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Market Pioneering Game for Symmetric Players

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Abstract

In this paper, we consider with a market pioneering game among symmetric firms in highly competitive situation. To describe the puzzling situation of timing competition, we construct a dynamic game model and explore the equilibrium solution. As a result, we find a subgame perfect mixed strategy Nash equilibrium conceptually defined by '$t_0 + \varepsilon$ ***equilibrium***'. Our major findings include: i) market entry will be occurred in sequential manner even though the condition of each firm is symmetric ii) the optimal timing of market pioneering will be advanced until almost all of the monopolist's profit is dissipated, iii) as the market position of the pioneer is stronger, the timings of the pioneer and the follower are separated, iv) and as the slope of the profit flow is steeper, the entry timing of the two players will be pooled together.

Key words : timing game, dynamic optimization, subgame perfect equilibrium

1. Introduction

The timing decision of market pioneering(new product introduction) involves two strategic aspects, the qualitative and the quantitative ones[6]. The qualitative side of the timing decision is whether to be a pioneer or not. On the other hand, the quantitative factor is to decide the right time of market pioneering. Even if the decision variable can be summarized by the one dimensional decision variable(that is, time), the outcome of such a decision must be considered in two dimensional point of view. There are trade-offs in advancing and delaying the timing of market pioneering. While most of the firms want to be a pioneer in a R&D race, they still do not like launching a new product too early(it is widely known by a winner's curse).

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The advantages of pioneering a new market include the monopoly profit, learning effect and other preemptive effects such as patent right, brand loyalty etc. On the contrary, the early launcher cannot help confronting the undesirable effects such as heavy R&D cost, the risk of imitation, demand uncertainty and market cannibalization etc. If we consider the influences of rival firm's strategy, the optimal timing problem becomes even more complicated. To describe such a puzzling situation, we construct a game theoretic model of the market pioneering competition and explore the equilibrium solution for symmetric firms.

Classical works related to this topic are carried out by Loury[7], Dasgupta and Stiglitz[2] and Lee and Wilde[5]. They deal with the R&D effort and the expected duration of R&D race in a view of auction. In the sense of timing decision, Scherer[11] proposes the joint adoption equilibrium among firms in the fear of imitations. Reinganum[9] shows the existence of precommitment equilibrium which entails the innovation diffusion process. Further, she[10] constructs a symmetric leader-follower model and shows that the expected payoff of a closed-loop strategy is always greater than that of an open-loop strategy for both of players. Fudenberg and Tirole[3] consider with the optimal timing of technology adoption and suggest that if the expected amount of first mover's advantage is large, a diffusion equilibrium will exist but if it is negligible, a symmetric outcome(late adoption) will arise. More recently, the market pioneering competition for firms with asymmetric conditions are studied([1],[4]).

2. THE MODEL

To construct the payoff structure of the model, the advantages and disadvantages of market pioneering should be examined. For the simplicity of analysis, we will summarize the advantageous factors of market pioneering to the revenue function R(t), and the disadvantageous factors to the cost function C(t). Both of them will be defined by flow variables to reflect the life cycle effect.

〈Basic assumptions〉

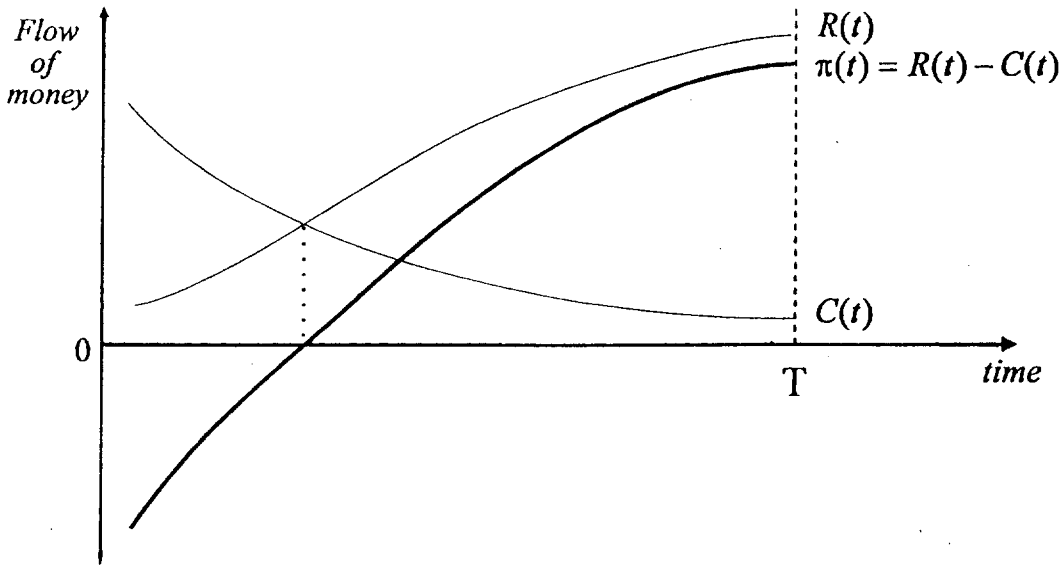
$C(t)$: the cost flow of market pioneering, $C>0$, $C'<0$, $C''>0$

