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A new interpretation of two-beam energy coupling in terms of Bragg diffraction in a photorefractive crystal

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Bragg diffraction of a strong reference beam from a steady-state photorefractive grating is measured experimentally and an analytic prediction is derived from the coupled wave equations of two-beam energy coupling. The relation between Bragg diffraction and two-beam coupling is used to check the mechanism of photorefractive grating formation.

The formation of a photorefractive grating is well explained by the band-conduction model[1] and the readout of the grating, or measurement of the Bragg diffraction, can be predicted by Kogelnik's formula[2] when the grating amplitude is assumed to be uniform in the photorefractive material. However, when two incident beams create a photorefractive grating, each beam is Bragg diffracted from the grating and added to or subtracted from the other beam, resulting in an energy transfer from one beam to the other. Moreover, due to two-beam energy coupling, modulation of the intensity pattern and the amplitude of the resulting photorefractive grating vary inside the photorefractive material. In this case the diffraction efficiency of a third probe beam depends not only on intensity modulation but also the two-beam coupling coefficient.

In the presence of two-beam energy coupling the diffraction efficiency of a third weak probe beam was first obtained in Ref. 1. In general, the diffraction efficiency can be obtained from the coupled wave equations of four-wave mixing[3,4]. In a forward four-wave-mixing geometry the diffraction efficiency of the third beam of arbitrary strength was derived by assuming an arbitrary phase shift between the photorefractive grating and the intensity pattern[5]. When the probe beam had a different polarization from the writing beams, the diffraction efficiency in the presence of two-beam coupling was obtained analytically and measured experimentally[6]. When it is assumed that the probe beam is weak enough not to erase the photorefractive grating and the phase shift between the photorefractive grating and the intensity pattern is exactly $\pi/2$ and the coupling coefficients at writing and reading the grating are same, the diffraction efficiencies in Ref. 1, 5 and 6 become identical.

In this paper, we derive the diffraction efficiency of the writing beam off the steady-state photorefractive grating from the coupled wave equations of two-beam energy coupling, not from a four-wave-mixing geometry, by considering the anti-symmetry in the Bragg diffraction of two writing beams. We measure, for the first time to our knowledge, the Bragg diffraction of the writing beam almost without any grating erasure and compare it with the analytic prediction. We also obtain the analytic prediction for the Bragg diffraction when the amplitude of the photorefractive grating is given by a nonlinear function of the intensity modulation due to higher order harmonics[1,7,8].

When two writing beams intersect in a photorefractive crystal with the electric field of the form of $\vec{\varepsilon}_1=\frac{1}{2}\vec{E}_1\exp[i\vec{k}_1\cdot\vec{r}-i\omega t]+c.c$ and $\vec{\varepsilon}_2=\frac{1}{2}\vec{E}_2\exp[i\vec{k}_2\cdot\vec{r}-i\omega t]+c.c$ such that the bisector of the beams is parallel to the entrance surface normal of the crystal, the x-axis, the electric fields can be obtained from the coupled wave equations,

$$\frac{\partial E_1}{\partial x}=\frac{1}{2}\Gamma\frac{m}{2}E_2, \tag{1}$$

$$\frac{\partial E_2}{\partial x}=-\frac{1}{2}\Gamma\frac{m^*}{2}E_1, \tag{2}$$

where $\vec{k}_1$ and $\vec{k}_2$ are wavevectors, $\vec{r}$ is position vector, ω is angular frequency, Γ is two-beam coupling coefficient, which is real for the $\pi/2$ phase shifted photorefractive grating, c.c and * stand for complex conjugate, and m is the intensity modulation given by $m=2\vec{E}_1\vec{E}_2^*/(|\vec{E}_1|^2+|\vec{E}_2|^2)$. Then, the intensities of two writing beams are obtained as functions of x as

$$I_1=I_{1o}\frac{(s+1)e^{\Gamma x}}{(s+e^{\Gamma x})}, \tag{3}$$

$$I_2=I_{2o}\frac{(s+1)}{(s+e^{\Gamma x})}, \tag{4}$$

where I_{1o} and I_{2o} are beam intensities measured at the entrance surface of the crystal and s is their ratio defined by $s = I_{2o}/I_{1o}$. In a steady state the intensity modulation in the crystal can be obtained from the beam intensities as

$$m = 2\sqrt{s}\frac{e^{\Gamma x/2}}{s+e^{\Gamma x}}. \tag{5}$$

Since the intensity modulation is invariant in time in the steady state, the two writing beams then seem to pass through a fixed refractive index grating with the amplitude of $\Gamma m/2$, and to experience Bragg diffraction.

