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Channel Allocation for the Low Earth Orbit Satellite Systems

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Abstract

We consider the channel allocation problem for the low earth orbit (LEO) satellite systems. This problem is known to be NP-complete and a couple of heuristic algorithms have been developed. In this paper, we convert the problem into a simpler form through the concept of pattern. And we suggest another algorithm based on Simulated Annealing for this simplified problem. The results of perfomance comparison show that our method works very well. Simulation results are reported.

1. Introduction

Terrestrial cellular systems were originally intended to improve the capacity of mobile telephone networks by the reuse of spectrum. Such an approach is economically viable in high population density areas, but to be so in low density rural areas, a different approach is required. Various concepts may achieve the desired result, however herein, we restrict our attention to systems employing Low Earth Orbit (LEO) constellations. Iridium system is a LEO satellite communication systems utilizing concepts from terrestrial cellular networks. Systems such as Iridium utilize spot beams to form cells on the earth's surface similar to terrestrial cellular radio systems. A multiple beam can be used to form the number of cells required on each satellite. The cellular system is formed on a grid of cells established by satellite systems.

Due to the increasing demand for mobile communication services and a finite spectrum allocated to

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such services, an efficient method of frequency reusing in different cells is essential [14]. For a fixed spectrum assigned and a specific multiplexing technology used, the traffic-carrying capacity of a cellular system depends on how the frequency channels are managed [12].

There are two aspects of channel management. A set of channels is allocated to each cell on a nominal basis and each cell assigns a specific channel to carry a specific call. The specific channel being assigned normally is a nominal channel of that cell. But if none is available, borrowing from a neighboring cell may be allowed. A variety of channel assignment strategies are available.

Besides the channel borrowing strategies, a good allocation of nominal channels to the cells can also significantly increase the traffic-carrying capacity of a system [19]. This is because with good nominal channel allocation, the need for channel borrowing will be reduced. Given the traffic distribution of a cellular mobile system, searching the optimal nominal channel allocation pattern is known as an NP-complete graph coloring problem. As it is infeasible to search for the optimal allocation pattern, heuristic algorithms that can give near-optimal solutions are needed.

Most previous investigations concerning the channel allocation problem were based on graph theoretic or heuristic approaches [1, 2, 4, 8]. The graph theoretic approach has several disadvantages in applicability and flexibility [10]. Investigations based on neural networks [10] yield excellent results in special cases, but under certain conditions, principally only suboptimal solutions can be found [11].

In this study, we propose a quite satisfactory approach called SA based on *Simulated Annealing* for the allocation of nominal channels. This method has been applied by Duque-Antn et al. [3], but they use a completely different model. In Section 2, the optimal channel allocation problem is defined and formulated mathematically. Also a reduced problem using the concept of pattern is suggested. The solution procedure using simulated annealing is described in Section 3. In Section 4, heuristic procedures for generating candidate patterns considered in the reduced problem are suggested. The results of performance comparison are reported in Section 5. Section 6 concludes the study and describes how this study can be used for the LEO satellite systems.

2. The Optimal Channel Allocation Problem

Suppose that we have a satellite based cellular system consisting of N cells and M channels. Let λ_i be the traffic demand in erlangs of cell i and let the number of channels available in the cell be m_i, then the call blocking probability in the cell is given by the Erlang B formula as

$$B(\lambda_i, m_i) = [\sum_{k=0}^{m_i} \frac{\lambda_i^k}{k!}]^{-1} \frac{\lambda_i^{m_i}}{m_i!}.$$

The weighted average blocking probability in the cellular system is given by

$$R = \sum_{i=1}^{N} w_i B(\lambda_i, m_i)$$

where $w_i = \lambda_i / \sum_{i=1}^{N} \lambda_i$ is the traffic weighting factor [6, 19]. The channel allocation problem *(CAP)* which minimizes the weighted average blocking probability subject to cochannel interference constraints is as follows [6].