$R(t)$: the revenue flow of market pioneering, $R>0$, $R'\geq 0$

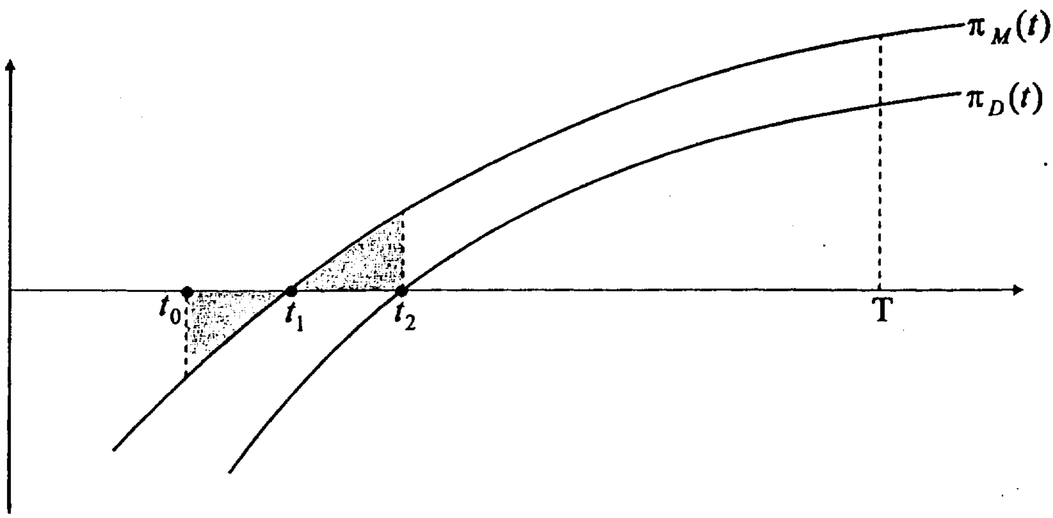
$\pi_M(t)=R(t)-C(t)$: the monopoly profit flow, $\pi_M(0)<0$, $\pi_M'>0$

$\pi_D(t)$: the duopoly profit flow, $\pi_M(t)-\pi_D(t)>0$, $\forall t$

In general, the cost flow of pioneering is apt to decline against time because of the technological pervasiveness. On the other hand, the potential revenue flow has a tendency of increasing due to the expanding market opportunities. With these functional properties, we can draw the net profit flow to be a strictly increasing function with its initial point strongly negative(See [Figure 1]).



[Figure 1] The profit flow of market pioneering



[Figure 2] The monopoly and the duopoly profit flows

In [Figure 2], π_M and π_D denote the symmetric profit flow of the monopolist and the duopolist, respectively. Since the profit flow of the duopolist will be less than that of monopolist, the π_D curve is drawn strictly below the π_M curve. Disregarding the discount rate, the gross payoff of the monopolist pioneering at time t can be calculated by the equation (1).

$$V_M(t) = \int_t^T \pi_M(\tau)\,d\tau \tag{1}$$

With this payoff structure, it is very easy to find that t1 is the optimal strategy for monopolist in [Figure 2]. For the symmetric duopolist, the positive profit flow is given only after the time t_2. However, t_2 is never an optimal strategy for duopolist because anyone can get more profit by advancing the timing of market entry. In the case of competition, since the payoff of each firm depends on the other's strategy, it is not easy to derive an optimal strategy in a simple way.

