity distribution was used in deriving the concentration distribution. The first attempt to resolve this inconsistency was a change in the assumption for the mixing length to:

$$l=\kappa\ \mathrm{y}^{\gamma} \tag{23}$$

Using Laursen's other assumptions (Eqs. (15), (16), (17), and (18)), combining terms, and integrating results in a power velocity distribution

$$\bar{u}=\left(\frac{1}{\kappa}\right)\sqrt{\frac{\tau_0}{\rho}}\ \frac{1}{(-\gamma+1)}\mathrm{y}^{-\gamma+1} \tag{24}$$

If γ is taken as 3/4, $(-\gamma+1)$ is 1/4; if γ is 6/7, $(-\gamma+1)$ is 1/7. These are the typical values found in the literature for rough and smooth boundaries, and

$$\bar{u}=\left(\frac{4}{\kappa}\right)\sqrt{\frac{\tau_0}{\rho}}\ \mathrm{y}^{1/4} \tag{25}$$

or

$$\bar{u}=\left(\frac{7}{\kappa}\right)\sqrt{\frac{\tau_0}{\rho}}\ \mathrm{y}^{1/7} \tag{26}$$

Note that κ values in this power equation do not have to be the same as the κ value in the logarithmic distribution equation (usually taken as about 0.4 or less). A comparison of the power equation (Eq. (26)) and the logarithmic equation, and the universal equation indicates the power distribution with 1/7 exponent fits well with logarithmic velocity distribution, as shown in Fig. 2.

Power-law formulas were used before logarithmic equations were used (Daily and Harleman, 1966). The general power velocity distribution equation (Izbash and Khaldre, 1971) is:

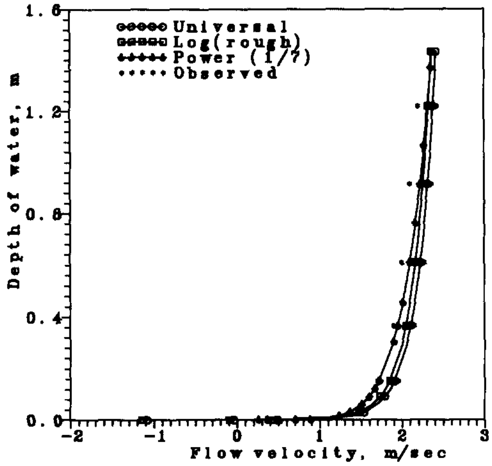

Fig. 2. Comparison of Velocity Distributions

$$\frac{\bar{u}}{U_{\max}}=\left(\frac{y}{D}\right)^{x} \tag{27}$$

where $U_{\max}$ = velocity at the water surface; $\bar{u}$ = time-averaged flow velocity at a distance y from the bed; and x = a power index, usually between 1/4 to 1/7.

The velocity $\bar{u}$ at level y is given by Eq. (27), and since the mean velocity $\bar{U}$ times D is equal to the discharge per unit width, q, is:

$$q=\bar{U}D=\int_0^D \bar{u}dy \tag{28}$$

$$q=\bar{U}D=\int_0^D U_{\max}\left(\frac{y}{D}\right)^{x}dy \tag{29}$$

which results in:

$$q=\bar{U}D=\left(\frac{1}{x+1}\right)U_{\max}D \tag{30}$$

So,

$$U_{\max}(x+1)\bar{u} \tag{31}$$

Therefore,

$$\frac{\bar{u}}{(x+1)\bar{U}}=\left(\frac{y}{D}\right)^{x} \tag{32}$$

$$\frac{\bar{u}}{\bar{U}}=(x+1)\left(\frac{y}{D}\right)^{x} \tag{33}$$

Continuing to follow Laursen's assumption, the associated concentration distribution equation can be obtained. The mixing coefficients, the differential diffusion equation, and the final formulation are:

$$\in_m=\sqrt{\tau_0/\rho}\,(1-y/D)\kappa y^{\gamma} \tag{34}$$

$$\in_s=\frac{\beta\in_m}{(1-\frac{y}{D})}=\beta\sqrt{\tau_0/\rho}\;\kappa y^{\gamma} \tag{35}$$

$$\frac{dC}{C}=\frac{w}{\in_s}dy=\frac{w}{\beta\kappa\sqrt{\tau_0/\rho}}\frac{dy}{y^{\gamma}}=-\frac{z}{y^{\gamma}}dy \tag{36}$$

where

$$z=\frac{w}{\beta\kappa\sqrt{\tau_0/\rho}} \tag{37}$$

and

$$\frac{C}{C_a}=e^{\frac{z}{-\gamma+1}a^{-\gamma+1}}\Big/e^{\frac{z}{-\gamma+1}y^{-\gamma+1}} \tag{38}$$