(CAP)

$$\min \sum_{i=1}^{N} w_i B(\lambda_i, m_i)$$

$$s.\ t.\ m_i = \sum_{j=1}^{M} f_{ij}, \quad \text{for } j = 1, \ldots, N, \quad \ldots\ldots(1)$$

$$f_{sj} + f_{tj} \leq , \quad \text{for } j = 1, \ldots, M$$

and all interfering cell pairs *(s, t)*, ……(2)

$$f_{ij} = 0 \text{ or } 1, \quad \text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, M. \quad \ldots\ldots(3)$$

The decision variable f_{ij} is a binary integer variable indicating channel allocation where $f_{ij} = 1$ represents that channel j is allocated to cell i and $f_{ij} = 0$ otherwise. The decision variable m_i, the number of channels allocated to cell i, is determined by channel allocation f_{ij}'s as shown in equation (1). The equation (2) means that the same channel j should not be allocated to different cells s and t simultaneously if the cells s and t are within interference zone of each other.

The problem *(CAP)* is known to be NP-complete. The number of binary integer variables in the problem *(CAP)* is MN. In the case of $M=100$ and $N=50$, to get the optimal solution of the problem *(CAP)*, it is necessary to enumerate explicitly or implicitly all possible combinations of size $2^{MN} \approx 10^{1500}$. Even if all the infeasible choices are eliminated, the resulting number is still astronomical. Also, the cochannel interference constraints (2) are very complex. Thus it may be impossible to solve the problem *(CAP)* directly.

To deal with the proposed problem more conveniently, we introduce the concept of pattern. A channel cannot be allocated to adjacent cells simultaneously because of the cochannel interference. If an identical channel is allocated to a set of cells without causing cochannel interference between pairs

of cells, these cells are called cochannel cells. This set of cochannel cells forms a pattern.

Now, suppose that we generate P patterns such that every cell belongs to at least one of these patterns. Then the channel allocation problem reduces to the problem of allocating M channels to P patterns. Let the decision variable x_p denote the number of channels allocated to pattern p. Then using the P patterns, the problem (CAP) reduces to the following problem :

(CAP$_1$)

$$\min \sum_{i=1}^{N} w_i B(\lambda_i, m_i)$$

$$s.t.\ m_i = \sum_{p \in S_i} x_p, \quad \text{for } i = 1, \ldots, N, \quad \cdots\cdots(4)$$

$$\sum_{p=1}^{P} x_p \leq M, \cdots\cdots(5)$$

$$x_p : \text{nonnegative integer, for } p = 1, \ldots, P, \quad \cdots\cdots(6)$$

where S_i is the set of patterns which cover cell i. Due to the property of a pattern, a solution of the problem *(CAP$_1$)* always satisfies the cochannel interference constraints. If all feasible patterns are considered, the problem *(CAP$_1$)* is equivalent to the original problem *(CAP)*, and the combinations that these patterns generate are similar to the combinations generated by the original formulation.

However, this is not true if only special candidate patterns, for example, the patterns generated by the pattern generation procedures in Section 4, are considered. In the case of M=100, N=50, and P=50, to get the optimal solution of the problem *(CAP$_1$)*, it is necessary to enumerate explicitly or implicitly all nonisomorphic combinations of size $\binom{M+P-1}{P-1} \approx 6.7 \times 10^{39}$, which is significantly less than $2^{MN} \approx 10^{1500}$. This property is not dependent on the specifics of the cell topology, if only special candidate patterns are considered.

3. Simulated Annealing

This part will be devoted to finding the optimal solution via an efficient and robust optimization algorithm — *simulated annealing*. Details of this method for the minimization problem is described in Appendix. In order to apply simulated annealing to *(CAP$_1$)*, we have to define the corresponding

discrete solution space X, the cost function C and the neighborhood structure N_e.

A. Solution space

Suppose that we have P patterns generated by the pattern generation procedures in Section 4. The current state of the channel allocation is represented by a P-dimensional vector x.

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_p \\ \vdots \\ x_p \end{bmatrix}$$

A component x_p of vector x denotes the number of channels assigned to pattern p where $1 \leq p \leq P$, $0 \leq x_p \leq M$, and $\sum_{p=1}^{P} x_p = M$. In the problem (CAP_1), we can see that the objective function value comes to decrease as the total number of channels assigned to each pattern approaches to M. Therefore we set the total number of channels assigned to each pattern as M. In order to store the channels assigned to each pattern, we also define $CS(p), p = 1, \ldots, P$, as a set of channels assigned to pattern p.

