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An Analysis of Two-stage Manufacturing Systems with Random Processing Times

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Abstract

This paper analyzes a system which consists of two workstations that are separated by finite buffer storage. In this system, we assume that the processing time in each station is a random variable and each station is not vulnerable to failure. To control the in-process inventory in the serial production system we use the (R, r) policy which is similar to the (s, S) policy in the inventory theory. Under the (R, r) policy the preceding station is forced down when the inventory level in the buffer reaches R and starts operation again when the inventory level falls to r. For the model developed, we analyze the system characteristics and the system performances.

1. Introduction

In serial production lines, workpieces pass through successive stations with specific operations being performed at each station. A major cause of line inefficiency is unbalance of processing time at the stations in the line. Suppose that there are no storage buffers. Then, when the processing time at the preceding station is long, the succeeding station may be forced down, or starved, since the preceding station is unable to feed the downstream station. Similarly, the preceding station may be forced down, or blocked, when the processing time at the succeeding station is long and then the station cannot remove the semiprocessed items from the upstream station. An interstation storage buffers can be used to reduce the effect of strong interference between stations in the line. A buffer provides temporary storage space for the preceding station when the processing time at the succeeding station is long and it can be temporary supplier for the succeeding station when the

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processing time at the preceding station is long. Too much inventory, however, causes excessive inventory holding cost which consists of floor space costs, material handling costs, etc.

To operate the above system, we use the *(R, r)* policy which is familiar in the *(s, S)* inventory model. As a workpiece enters the preceding station, the station starts operating. The piece is then moved to the buffer and later to the succeeding station. When processing at the station is completed, the workpiece leaves the system. In this system, the maximum permitted level R (upper control parameter) of work-in-process between the two stations is predetermined. If the number of workpieces in the buffer is less than R units, the preceding station continues to operate and produce its output until R units are placed in the buffer. When the buffer is full, the preceding station stops the operation. From the moment, the inventory level will be decreasing. When the number of items in the buffer falls to r (lower control parameter) the preceding station restarts operating.

Such two-stage production systems have been extensively studied. Buzacott and Hanifin[2] have presented a survey paper on related topics. Elsayed and Hwang[3], Seidmann[10] and Wijngaard[11] studied the system with breakdowns at stations. The system of a random processing time without failure was analyzed by Hillier and Boling[4], Hokstad[5], Ohsone[9] and Truslove[11,12]. But all these studies deal with relatively simple control policy that has upper control parameter only.

The system with dual (upper and lower) control parameters was studied by Altiok and Shieu[1], Hopp *et al.*[6], Hwang and Koh[7], and Koh and Hwang[8]. Altiok and Shieu[1] studied a single stage and warehouse problem in which the stage is controlled by dual control parameter and the producing time and the demand process are a general random variable and Poisson process, respectively. Hopp *et al.*[6] deal with a system in which each station is vulnerable to failure and materials flow like continuous fluid. Hwang and Koh[7] studied discrete material flow, random failure and random processing time which are exponential random variables. The system with constant processing time and random failure analyzed by Koh and Hwang[8].

In this paper, we deal with a system consisting of two perfectly reliable workstations and one interstation storage buffer which is controlled by *(R, r)* policy. We assume that the processing time at the preceding station is a general random variable and Erlang random variable for the succeeding station.

Considering the processing time of the preceding station as the interarrival time, the buffer storage as the waiting room, and succeeding station as the server, the problem with upper control parameter only is the same problem as a single server queueing systems ($GI/E_k/1$ queue) with a finite waiting room and switch-off arrival process when the capacity of the queue is full.

Using the supplementary variable and the phase technique, we investigate the system with *(R, r)* control policy, or equivalently, $GI/E_k/1$ queue with finite waiting room which is controlled by dual control parameters.

2. Model Development

2.1 Assumptions and Notations

The system consists of two stations that are separated by a finite storage buffer. Although the stations are not vulnerable to failure, they cannot produce workpieces in two cases as follows:

1. No pieces are available to the succeeding station. In this case, we will say the station is *starved*.
2. If the preceding station completes its production when the inventory level in the buffer is R, the station cannot start new job until sum of the parts in the preceding station (0 or 1) and the buffer (0 through R) falls to restarting point r $(0 \le r \le R)$. In this case, we will say the preceding station is *blocked*.