If γ = 3/4 (rough boundary):

$$\frac{C}{C_a}=e^{4za^{1/4}}/e^{4zy^{1/4}}=e^{4z(a^{1/4}-y^{1/4})} \tag{39}$$

If γ = 6/7 (smooth boundary):

$$\frac{C}{C_a}=e^{7za^{1/7}}/e^{7zy^{1/7}}=e^{7z(a^{1/7}-y^{1/7})} \tag{40}$$

6. Jung's (1993) Second Revision of Laursen's Revision of Prandtl's Assumptions

Unfortunately the first attempt to find a simpler pair of equations resulted in a complex concentration distribution equation to go with the simpler power velocity distribution. In the second attempt at simplicity, the assumption R=(1-y/D) for the correlation coefficient was modified to:

$$R=(1-\frac{y}{D})y^{2\gamma-2} \qquad (41)$$

Retaining the previous assumption for mixing length, the velocity distribution becomes:

$$\bar{u}=\frac{1}{\kappa}\sqrt{\frac{\tau_0}{\rho}}\ \frac{1}{-2\gamma+2}\ y^{-2\gamma+2} \qquad (42)$$

which appears to be slightly different form Eq. (24), but when γ is 7/8 or 13/14 the exponent is 1/4 or 1/7 and results in Eqs. (25) and (26). The mixing coefficients, differential diffusion equation, and final concentration distribution equation become:

$$\in_m=\sqrt{\tau_0/\rho}\,(1-\frac{y}{D})\kappa\ y^{2\gamma-1} \qquad (43)$$

$$\in_s=\beta\ \kappa\ \sqrt{\tau_0/\rho}\ y$$

$$\frac{dC}{C}=-\frac{w}{\beta\ \kappa\ \sqrt{\tau_0/\rho}}\ \frac{dy}{y}=-z\frac{dy}{y} \qquad (44)$$

$$\frac{C}{C_a}=(\frac{a}{y})^{\frac{w}{\beta\ \kappa\ \sqrt{\tau_0/\rho}}} \qquad (45)$$

Eq. (45) is the same as Laursen's concentration distribution, Eq. (22). A comparison of the three concentration distribution equations (Rouse, Eq. (13); Laursen, Eq. (22); and Jung, Eq. (40)) is shown in Fig. 3.

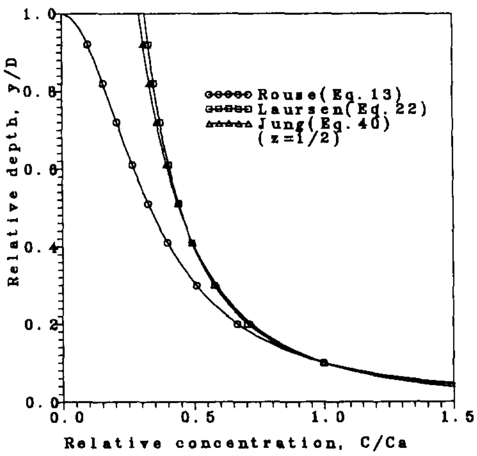

Fig. 3. Comparison of Eqs. (13), (22), and (40) (β=1.0, κ=0.4, a=0.1D)

7. Comparison of Laursen's Concentration Distribution with Rio Grande Measurements

Fig. 4 shows a comparison between the theoretical concentration curves using equation (45) and two sets of measurements made by the U.S. Geological Survey (USGS) at gaging stations 2243 and 2249 on the Rio Grande (Culbertson et al., 1972). The bed material and suspended sediment samples were divided into four fractions in the USGS data (Fig. 5), and these were used for the computations. The z-values were based on nominal values κ=0.4, β=1.0, visual accumulation tube fall velocity, and measured values of slope and depth.