B. Cost function

The channel allocation algorithm introduced here is aimed at minimizing the weighted average blocking probability. Therefore the cost function is as follows :

$$C(x) = \sum_{i=1}^{N} w_i B(\lambda_i, m_i).$$

This is the weighted average blocking probability defined in Section 2.

C. Neighborhood structure

Here, given an $x = (x_1, x_2, \ldots, x_p)^T$, the neighborhood $N_e(\mathrm{x})$ is defined as follows :

$$N_e(\mathrm{x}) = \{\bar{x} \| \bar{x} = (x_1, x_2, \cdots, x_p + 1, \cdots, x_g - 1, \cdots, x_P)^T\},$$

where $p \neq q$, $1 \leq p, q \leq P$, and $x_q - 1 \geq 0$. And we propose two strategies for picking a solution $\bar{x}$ in $N_e(x)$.

- (Random picking) :

Select $\bar{x}$ randomly in $N_e(x)$. That is, choose any two numbers p and q such that $p \neq q$, $1 \leq p, q \leq P$, and $x_q \geq 1$. Set $\bar{x} = x$. Update $\overline{x_p}$ and $\overline{x_q}$ as follows :

$$\overline{x_p} = \overline{x_p} + 1, \quad \overline{x_q} = \overline{x_q} - 1.$$

And get a channel out of $CS(q)$, insert the channel into $CS(p)$.

● (Proportional picking) :

Let NC_p be the number of cells covered with pattern p and let $TNC = \sum_{p=1}^{P} NC_p$. Then $\frac{NC_p}{TNC}$ is the proportion of the number of cells covered with pattern p to the total number of cells covered with all patterns. Since we can infer that the pattern p, which has the largest value of $\frac{NC_p}{TNC}$, may have more influence than other patterns, we determine the p and q as follows :

$$p = \text{Argmax}\left\{\frac{NC_p}{TNC} \; p = 1, 2, \ldots, P\right\},$$

$$q = \text{Argmin}\left\{\frac{NC_p}{TNC}, \; x_p \geq 1, p = 1, 2, \ldots, P\right\}.$$

Set $\bar{x} = x$, Update $\overline{x_p}$, $\overline{x_q}$, (SCP), and $CS(q)$ as stated above.

4. Pattern Generation

The problem (CAP_I) in Section 2 is an approximation to the original problem (CAP). The choice of patterns considered in the problem (CAP_I) may have an influence on the quality of solutions relative to the problem (CAP). Thus it is important to find good candidate patterns. In finding good patterns, we consider the mutual distances between adjacent cochannel cells and the traffic demand distribution.

Define $D(g, k)$ as the Euclidean distance between cell g and cell k. This is computed by

$$D(g, k) = \sqrt{(k_1 - g_1)^2 + (k_1 - g_1)(k_2 - g_2) + (k_2 - g_2)^2},$$

where (g_1, g_2) and (k_1, k_2) are center coordinates of cell g and k, respectively, in the plane

with inclined axes (see Figure 1) [4, 14]. The minimum reuse distance is denoted by δ.

Pattern Generation Procedure A

Step 0 : $I := \{1,...,N\}$; $i := 1$.

Step 1 : $I_P := \emptyset$; $I_P := I_P \cup \{i\}$.

Step 2 : $I_R = \{g \| \min \{D(g, k); k \in I_P\} \geq \delta; g \in I - I_P\}$;

Set $g^* =$ Argmin $\{\sum_{k \in I_P} D(g, k); g \in I_R\}$.

Step 3 : $I_P := I_P \cup \{g^*\}$; $I_R := I_R - \{g^*\}$; *If* $I_R = \emptyset$ then go to Step 4; Otherwise return to Step 2.

Step 4 : Check whether the pattern I_P is identical to one of patterns generated already or not; If so, discard the pattern I_P; If $i = N$ then Stop; Otherwise $i = i + 1$, return to Step 1. ◆

This procedure preferentially selects noninterfering close cells. The following procedure B is a simple modification of the procedure A. In this procedure, a cell which is nearest to the first chosen cell is selected repeatedly. The procedure B is identical to the procedure A except Step 2.