We assume that, on the other hands, the raw material is always available to the preceding station of the production line, that is, the preceding station can never be starved, and the completed workpieces at the succeeding station can always be deposited into a storage of infinite capacity, that is, the succeeding station cannot be blocked.

It is assumed that processing times at the preceding station is independent and identically distributed nonnegative random variables having a distribution function $F(x)$ $(x \ge 0)$ with probability density function $f(x)$ $(x \ge 0)$ and mean $E(X)$. For notational convenience, we define

$$\overline{F}(x) = 1 - F(x), x \ge 0,$$

and

$$h(x) = f(x) / \overline{F}(x), x \ge 0.$$

And let T be the elapsed time since the last input to the preceding station at an arbitrary moment.

The total time a workpiece spends in the succeeding station is a random variable distributed according to the k-phase Erlang distribution with the p.d.f.,

$$g(x) = \frac{(k\mu)^k}{(k-1)!} x^{k-1} e^{-k\mu x}.$$

Since the staying time of a workpiece in the succeeding station is the k-phase Erlang random variable, the time for each workpiece can be decomposed into k independent phases which are distributed exponentially.

Let Q be the steady-state number of workpieces in the buffer plus the number of workpieces in the succeeding station (0 or 1), M the number of workpieces in the preceding station(0 or 1) and the buffer, N the number of phases that the succeeding station has to process in the buffer and in itself, and J the number of phases remaining in the succeeding station for a workpiece in process. Then we have the relation

$$N = \begin{cases} (Q-1)k + J, & \text{if } 1 \le Q \le R+1, \\ 0, & \text{if } Q = 0. \end{cases}$$

Now we define the system states which perfectly define the system and which have two forms according to the condition of the preceding station.

1) When the preceding station is not blocked.
   (N, T), where $N = 0, 1, \ldots, (R+1)k$ and $T \ge 0$.
2) When the preceding station is blocked.
   $[M, J]$, where $M = r+1, r+2, \ldots, R+1$ and $J = 1, 2, \ldots, k$.

2.2 Steady State Probabilities

Define, when the preceding station is not blocked, the steady-state joint distribution of the number of phases present in the buffer and the succeeding station and the elapsed time since the last input to the preceding station as

$$P_n(x)dx = \Pr\{N = n,\ x < T \quad x + dx\}, \quad \text{for } x \ge 0 \text{ and } n = 0, 1, \ldots, Rk+k,$$

and the marginal distribution of the number of phases present as

$$\begin{aligned} p_n &= \Pr\{N = n\} \\ &= \int_0^\infty p_n(x)dx, \text{ for } n = 0, 1, \ldots, Rk+k. \end{aligned}$$

And let, when the preceding station is blocked, the joint distribution of the number of workpieces in the buffer and the preceding station and the number of phases to be processed on the succeeding station for the workpiece in process be

$$q_m(j) = \Pr\{M = m,\ J = j\} \quad \text{for } m = r+1, r+2, \ldots, R+1 \text{ and } j = 1, 2, \ldots, k.$$

and the marginal distribution of the number of phases to be processed on the succeeding station for the workpiece in process be

$$q_m = \Pr\{M = m\}$$
$$= \sum_{j=1}^{k} q_m(j), \text{ for } m = r+1, \dots, R+1.$$

Then we have for $x > 0$ that

$$\frac{d}{dx} p_0(x) + h(x)p_0(x) = k\mu p_1(x), \tag{1}$$

$$\frac{d}{dx} p_n(x) + \{h(x) + k\mu\}p_n(x) = k\mu p_{n+1}(x), \text{ for } n = 1,2, \dots, Rk+k-1, \tag{2}$$

$$\frac{d}{dx} p_{Rk+k}(x) + \{h(x) + k\mu\}p_{Rk+k}(x) = 0, \tag{3}$$

$$k\mu q_{R+1}(j) = k\mu q_{R+1}(j+1) + \int_0^\infty h(x)p_{Rk+j}(x)dx, \text{ for } j = 1, 2, \dots, k, \ q(k+1) = 0, \tag{4}$$

and

$$q_m(j) = q_m(j+1) \text{ and } q_m(k+1) = q_{m+1}(1), \text{ for } m = r+1, r+2, \dots, R, \ j = 1,2, \dots, k. \tag{5}$$