The reference concentration used in Fig. 4 was at the lowest level sampling point. If the mid-depth sampling point had been used, the theoretical curve would seem to go through the

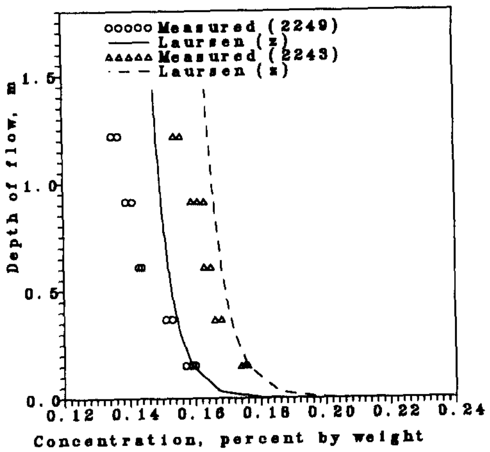

Fig. 4. Comparison of Concentration Distribution (Eq. 45) with measurements at San Marcial, Rio Grande River.

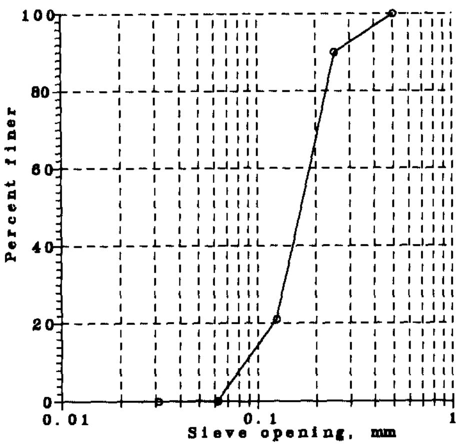

Fig. 5. Typical Bed Material Size Distribution at San Marcial, Rio Grande River, Station 2249.

measured data much better. However, while this better fit is more apparent than real, the better fit does lead to a relevant question: where should the sediment sample be taken? For finesediment the reference level should approach mid-depth; for coarse sediment it should approach the bed, for mixed sediment it is somewhere in between and depends partly on the sediment problem to be evaluated.

In Fig. 6, the same data are used as in Fig. 4, and the value of z is arbitrarily doubled. The Laursen curves now go through the measured points as well as one could expect. Note that the lowest sampling elevation is still the reference level. The question, then, is why the "correct" z value might be twice the nominal z value. The z value would be larger if the κ value was smaller. The κ value could be in the order of 0.32; this would increase z by a factor of 1.25 (0.4/0.32). The diameter determined by the visual accumulation(VA) tube is presumed to be the diameter of a sphere falling by itself, and the sediment particles are presumed to have the same fall velocity (or time of fall) as measured by the VA tube. This should give the "correct" fall velocity if the concentration of the particles in the tube is the same as the concentration in the flow, and if the turbulence in the tube is the same as the turbulence in the stream. If the concentration in the tube is greater than in the stream, a factor larger than unity should be applied. Turbulence in the tube is caused by the falling particles and can probably be safely neglected. Not enough has been done to permit estimating the effect of stream turbulence on the fall velocity of suspended sediment particles. One can argue that since non-spherical particles tend to fall such that the drag forces are at the maximum, turbulence of a scale which would rotate particles would result in a larger fall velocity. If these two velocity effects need multiplying factors of 1.20 and 1.33, respectively, the three factors together would increase the nominal value of z by a factor of two (1.25 * 1.2 * 1.33). A value of β of less than unity would have a similar effect, but there is no evidence that this would be a reasonable supposition.

More research to clarify these issues in needed, but is not unreasonable to believe that the z used in Fig. 6 is just as valid as the z used Fig. 4, if not more so.

It is probably not reasonable to expect Prandtl's mixing length theory (or any variation of it) to be entirely satisfying in describing either the velocity distribution or the concentration distribution of a real river or, even, of a laboratory flume. There are other aspects of the flow that can have substantial effects: secondary currents that are quite steady and persistent, such as the spirals induced by bends, and vortices that are more random, close to the scale of the river depth, and which could also be considered very large-scale turbulence – but which are not the phenomenon described by Prandtl's turbulence.