Pattern Generation Procedure B

Step 2 : $I_R = \{g \| \min \{D(g, k); k \in I_P\} \geq \delta ; g \in I - I_P\}$;

Set $g^* =$ Argmin $\{D(g,i); g \in I_R\}$. ◆

The procedure A and B do not consider the traffic demand distribution. It may be desirable to generate a pattern considering traffic demands. We suggest a heuristic procedure C to find these patterns. Let d_g be the traffic demand of cell g.

Pattern Generation Procedure C

Step 0 : $I := \{1,...,N\}$; $i := 1$.

Step 1 : $I_P := \emptyset$; $I_P := I_P \cup \{i\}$.

Step 2 : $I_R = \{g \| \min \{D(g,k); k \in I_P\} \geq \delta, g \in I - I_P\}$;

$I_D = \{g \| \min \sum_{k \in I_P} |d_g - d_k|; g \in I_R\}$;

set $g^* = \text{Argmin} \{ \sum_{k \in I_P} D(g,k); g \in I_D \}$.

Step 3 : $I_P = I_P \bigcup \{g^*\}$; $I_R := I_R - \{g^*\}$; If $I_R = \emptyset$ then go to Step 4; Otherwise return to Step 2.

Step 4 : Check whether the pattern I_P is identical to one of patterns generated already or not: If so, discard the pattern I_P; If $i = N$ then Stop; Otherwise $i = i + 1$, return to Step 1. ◆

This procedure selects, among cells satisfying the cochannel interference constraints, a cell whose demand-difference from already selected cells is as small as possible. If a tie happens, select the closest cell from already selected cells. The procedure A, B, and C can be used separately or together. In the test of Section 5, we will use all these procedures together.

5. Performance Comparison

As an example we have considered a cellular system like Figure 1 which consists of $N = 49$ cells and $M = 70$ channels. The number in each cell represents the call arrival rates and ranges from 20 calls/hour to 200 calls/hour from cell to cell.

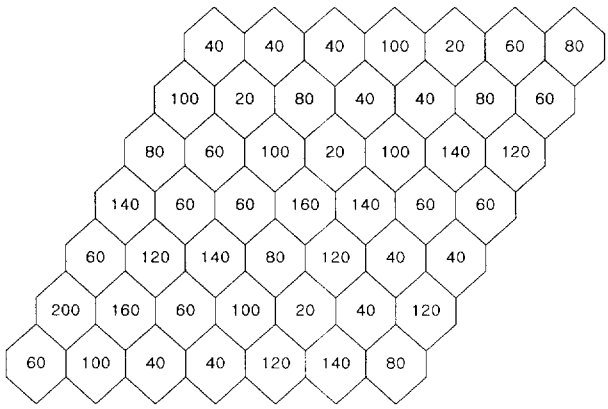

Figure 1 A Cellular system and a traffic distribution

Assume the arrival of calls is a Poisson process and the call duration is exponentially distributed with a mean of 3 min. We also assume that the minimum reuse distance δ is $\sqrt{21}r_c$, where r_c is the cell radius. Cells having a mutual distance not smaller than δ can use the same channels.

In order to apply simulated annealing, initial t has been set to 10 and r has been set to 0.65. The simple approach of permitting 100 state transitions at each t level has been used. When t is less than 10^{-12}, we conclude it is frozen. We have used the proportional picking strategy for picking a next solution. The number of patterns, P, generated using procedures in Section 4 is 38. Even though the computation time is not critical because this (CAP) is a design problem, too much time-consuming methods are undesirable. The computation time of our method in the case of the examples in this section was within a few minutes.

Let us start with a simple uniform traffic distribution of 100 calls/hour in all 49 cells. Table 1 shows the results of this case. Using the allocation strategies proposed in this study and in [18], the number of channels allocated in each cell is found. The total number of simultaneously usable channels z for each allocation strategy is obtained by adding up the number of allocated channels in the 49 cells and the weighted average blocking probability R is found by using the Erlang B formula.

Table 1 Channel allocation with uniform traffic

Strategy	R	z
Uniform*	1.84%	490
Compact*	1.84%	490
Hybrid/CB*	1.97%	490
SA	1.81%	513

* : the allocation strategies proposed in [18].