Also we have the boundary conditions

$$p_n(0) = 0, \text{ for } n = 0, 1, \dots, k-1, \tag{6}$$

$$p_n(0) = \int_0^\infty h(x)p_{n-k}(x)dx, \text{ for } n = k, k+1, \dots, Rk+k \text{ but } n \neq rk+k, \tag{7}$$

and

$$p_{rk+k}(0) = \int_0^\infty h(x)\, p_{rk}(x)dx + k\mu\, q_{r+1}(1). \tag{8}$$

The normalizing condition

$$\sum_{n=0}^{Rk+k} \int_0^\infty p_n(x)dx + \sum_{m=r+1}^{R+1} \sum_{j=1}^{k} q_m(j) = 1 \tag{9}$$

should be imposed on this probability distribution.

It can be shown inductively that the general solutions of the system of linear differential equations (1) and (2) are given by

$$p_n(x) = \begin{cases} \overline{F}(x)\left[c_0 - \sum_{i=0}^{Rk+k-1} c_{i+1} \sum_{l=0}^{i} (k\mu x)^l e^{-k\mu x} / l!\right], & \text{for } n = 0, \\ e^{-k\mu x}\overline{F}(x)\sum_{i=0}^{Rk+k-n} (k\mu x)^i c_{n+i} \; / \; i!, & \text{for } n = 1,\ldots, Rk+k \end{cases} \quad (10)$$

where c_i (i = 0,...,$Rk+k$) are constants to be determined.

Letting $x = 0$ in (10), we can easily get that

$$p_n(0) = \begin{cases} c_0 - \sum_{i=1}^{Rk+k} c_i, & \text{for } n = 0, \\ c_n, & \text{for } n = 1, \ldots, Rk+k. \end{cases} \quad (11)$$

From (6) and (11),

$$c_0 - \sum_{i=1}^{Rk+k} c_i = 0$$

and

$$c_n = 0, \quad \text{for } n = 1, \ldots, k-1. \quad (12)$$

Now let us define that for j = 0,1,...,$Rk+k-1$,

$$F^*_j(s) = \int_0^\infty x^j f(x) e^{-sx} dx$$

$$= (-1)^j \frac{d^j}{ds^j} \int_0^\infty e^{-sx} f(x) dx$$

$$= (-1)^j \frac{d^j}{ds^j} E(e^{-sx}),$$

$$\phi_j = (k\mu)^j F^*_j(k\mu)/j!,$$

and

$$\theta_j = \sum_{i=0}^{j} \phi_i, \quad (13)$$

then

$$\int_0^\infty h(x)\ p_n(x)dx = \int_0^\infty e^{-k\mu x} f(x) \sum_{i=0}^{Rk+k-n} \frac{(k\mu x)^i}{i!}\ c_{n+i}dx$$
$$= \sum_{i=0}^{Rk+k-n} \frac{(k\mu)^i}{i!}\ c_{n+i} \int_0^\infty x^i\ e^{-k\mu x} f(x)dx \tag{14}$$
$$= \sum_{i=0}^{Rk+k-n} \phi_i\ c_{n+i}.$$

Therefore from (4) we find

$$q_{R+1}(j) = q_{R+1}(j+1) + \frac{1}{k\mu} \sum_{i=0}^{k-j} C_{Rk+j+i}\ \phi_i\ , \quad \text{for } j=1,2,\dots,k$$

and

$$q_{R+1}(k+1) = 0. \tag{15}$$

And from (7),

$$p_n(0) = \begin{cases} c_0 - \sum_{i=0}^{Rk+k-1} c_{i+1}\theta_i, & \text{for } n = k, \\ \sum_{i=0}^{Rk+2k-n} c_{n-k+i}\phi_i, & \text{for } n = k+1,\dots,Rk+k \text{ but } n \neq rk+k. \end{cases} \tag{16}$$