The masses of fluid moving up and down lead

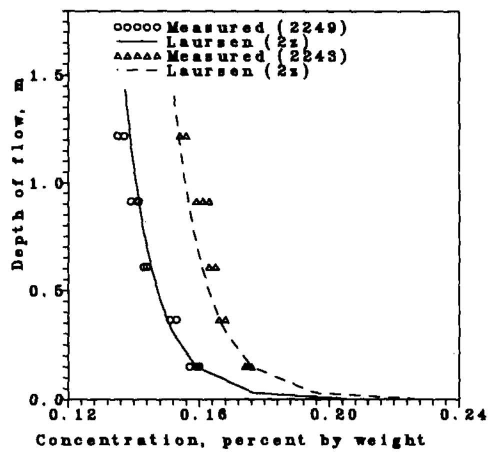

Fig. 6. Comparison of Concentration Distributions (Eq. (45)) Using Corrected z with Measurement at San Marcial, Rio Grande River.

to Prandtl's theory of mixing length. The large-scale eddies and vortices are a different phenomenon and are not approximated by Prandtl's theory. Secondary flows, such as vertical-and horizontal-axis vortices, can be very important in sediment problems. They could be more important in the turbulent mixing of sediment than in the turbulent mixing of velocity (momentum).

8. Average Sediment Concentration Based on the Simpler Concentration Distribution Equation and a Power Velocity Distribution

From the general relationship among suspended lead, concentration, velocity, and depth,

$$q_s = \eta \int_a^D \bar{u} C dy = \eta\, C_m \bar{U} D \quad (46)$$

where η is a coefficient related to units of the variables, and

$$C_m = \int_a^D \frac{\bar{u}}{\bar{U}} C \frac{dy}{D} \quad (47)$$

where a is equal to $2d_{50}$, a value that may be changed with further research. From Eq. (45) for the sediment concentration distribution,

$$C = C_a \left(\frac{a}{y}\right)^{\frac{w}{\beta \kappa \sqrt{\frac{\tau_0}{\rho}}}}$$

Finally from Eqs. (33), (45), and (47),

$$C_m = \int_a^D \frac{\bar{u}}{\bar{U}} C \frac{dy}{D} \quad (48)$$

$$C_m = \int_a^D (x+1)\left(\frac{y}{D}\right)^x \left(\frac{1}{D}\right) C_a \left(\frac{a}{y}\right)^z dy \quad (49)$$

$$C_m = (x+1) D^{-x-1} C_a a^z \left[\frac{1}{x-z+1}\, y^{x-z+1}\right]_a^D \quad (50)$$

or, the average sediment concentration in a vertical is

$$C_m = \frac{JC_a a_z}{(J-z)D^J}\,(D^{J-z} - a^{J-z}) \qquad (51)$$

where

$$J = x+1 = 5/4 \sim 8/7 \qquad (52)$$

In Eq. (51), in this study at this time, is assumed to have a very small value near the bed of the river at a level equal to $2d_{50}$.

9. Comparison of Eqs. (40) and (45)

Computed values for the vertical concentration using Eq. (40) and Eq. (45), which is the same as Eq. (22), are compared in Fig. 3. The two curves are quite consistent, but there are interesting differences. When computed values for a standard rectangular channel are considered, the C/C_a values using Eq. (45) are always less than the C/C_a values using Eq. (40), but the difference is negligible. Also, there is no big difference in computed values between when 3/4 or 6/7 is used as the γ value, (or velocity power of 1/4 or 1/7), as shown in Fig. 7. Generally, these two curves have good consistency. It is concluded, therefore, that computational ease is an appropriate factor to consider in choosing an approximate equation.

Also correlation coefficients (Eqs. (18) and (41)) and mixing lengths (Eqs. (4) and (23)) were examined graphically, as shown in Figs. 8 and 9, respectively. Differences are apparent in the correlation coefficients in the lower level of flow, while the two mixing lengths do not agree above the mid-point of the flow. However, the values of these two quantities, ι and R, cannot be directly measured, and it is not possible to say which is a better approximation. Either pair gives velocity and concentration distributions which are acceptable approximations.

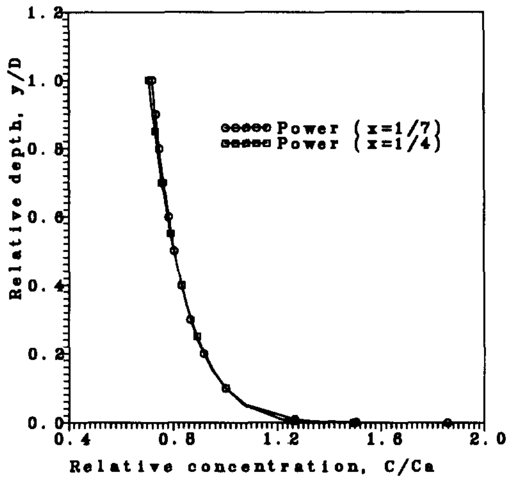

Fig. 7. Comparison of Concentration Distribution Using x=1/4 and x=1/7, β=1.0, κ=0.4, a=0.1D, and z=0.1874.