Fig. 1 shows transmission and Bragg diffraction of two incident beams from the fixed refractive index grating. In this case the transmission and Bragg diffraction of one beam is independent of the other beam. Then the electric fields of the transmitted and the Bragg diffracted of beam 1 can be obtained from the same coupled wave equations (1) and (2), but with m replaced by Eq. (5).

$$\frac{\partial E_1^t}{\partial x} = \frac{1}{2}\Gamma\frac{m}{2}E_1^d, \tag{6}$$

$$\frac{\partial E_1^d}{\partial x} = -\frac{1}{2}\Gamma\frac{m}{2}E_1^t. \tag{7}$$

Similarly, for beam 2 one can obtain

$$\frac{\partial E_2^t}{\partial x} = -\frac{1}{2}\Gamma\frac{m}{2}E_2^d, \tag{8}$$

$$\frac{\partial E_2^d}{\partial x} = \frac{1}{2}\Gamma\frac{m}{2}E_2^t. \tag{9}$$

Next both sides of Eqs. (6) and (8) are divided by E_{1o} and E_{2o}, respectively, which are the electric fields of beam 1 and 2 at the entrance surface of the crystal. Then it is noted that the derivatives of the normalized electric fields, E_1^t/E_{1o} and E_2^t/E_{2o}, should be identical in the crystal because the Bragg diffraction changes only the amplitude of the transmitted beam with no change in its phase for the $\pi/2$ phase shifted photorefractive grating. Eqs. (6) and (8) therefore result in

$$E_1^d = -\frac{E_{1o}}{E_{2o}}E_2^d. \tag{10}$$

Note that E_1^d are E_2^d functions of x in general.

If one interprets the situation of Fig. 1 in terms of the steady-state two-beam coupling, the electric field of one beam, $E_1^t + E_2^d$, should be related to the other, $E_1^d + E_2^t$, by the coupled wave equations (1) and (2) as

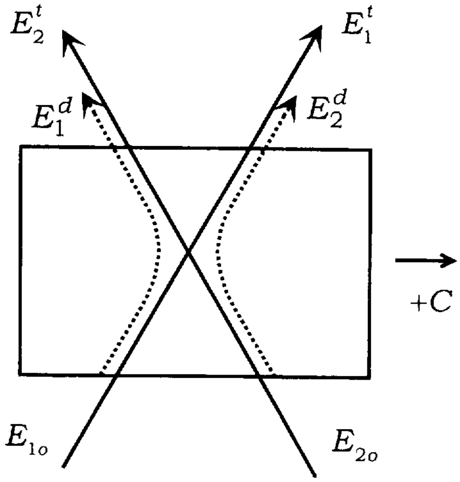

Fig 1. Two incident beams are Bragg diffracted from a fixed refractive index grating that is assumed to be the photorefractive grating formed at the steady-state of the two-beam energy coupling. t stands for transmission and *d* Bragg diffraction.

$$\frac{\partial}{\partial x}(E_1^t + E_2^d) = \frac{1}{2}\Gamma\frac{m}{2}(E_1^d + E_2^t), \tag{11}$$

$$\frac{\partial}{\partial x}(E_1^d + E_2^t) = -\frac{1}{2}\Gamma\frac{m}{2}(E_1^t + E_2^d), \tag{12}$$

At the steady state the equations are already solved to yield the beam intensities of Eqs. (3) and (4). When Γ is real, the electric fields of beam 1 and 2, $E_1^t + E_2^d$ and $E_1^d + E_2^t$, can be assumed to be $\sqrt{I_1}$ and $\sqrt{I_2}$ in the left sides of Eqs. (11) and (12). Using the relation $E_1^t/E_{1o} = E_2^t/E_{2o}$ in the right sides of Eqs. (11) and (12) together with Eq. (10) one can obtain the electric fields of the Bragg diffracted and the transmitted beams as

$$E_1^d = E_{1o}\left(\frac{s}{s+1}\right)^{1/2}\frac{(1-e^{\Gamma x/2})}{(s+e^{\Gamma x})^{1/2}} \tag{13}$$

$$E_1^t = E_{1o}\left(\frac{1}{s+1}\right)^{1/2}\frac{(s+e^{\Gamma x/2})}{(s+e^{\Gamma x})^{1/2}} \tag{14}$$

Hence the diffraction efficiency of beam 1 and beam 2 can be derived as $|E_1^d/E_{1o}|^2$; see Eq. (10), which is identical to the results of Refs. 1, 5 and 6 for the same limits used in this derivation.