From (5) we can put that

$$q = q_m(j), \quad \text{for } m=r+1,\ r+2,\ \dots,\ R \text{ and } j=1,\ 2,\ \dots,\ k,$$
$$q = q_{R+1}(1). \tag{17}$$

Thus, from (8)

$$p_{rk+k}(0) = \sum_{i=0}^{(R-r+1)k} c_{rk+i}\ \phi_i + k\mu q. \tag{18}$$

But we know that

$$\sum_{n=0}^{Rk+k} \int_0^\infty p_n(x)dx = \int_0^\infty \sum_{n=0}^{Rk+k} p_n(x)dx$$
$$= \int_0^\infty c_0 \overline{F}(x)dx \tag{19}$$
$$= c_0 E(X)$$

And from (15) one can show inductively that

$$q_{R+1}(j)=\frac{1}{k\mu}\sum_{i=0}^{k-j} c_{Rk+j+i}\,\theta_i, \text{ for } j=1,\ldots,k, \tag{20}$$

and

$$q = q_{R+1}(1)$$

$$=\frac{1}{k\mu}\sum_{i=1}^{k} c_{Rk+i}\theta_{i-1}. \tag{21}$$

Now we are able to derive a system of equations to determine c_i, i = 0, 1, ..., $Rk+k$. But from (12) we know that $c_i=0$, for i = 1, 2, ..., k-1. Therefore we have the following system of equations with Rk+2 variables. From (12), (16) and (18), we have, for $0<r<R$,

$$c_0-\sum_{i=k}^{Rk+k} c_i=0, \tag{22}$$

$$c_0- c_k-\sum_{i=k}^{Rk+k}\theta_{i-1}\, c_i=0, \tag{23}$$

$$c_n-\sum_{i=k}^{Rk+k}\phi_{i-n+k}\, c_i=0, \text{for } n=k+1, k+2, \ldots 2k-1, \tag{24}$$

$$c_n-\sum_{i=n-k}^{Rk+k}\phi_{i-n+k}\, c_i=0, \quad \text{for } n=2k,\ldots,Rk+k \text{ but } n\neq rk+k, \tag{25}$$

and

$$c_{rk+k}-\sum_{i=rk}^{Rk=k}\phi_{i-rk}\, c_i-\sum_{i=1}^{k}\theta_{i-1}\, c_{Rk+i}=0. \tag{26}$$

In two extreme cases, equation (26) should be changed as follows:

1) If r = 0 (that is $rk + k = k$)

$$c_o- c_k-\sum_{i=k}^{Rk+k}\theta_{i-1}\, c_i+\sum_{i=1}^{k}\theta_{i-1}\, c_{Rk+i}=0. \tag{27}$$

2) If r = R (that is $rk + k = Rk + k$),

$$c_{Rk+k}-\sum_{i=Rk}^{Rk+k}\phi_{i-Rk}c_i-\sum_{i=1}^{k}\theta_{i-1}\, c_{Rk+i}=0. \tag{28}$$

It is well known that the above system of equations has a redundant equation and one equation has to be changed with the following normalizing equation:

$$E(X)c_{o}+\frac{1}{k\mu}\sum_{i=Rk+1}^{Rk+k}\{(R-r)k\theta_{i-RK-1}+\sum_{l=0}^{i-Rk-1}\theta_{l}\}c_{i}=1 \quad (29)$$