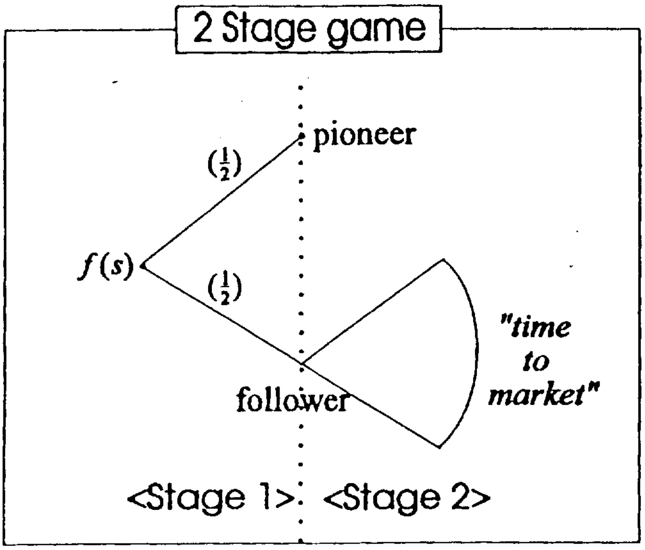
3. THE EQUILIBRIUM SOLUTION

Let Γ be the two person symmetric game constructed by the payoff structure shown in [Figure 2]. And let $s,t \in [0,T]$ be the strategy variables of firm 1 and firm 2 in the game Γ. Then it is easy to find that the pure strategy Nash equilibria of the game Γ are $(s,t)=(t_1,t_2)$ and $(s,t)=(t_2,t_1)$. Because t_1 is the best response to t_2 and vice versa. But these equilibrium pairs have some problems in the context of implementability. Since choosing t_1 is more profitable than t_2, it may be hard to believe that such a harmonic settlement can be attained in a noncooperative situation[12]. In other words, these equilibrium pairs fail to get rationality in a dynamic sense. In the theoretical point of view, we can prove that there exists no subgame perfect pure strategy equilibrium in the game Γ. This implies that we cannot imagine any obviously certain outcome of the game Γ in a rational way.

So, we must hire the mixed strategy concept to solve the equilibrium timing. Let $f(s)$, $s \in [0,T]$ be the mixed strategy of firm 1, where $\int_0^T f(s)ds = 1$. Since our model assumes the symmetry between two firms, it is sufficient to examine the case of firm 1 only. The functional form of the payoff with the mixed strategy can be written by equation (2). In the payoff function, if one wins the game, he will enjoy the monopoly profit flow until the rival enters the market. But if he loses, he can obtain only a competitive profit flow.

$$\Phi(s,t) = \begin{cases} \int_s^t \pi_M(\tau)\,d\tau + \int_t^T \pi_D(\tau)\,d\tau, & 0 \le s < t \\ \int_s^T \pi_D(\tau)\,d\tau, & t \le s \le T \end{cases} \quad (2)$$

In solving the game Γ, we can reduce the complexity by using 'two-stage approach'. [Figure 3] shows the sequential decision problem.



[Figure 3] Two-stage game

In [Figure 3], the first stage is to decide whether to be a pioneer or not in their own mixed strategies. Only if one loses in the first stage(that is, he turns out to be a follower), he will be engaged in the second stage. In the second stage, there remains only a solitary decision problem and it is easy to find that choosing t2 is follower's optimal strategy. With this result, we can reduce the original payoff function to the simpler form as equation (3).

$$\Phi'(s,t) = \begin{cases} \int_s^{t_2} \pi_M(\tau)\,d\tau & , 0 < s < t \\ 0 & , t < s < t_2 \end{cases} \quad (3)$$

The reduced form of payoff function can be obtained by the linear transformation of $\Phi' = \Phi - \int_s^T \pi_0(\tau)\,d\tau$. It has a similar structure to that of 'War of attrition' proposed by Maynard Smith Jr.[11].

〈Theorem〉

The subgame perfect mixed strategy Nash equilibrium of firm 1 in the game Γ is

$$\begin{cases} s = t_2 & , s > t \\ r(s) = \dfrac{f(s)}{1-F(s)} = \dfrac{\pi_M(s)}{\int_{t_0}^{s} \pi_M(\tau)d\tau} , & s \in (t_0, t_1] , \ otherwise \end{cases} \tag{4}$$

$$, \text{where} \quad t_0 = \left\{ x \mid \int_x^{t_2} \pi_M(\tau)\, d\tau = 0 \right\}.$$

(proof) The expected payoff of firm 1 decide to pioneer the market at time s can be written by the equation (A.1).

$$E_N = \int_s^{t_2} \pi_M(\tau)\, d\tau \tag{A.1}$$

But if firm 1 decide to delay the market pioneering for time h, the expected payoff can be calculated by the equation (A.2).

$$\begin{aligned} E_W &= 0 \cdot \int_s^{s+h} \frac{f(t)}{1-F(s)}\, dt \\ &\quad + \int_{s+h}^{t_2} \pi_M(\tau)\, d\tau \left[1 - \int_s^{s+h} \frac{f(t)}{1-F(s)}\, dt \right] \end{aligned} \tag{A.2}$$