Fig. 2 shows the experimental setup. A 515nm laser beam of ordinary polarization is split into two by a beam splitter (BS) and made to intersect in a $BaTiO_3$ crystal at an angle of 20 degree measured outside the crystal. The on and off of the two beams is controlled by shutters S1 and S2, and the intensity modulation by neutral density filter (ND). Since ND may be inserted either into beam 1 or beam 2, the beam with the reduced intensity is called signal and the other reference. Sig-

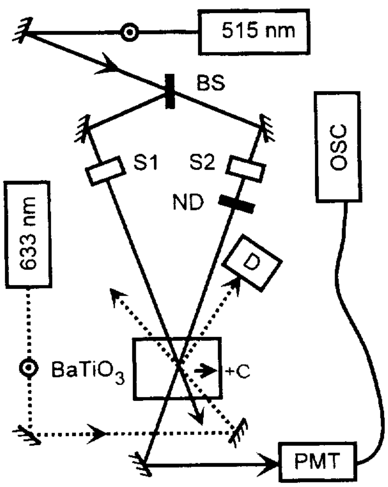

Fig 2. Experimental setup to measure Bragg diffraction of the strong reference and that of the weak probe beam.

Table 1. Shutter control for two writing beams during the experiment

	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Signal beam	off	off	on	on	off
Reference beam	on	off	off	on	on

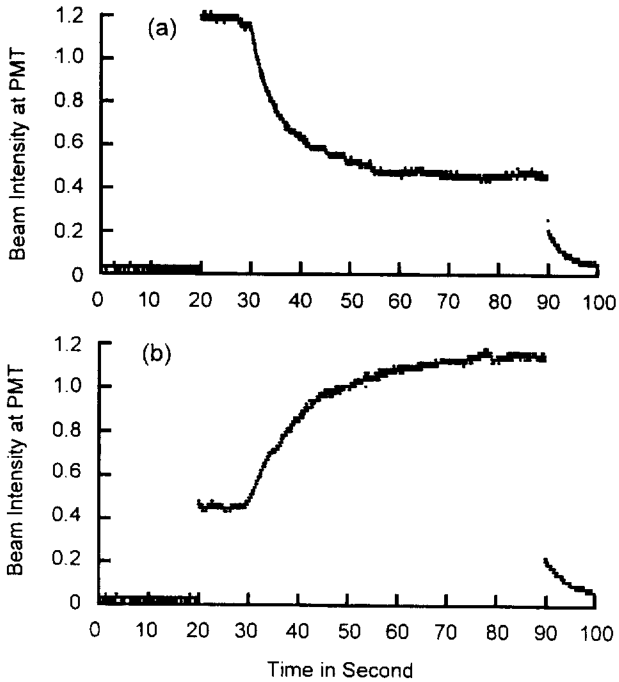

Fig 3. Traces of the digital storage oscilloscope. The weak signal experiences energy loss in the crystal in (a) and energy gain in (b).

nal is then inserted into photomultiplier tube(PMT) and its intensity is measured with digital storage oscilloscope(OSC). We also measure the diffraction efficiency of a 633nm beam of ordinary polarization with a photodetector (D) in the conventional four-wave mixing.

In the experiment two shutters are made on or off according to Table 1. At Stage 1 the residual gratings in the crystal is erased by the strong reference. At Stage 2 both beams are turned off and the background noise is measured. At Stage 3 the intensity of the incident signal is measured. At Stage 4 both beams are on and a photorefractive grating is written in the crystal until a steady state is reached. At Stage 5 the weak signal is turned off suddenly so that PMT can detect the Bragg diffraction of the strong reference from the photorefractive grating. Fig. 3 shows typical oscilloscope traces of the measured beam intensity v.s. time. The duration of each stage is approximately 10 seconds, except for 60 seconds at Stage 4. The figure shows that the signal experiences energy loss due to two-beam coupling in (a) and energy gain in (b). In the experiment the sampling period of the digital oscilloscope is 50 msec and the transit time of the shutter is less than 1 msec, and the response time of PMT is less than 0.1 msec. The grating erasure time constant is approximately 10 seconds as can be seen from the figure. Since the grating erasure time constant is much longer than other time scales, at the starting point of Stage 5 we can read off the Bragg diffraction of the strong reference from the steady-state photorefractive grating.

The measured diffraction efficiency, compared with the analytic prediction, is obtained as follows. First, from the oscilloscope trace intensities of the weak signal are measured at Stages 3 and 4 to find Γ using Eq. (3) or (4): Eq. (3) when I_1 is the weak signal experiencing energy gain(s>1), Eq. (4) when I_2 is the weak signal experiencing energy loss(s<1). Inserting Γ into Eq. (13) the analytic prediction of the diffraction efficiency of the strong reference can be obtained from the steady-state photorefractive grating.