After calculation of c_i, we can derive the probability distribution of the system states as follows:

$$\begin{aligned} p_n &= \int_0^\infty p_n(x)dx \\ &= \int_0^\infty e^{-k\mu x}\overline{F}(x)\sum_{i=0}^{Rk+k-n}\frac{(k\mu x)^i}{i!}c_{n+i}dx \\ &= \sum_{i=0}^{Rk+k-n}\frac{(k\mu)^i}{i!}c_{n+i}\int_0^\infty e^{-k\mu x}\overline{F}(x)\,x^i dx \\ &= \frac{1}{k\mu}\sum_{i=0}^{Rk+k-n}c_{n+i}(1-\theta_i),\ \text{for } n=1,2,\ldots,Rk+k \end{aligned} \quad (30)$$

and

$$\begin{aligned} p_0 &= c_0\int_0^\infty \overline{F}(x)dx-\sum_{i=0}^{Rk+k-1}c_{i+1}\sum_{j=0}^{i}\frac{(k\mu)^j}{j!}\int_0^\infty e^{-k\mu x}x^j\overline{F}(x)dx \\ &= E(X)\,c_0-\frac{1}{k\mu}\sum_{i=0}^{Rk+k-1}c_{i+1}\sum_{j=0}^{i}(1-\theta_j). \end{aligned} \quad (31)$$

And for the blocking states,

$$q_{R+1}(j) = \frac{1}{k\mu}\sum_{i=0}^{k-j}c_{Rk+j+i}\,\theta_i,\ \text{for } j = 1, 2, \ldots, k, \quad (32)$$

$$q_m(j) = \frac{1}{k\mu}\sum_{i=1}^{k}c_{Rk+i}\,\theta_{i-1},\ \text{for } m = r+1, \ldots, R \text{ and } j = 1, 2, \ldots, k. \quad (33)$$

2.3 Performance Measures

Once the steady state probabilities are calculated, we can derive several performance measures as follows:

1) Utilization ratio of each station (e_1 for the preceding station and e_2 for the succeeding one)

$$e_1 = \sum_{n=0}^{Rk+k} p_n \quad (34)$$

$$e_2 = \sum_{n=1}^{Rk+k} p_n + \sum_{m=r+1}^{R+1} \sum_{j=1}^{k} q_m(j)$$

$$= 1 - p_0 \tag{35}$$

2) System productivity

$$P = k\mu \left\{ \sum_{n=0}^{R} p_{nk+1} + \sum_{m=r+1}^{R+1} q_m(1) \right\}$$

$$= k\mu \left\{ \sum_{n=0}^{R} p_{nk+1} + (R-r+1)q \right\}, \quad \text{where } q = q_{R+1}(1) \tag{36}$$

3) Average work-in-process in the buffer

$$\overline{I} = \sum_{n=1}^{R} n \sum_{i=1}^{k} p_{nk+i} + \sum_{m=r+1}^{R+1} m \sum_{j=1}^{k} q_m(j)$$

$$= \sum_{n=1}^{R} n \sum_{i=1}^{k} p_{nk+i} + (R+1) \sum_{j=1}^{k} q_{R+1}(j) + (R-r)(R+r+1)kq/2 \tag{37}$$

2.4 Some Examples

(1) Exponential / Exponential case

Assuming that the processing time of the preceding station follows exponential distribution with mean $1/\lambda$ and k, phase number of the Erlang distribution, is one, it is clear that:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, x > 0, \\ 0, \text{elsewhere} \end{cases} \tag{38}$$

$$F^*_j(s) = j!\lambda/(s+\lambda)^{j+1}, \tag{39}$$

and

$$\phi_j = \lambda\mu^j/(\mu+\lambda)^{j+1} \tag{40}$$

(2) Erlang / Erlang case

For the case when the processing time of the preceding station follows δ-phase Erlang distribution with mean $1/\lambda$, we have that:

$$f(x) = \begin{cases} \dfrac{(\delta\lambda)^{\delta}}{(\delta-1)!} x^{\delta-1} e^{-\delta\lambda x}, x > 0, \\ 0, \text{ elsewhere}, \end{cases} \tag{41}$$

$$F^*j(s) = \frac{\{1 + s/(\delta\lambda)\}^{-\delta-j}}{(\delta\lambda)^j} \prod_{l=0}^{j-1}(\delta + l), \tag{42}$$

and

$$\phi_j = \frac{\{1 + s/(\delta\lambda)\}^{-\delta-j}}{j!} \left(\frac{k\mu}{\delta\lambda}\right)^j \prod_{l=0}^{j-1}(\delta + l) \tag{43}$$