By the principle of optimality, the subgame perfect equilibrium must satisfy the condition EN = EW. Thus the equilibrium strategy must satisfy equation (A.3).

$$\int_s^{t_2} \pi_M(\tau)\, d\tau - \int_{s+h}^{t_2} \pi_M(\tau)\, d\tau \left[1 - \int_s^{s+h} \frac{f(t)}{1-F(s)}\, dt \right] = 0 \tag{A.3}$$

Divide both sides of (A.3) by h and taking the limit of $h \to 0$, we can get equation (A.4).

$$\lim_{h \to 0} \frac{1}{h} \left[\int_s^{t_2} \pi_M(\tau)\, d\tau - \int_{s+h}^{t_2} \pi_M(\tau)\, d\tau \left(1 - \int_s^{s+h} \frac{f(t)}{1-F(s)}\, dt \right) \right] = 0 \tag{A.4}$$

Since

$$\lim_{h \to 0} \frac{1}{h} \left[\int_s^{s+h} \frac{f(t)}{1-F(s)}\, dt \right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\int_{-\infty}^{s+h} \frac{f(t)}{1-F(s)}\,dt - \int_{-\infty}^{s} \frac{f(t)}{1-F(s)}\,dt\right]$$

$$= \frac{f(s)}{1-F(s)} \tag{A.5}$$

Then we can rewrite (A.4) as (A.6).

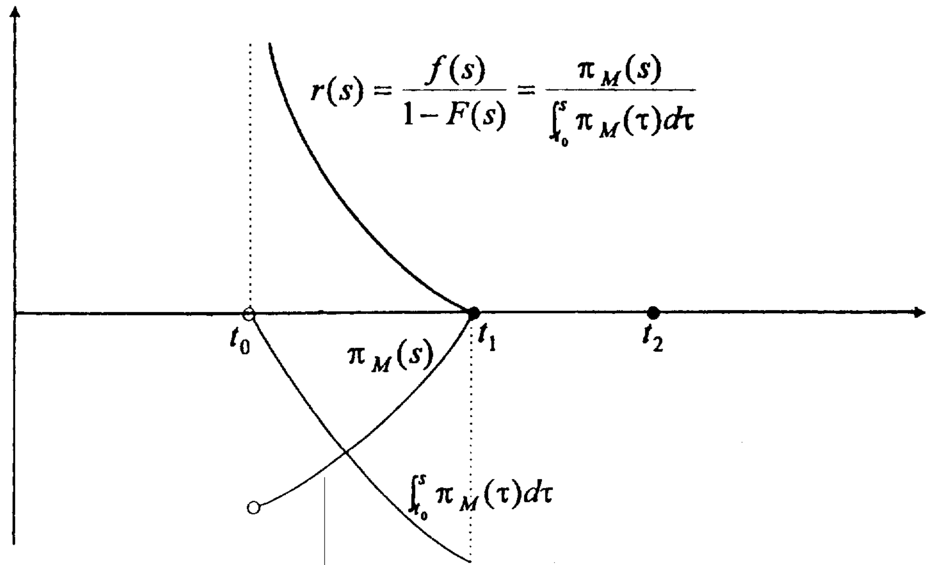
$$\lim_{h\to 0}\frac{1}{h}\left[\int_{s}^{t_2} \pi_M(\tau)\,d\tau - \int_{s+h}^{t_2} \pi_M(\tau)\,d\tau\right] + \frac{f(s)}{1-F(s)}\int_{s}^{t_2} \pi_M(\tau)\,d\tau = 0 \tag{A.6}$$

Finally, we can obtain the equilibrium condition of the form (A.7).

$$\frac{f(s)}{1-F(s)} = \frac{-\pi_M(s)}{\int_{s}^{t_2} \pi_M(\tau)\,d\tau} = \frac{\pi_M(s)}{\int_{t_0}^{s} \pi_M(\tau)\,d\tau} \tag{A.7}$$

Q.E.D. ∎

In the theorem, r(s) is so-called the 'failure rate' function which implies the probabilistic intensity that the event(market pioneering) will occur just at time s, while no event has occurred until the time s. [Figure 4] shows the shape of equilibrium strategy.