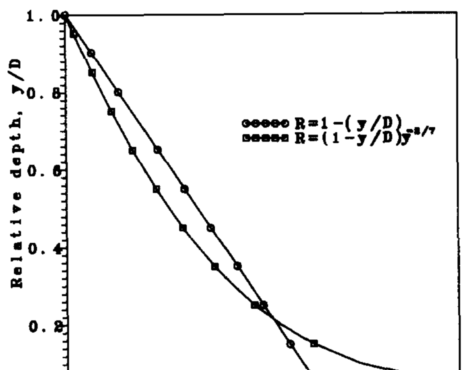

Fig. 8. Comparison of Correlation Coefficients (γ=6/7).

This study demonstrates that simple forms of the velocity and concentration distribution equa-

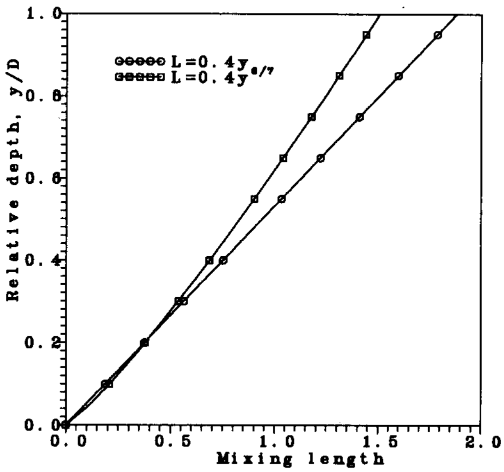

Fig. 9. Comparison of Mixing Lengths (κ =0.4).

10. Discussion and Conclusions

tions are sufficiently accurate. The most uncertain step in the computational procedure presented is related to the value of the Rouse number $z=w/(\kappa\beta\sqrt{\tau_0/\rho})$. Variables in this parameter are not yet completely understood. The fall velocity value needed in the computation procedure is the fall velocity of a natural sediment particle in turbulent flow and in the presence of many other particles. The κ needed is for flow in a natural channel of complex planform and with complex (and variable) roughness elements. Note that the value of κ is likely to be different for different assumptions of the turbulence structure and relation to the mean velocity gradient. The β factor needed is a measure of the difference between the mixing of momentum and sediment in suspension over and above the correlation coefficient used in the derivations presented in this study. The scant evidence cited here suggests that the restricted β might have a value of about one, and that any variation would be small. Subsequent companion papers will examine: (1) the ability to predict suspended sediment loads by the integration of these proposed equations, and (2) possible errors to be guarded against in single-point sampling.

The velocity distribution and the concentration distribution can be written with parameters containing variables generally accepted. Better agreement with measurements can be obtained by using coefficients and exponents derived from measurements at the specific gaging site. The difference between nominal "theoretical" values of the requisite parameters and variables and the empirical values based on measurements can sometimes, and to some extent, be explained by known variations in such things as and fall velocity. Mixing length theory is obviously a simplified concept which does not fully describe flow behavior and mixing by large-scale secondary and tertiary flow phenomena which are something between mean flow behavior and random turbulent eddies.

Further research on the characteristics and effects of secondary components of the mean flow and of transient vortices, which might be classified as large-scale turbulence, would be interesting and challenging. However, each gaging site would be unique at least in some way, and it is very doubtful that the behavior of these flow features could ever be well predicted for specific sites. Field measurements of velocity and concentration distributions should be sufficient to permit adjustment of the coefficients and exponents of the proposed equations.

Prandtl's basic concepts are still useful, but his concepts can be revised slightly and refined in order to better describe fluid flow phenomena. The refinements offered herein result in a better, but not perfect, description of the sediment concentration distribution in the vertical. With more and better measurements, other assumptions of the mixing length and the correlation coefficient might be made to give better results. There is always a temptation to say it would be wise to wait for better measurements and understanding of all the secondary factors involved, such as the fall velocity of particles in a turbulent field and in the presence of other particles of various sizes and the effects of secondary flow in a supposedly two-dimensional flow. However, measurements made when establishing a gaging site, and occasional measurements subsequently, as is done today, should be good enough for acceptable determination of suspended sediment loads by single-point sampling.

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