(3) 3-para. Erlang / Erlang case

The processing time of preceding station is a random variable with the following p.d.f:

$$f(x) = \begin{cases} \frac{(\delta\lambda)^\delta}{(\delta-1)!}(x-\nu)^{\delta-1} e^{-\delta\lambda(x-\nu)}, & x > \nu \geq 0, \\ 0, \text{ elsewhere}, \end{cases} \tag{44}$$

$$F_j^*(s) = e^{-\nu s} \sum_{i=0}^{j} \binom{j}{i} \frac{\nu^{j-i}\{1+s/(\delta\lambda)\}^{-\delta-i}}{(\delta\lambda)^i(\delta+i)} \prod_{l=0}^{i}(\delta+l). \tag{45}$$

and

$$\phi_j = (k\mu)^j e^{-\nu k\mu} \sum_{i=0}^{j} \frac{\nu^{j-i}\{1+k\mu/(\delta\lambda)\}^{-\delta-i}}{i!(j-i)!(\delta\lambda)^i(\delta+i)} \prod_{l=0}^{i}(\delta+l). \tag{46}$$

3. Numerical Results

In this section, we describe the behavior of the model, in terms of productivity and average work-in-process inventory. In other words, we observe the effects of some system parameters (λ, μ and buffer size) to the system performance (productivity and average inventory level). Observing the effects, we use 3-parameter Erlang distribution as the processing time of the preceding station.

The results are depicted in Figures 1, 2, 3, and 4. In these figures, one parameter was varied over a range, while all other parameters were held constant. We set $\nu = 1.0$, $\delta = 1$, $\lambda = 1.0$, $k=3$, $\mu = 0.5$, $R=20$, and $r=10$ as the standard values.

Figure 1 shows that as λ and μ increase, that is, the average processing time of each station decreases, system productivity increases. But the system productivity does not increase when μ is greater than 0.5, that is, the average processing time of the succeeding station is less than 2, since the average processing time of the preceding station is 2 ($= \nu + 1/\lambda$). By the similar reason, the system productivity does not increase when λ is greater than 1.

The graphs of the average work-in-process are plotted in Figure 2. The figure shows that as λ increases (the processing time of the preceding station decreases), the average work-in-process $\bar{I}$ increases. On the other hand, as μ increases (processing time of the succeeding station decreases), $\bar{I}$ decreases.

Figure 3 and 4 show that system productivity and the average inventory level increase as the buffer capacity R (we set r as $R/2$) increases. In these figures, we can find that the system productivity has little changes at relatively high values of R while the average inventory level is directly proportional to the buffer size R. This says that the increment of the inventory holding cost is greater than the increment of productivity when the buffer size is relatively large.