For the nonuniform traffic distribution shown in Figure 1, the weighted average blocking probabilities and the total number of available channels for each allocation strategy are listed in Table 2.

Tables 1 and 2 show that SA is superior to other strategies in respect to the weighted average blocking probability and the total number of available channels. Especially, with regard to the total number of available channels, SA shows very good performance.

Table 2 Channel allocation with nonuniform traffic distribution

Strategy	R	z
Uniform*	4.45%	490
Compact*	1.94%	490
Hybrid/CB*	1.28%	489
SA	1.20%	516

* : the allocation strategies proposed in [18].

We now vary the traffic load by multiplying the nominal arrival rates in each cell by a factor k. We vary k from 1.0 to 2.0 for the nonuniform distribution and plot R for various allocation strategies in Figure 2. The relative quality of our solutions is better as the traffic load increases.

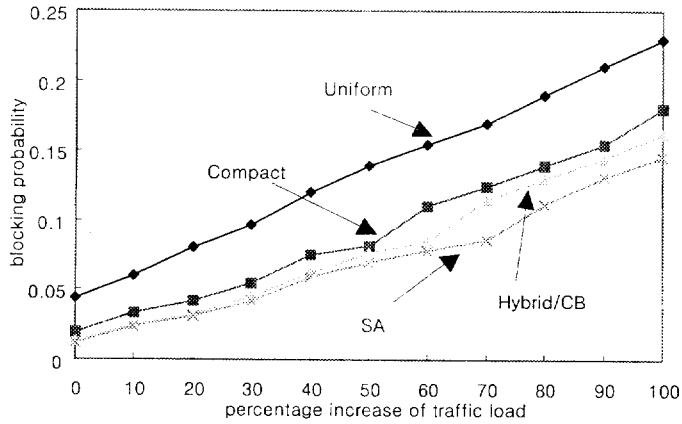

Figure 2 Comparison of channel allocation strategies

6. Conclusions

An optimal channel allocation problem which minimizes the weighted average blocking probability subject to cochannel interference constraints in a satellite based cellular system has been suggested. The problem is converted into a simpler form through the concept of pattern. We have applied a simulated annealing procedure to the simplified problem. It is found that our method provides the best performance among the strategies considered in this study.

The problem we have considered is for one satellite based cellular system. However, most of LEO satellite systems consists of a number of LEO satellites. Iridium system consists of 66 satellites, orbiting the earth in 413 nautical mile circular polar orbits. The satellites are equally distributed in 6 orbital planes, with 11 satellites per plane. The planes of satellites co-rotate towards the North Pole on one side of the earth, "crossover" at the polar region and travel towards the South Pole on the other side of the earth. Figure 3 shows the Iridium system.

Channel management for each satellite is achieved by using a look-up table stored on board the satellite. Generation of this table is contingent upon knowledge of the relative latitude and longitude of cell centers. In Figure 3, the distance between Satellite No. 1 and Satellite No. 2 is 32.727 degrees in latitude. This represents one eleventh of the full orbit of 360 degrees. Satellite No. 2, after the movement of 32.727 degrees, will be in the exact same position as Satellite No. 1 is.

A channel management table is created for each tenth of a degree for 32.7 degrees in latitude by using the method presented in this study. These tables are stored in memory on board each satellite for channel management utilization. After the movement of 32.727 degrees, Satellite No. 2 replaces the channel management table with the table received by Satellite No. 1 through the intersatellite links (ISLs).

An Iridium satellite has 48 cells. Though we have tested the system with 49 cells in Section 5, the number of cells makes no difference. Because of the polar orbits and the positioning of the Iridium satellites, the cells merge at the polar regions. As the satellites approach a pole, the distance between satellites in adjacent planes decreases, causing cell overlap. When cells begin to overlap, selected spot beam antennas are deactivated to prevent channel interference. Then, the number of cells which are activating is less than 48. However, channel management is necessary in these regions, too.