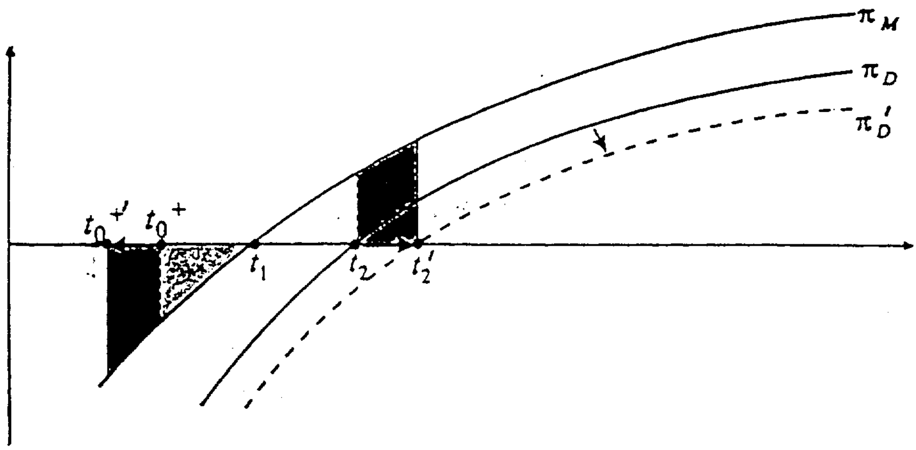


[Figure 4] The equilibrium mixed strategy of the game Γ

According to the equilibrium strategy, the market pioneering will occur at some time just after t_0. We define this outcome to be a '$t_0+\varepsilon$ equilibrium'. From the equilibrium outcome, we find that the optimal timing of market pioneering in a competitive situation will be advanced until almost all of the pioneer's expected profit is dissipated. And the expected profits are equalized for both of the symmetric players despite the asymmetric timing outcomes.

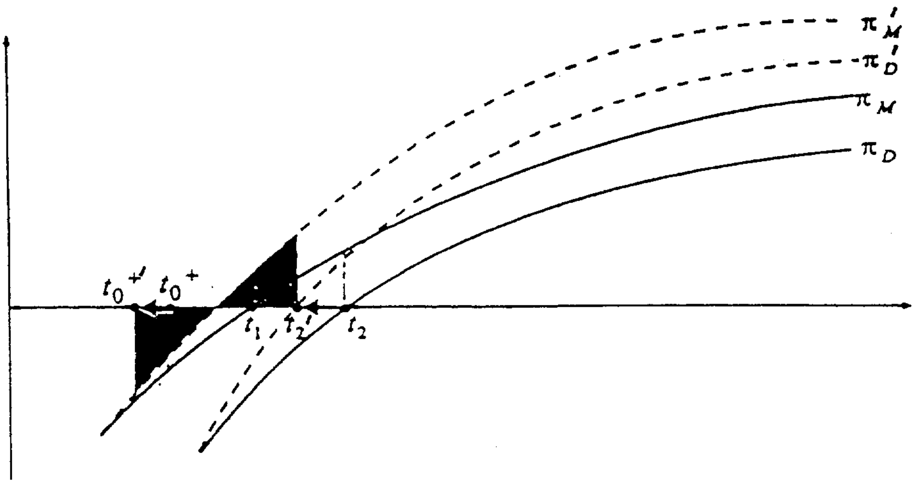
4. COMPARATIVE STATICS

Two kinds of comparative statics are carried on the equilibrium strategy. [Figure 5] shows the comparative statics result with resets to the pioneer's market position which is related to the strictness of patent protection and the value of brand loyalty. It is shown that if the advantage of the market pioneer is more attractive, the pioneering date will be advanced and the entry timings will be separated.



[Figure 5] Comparative statics with respect to the pioneer's market position

[Figure 6] shows the comparative statics result with respect to the changing rate of profit flow which is dependent upon the speed of cost reduction or market expansion. As a result, it is shown that if the changing rate of profit flow is more radical, the pioneering date will be advanced or not but the entry timings will be pooled together.



[Figure 6] Comparative statics with respect to the changing rate of profit flow

5. CONCLUSIONS

In this paper, we construct and analyze a dynamic two stage game model to describe the optimal timing decision of market pioneering among symmetric firms. As a solution of the game, we derive a subgame perfect mixed strategy Nash equilibrium and define it to be a '$t_0+\varepsilon$ equilibrium'. Our major findings include: i) there exist no pure strategy subgame perfect equilibrium in the game Γ, ii) almost all of the monopolist's profit is dissipated by the closed-loop equilibrium, iii) the stronger the market position of the pioneer, the wider the interval of market entry timing between the pioneer and the follower, iv) and the more faster the market environment changes, the closer the timing of market entry between two firms.

Related to this topic, an asymmetric extension of the timing game will be needed to be analyzed further. Also, some sort of empirical test may be needed to support the result of the analytic studies. The major theme of the empirical test will be about 'How much and in what direction the exogeneous factors such as the characteristics of technology, the number of potential competitors and the cost condition of R&D influence the economic performances such as the speed of technology progress, the efficiency of market, market concentration and so forth.

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