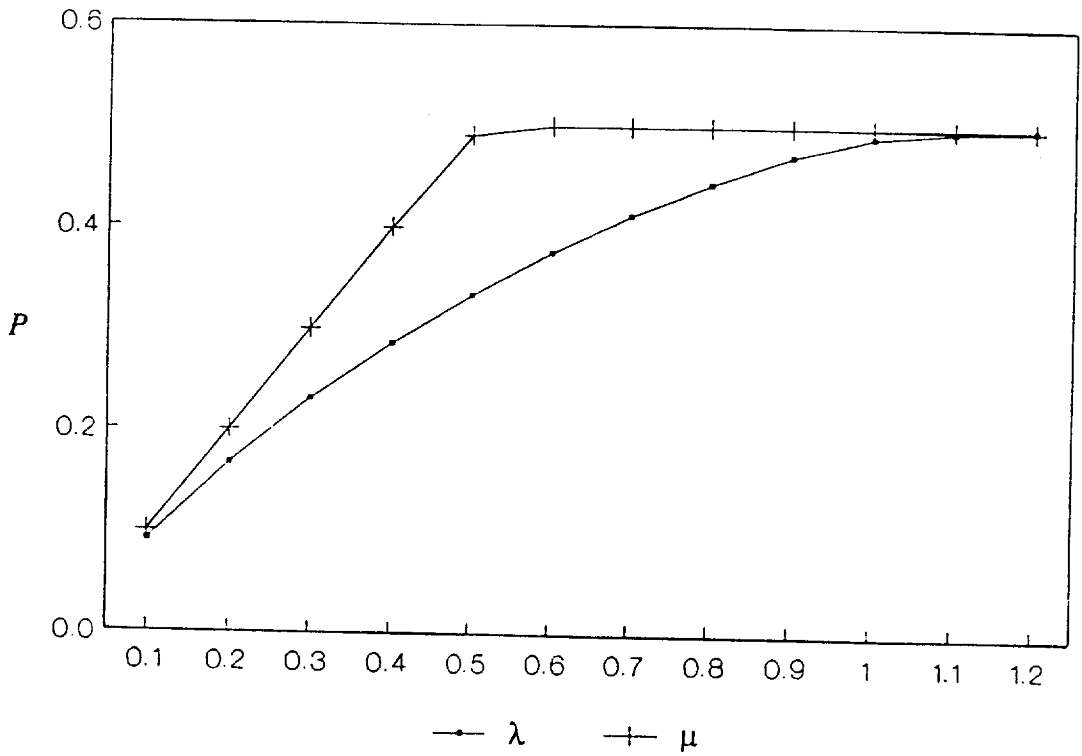

Figure 1. Effects of λ and μ to the system productivity

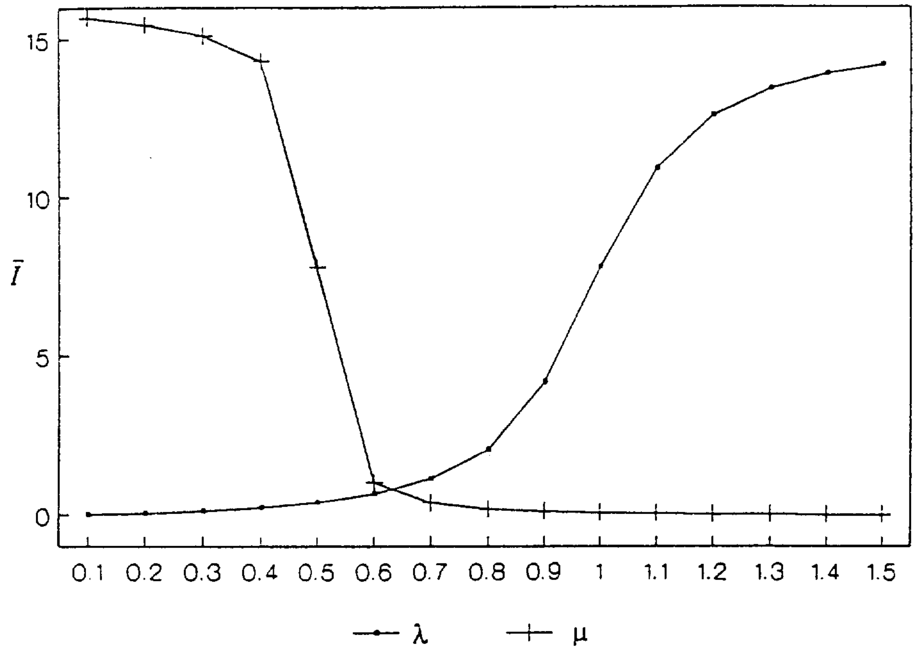

Figure 2. Effects of λ and μ to the average inventory

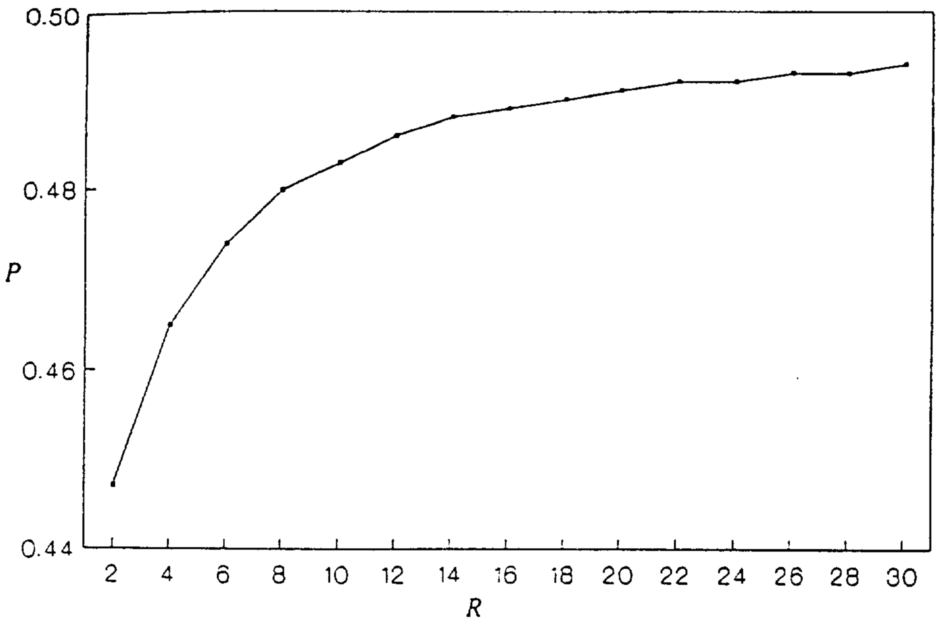

Figure 3. Effects of buffer size to the system productivity

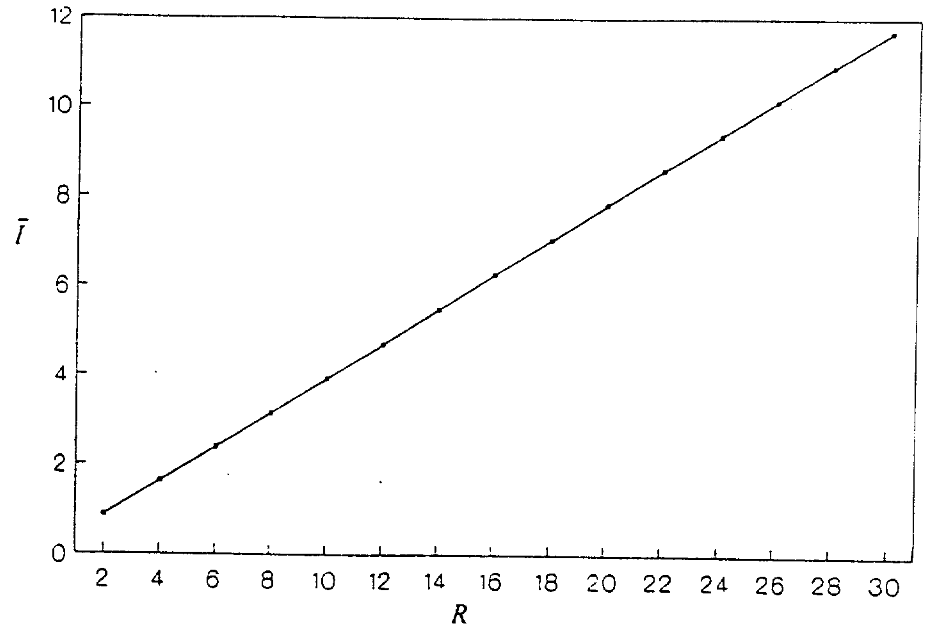

Figure 4. Effects of buffer size to average inventory

4. Conclusions

We studied a two-stage production line system in which each station is absolutely reliable but has random processing time. We assume the processing times of the preceding station and the succeeding station follow a general distribution and k-phase Erlang distribution, respectively. Using supplementary variable and phase technique, the system was analyzed.

The system can be considered as an approximation to a system in which each station has deterministic processing times and random failure and repair since the total time a workpiece spends in a station, including any necessary repair time but excluding blocking or starving time, can be considered as another random variable.

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