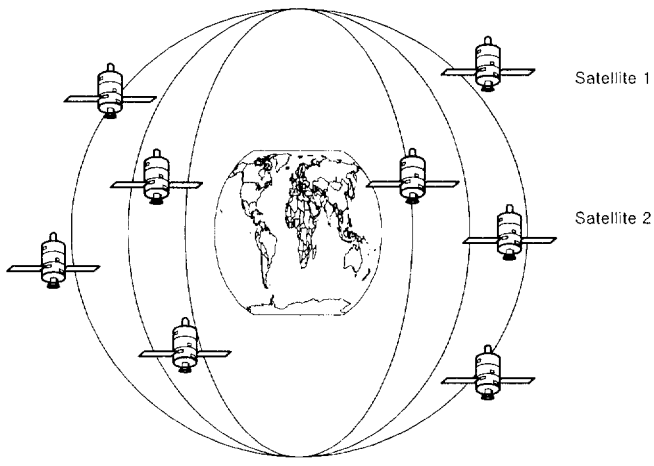

Figure 3 LEO satellite systems

APPENDIX

Simulated Annealing

Simulated annealing [7] is a general method for the approximate solution of difficult (i.e., NP-complete) combinatorial optimization problems. It has been applied in such diverse areas as computer aided design of integrated circuits, imageprocessing, code design, etc.

Generally, a combinatorial optimization problem consists of a set X of solutions and a cost function C which determines for each $x \in X$ the cost $C(X)$, i.e., a real number. Simulated annealing can be considered as a generalization of the iterative improvement scheme (local search). For performing a local search one needs to know the neighbors $\bar{x}$ of x Thus one has to define a neighborhood structure N_e on X, $N_e(x)$ determines, for each solution x, a set of possible transitions which can be proposed by x.

For local search, starting from an arbitrary solution x, in each step of iterative improvement a neighbor $\bar{x}$ of x is proposed at random. Then, x is replaced by $\bar{x}$ only if cost does not rise, i.e., $C(\bar{x}) \leq C(x)$. Obviously, this procedure terminates in a local minimum, i.e., in a solution whose neighbors do not offer any improvement in cost. Unfortunately, such a local minimum may have a substantially higher cost than the global one.

To avoid this trapping in poor local optima, simulated annealing occasionally allows solutions of higher cost according to the Metropolis criterion [16]. More precisely, if x and $\bar{x} \in N_e(x)$ are the two solutions, then the algorithm continues with solution $\bar{x}$ with a probability given by min $\{1, \exp(-(C(\bar{x}) - C(x))/T)\}$ (acceptance probability), where T is a positive control parameter called "temperature" and is gradually decreased to zero during the execution of the algorithm. Note that the acceptance probability decreases for increasing values of $C(\bar{x}) - C(x)$ and for decreasing values of T, and that cost-decreasing transitions are always accepted. Parameter T is initially set to a relatively large value so that the transition from x to $\bar{x}$ occurs more frequently, and then it is gradually decreased as the search proceeds. When T becomes sufficiently small and the solution does not change for many iterations, it is concluded to be "frozen", and the best solution available by then is outputted as the computed approximate solution.

The name "simulated annealing", as well as other terms such as "temperature" and "frozen", come from the resemblance of this procedure with the behavior of physical systems in the presence of a

heat bath, which has been studied in statistical mechanics. The probability exp $(-(C(\bar{x}) - C(x)) / T)$ was also chosen in analogy with the behavior of such systems. The entire scheme is now summarized as follows.

Step 1 : Determine an initial temperature T.

Step 2 : Given an x, pick randomly a solution $\bar{x}$ in $N_e(x)$ and let be the change in cost value of $\bar{x}$ from that of x

Step 3 : If $\triangle \leq 0$, then $x := \bar{x}$ Otherwise, let $x := \bar{x}$ with probability $e^{\frac{-\triangle}{T}}$.

Step 4 : If it is concluded that a sufficient number of trials have been made with the current T (*i.e., in equilibrium*), then go to Step 5. Otherwise return to Step 2 with the current x.

Step 5 : If the current T is concluded to be sufficiently small (i.e., frozen), then go to Step 6. Otherwise reduce the T (e.g., $T := rT$ with a constant r satisfying $0 < r < 1$) and return to Step 2.

Step 6 : Halt after outputting the best solution obtained so far as the computed approximate solution x. ◆

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