In Fig. 4 the circle and the square represent the diffraction efficiencies of the reference measured when the signal experiences energy gain and loss, respectively. The star and the cross represent the analytic predictions when the signal experiences energy gain and loss, respectively. The black circle and the black square represent the diffraction efficiencies of 633 nm beam measured in the conventional four wave mixing when the signal experiences energy gain and loss, respectively. Fig. 4 shows that the analytic predictions for the diffraction efficiency of the reference are in good agreement with the experimental data. It also shows that the diffraction efficiency of 633nm beam from the same photorefractive grating is smaller than that of the reference by a factor of 4 to 6. In the experiment the power of 633nm beam is made small (14 μW) in order not to erase the photorefractive grating

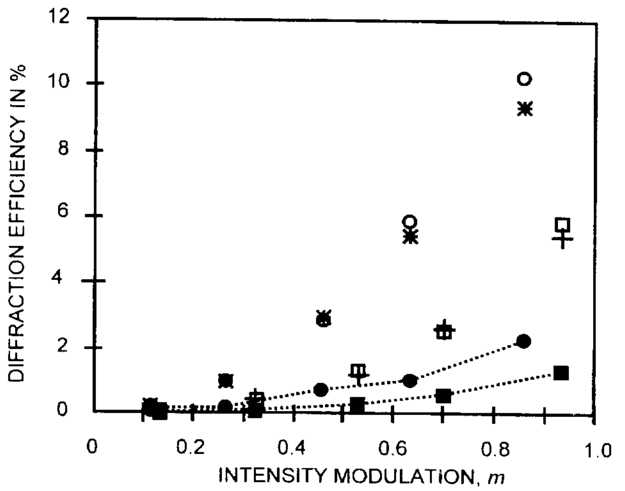

Fig 4. Diffraction efficiencies measured experimentally and predicted analytically.

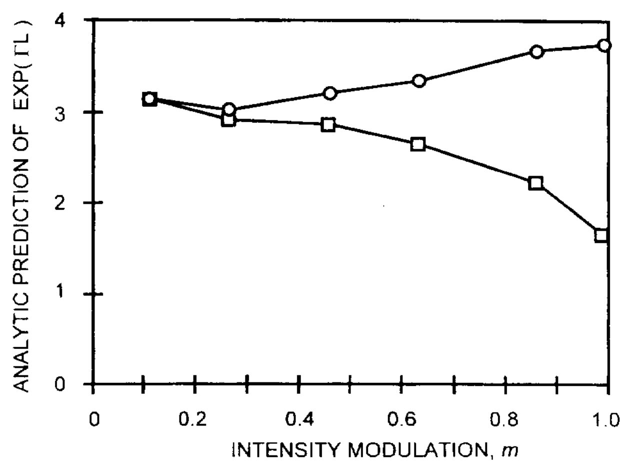

Fig 5. For two different dependencies of the photorefractive grating on the intensity modulation, m by circle and $2m/\left(1+\sqrt{1-|m|^2}\right)$ by square, different coupling coefficients are analytically predicted at large *m*.

written by 515nm beam, whose total power is maintained to be 7*m*W throughout the experiment. The smaller diffraction efficiency may result from the imperfect overlap of 633nm beam with the photorefractive grating due to the different incident angle of 633nm beam for the Bragg phase matching condition and the different Gaussian beam characteristics of 633nm and 515nm beams.

In the presence of higher order harmonics the fundamental photorefractive grating is given proportional to $2m/\left(1+\sqrt{1-|m|^2}\right)$ instead of m when the grating wavevector is much smaller than the Debye screening wavevector.[1,7,8] We next use this expression in the coupled wave equations (1) and (2) in place of m and follow the same procedure to derive the diffraction efficiency of the strong reference. In this case, due to the square root in the denominator, the coupled wave equations should be solved in two different regions. When $I_1 < I_2$ and I_1 is the weak signal experiencing energy gain, one can obtain $I_1 = I_{1o}\exp(\Gamma x)$ and $I_2 = I_{2o} + I_{1o}[1-\exp(\Gamma x)]$. Similarly, when $I_2 < I_1$ and I_2 is the weak signal experiencing energy loss, one can obtain $I_1 = I_{1o} + I_{2o}[1-\exp(-\Gamma x)]$ and $I_2 = I_{2o}\exp(-\Gamma x)$. From I_1 and I_2 one can obtain the steady-state intensity modulation and photorefractive grating. Hence, from the anti-symmetry in the Bragg diffraction of the two writing beams, one can obtain the diffraction efficiency of the reference as another function of *m* and Γ. It should be noted in this case that Γ has a different value from the previous derivation, even for the same energy transfer between two writing beams, because of the different functional forms of I_1 and I_2 from the previous derivation. When the intensity of the weak signal stays smaller than the strong reference throughout the crystal, it can be shown that the current analytic predictions for the diffraction efficiency are identical to the previous predictions.

When we try to explain a given energy transfer between writing beams using the two different grating formations, we note that the intensity modulation m should be identical in the crystal for the two cases even if the functional forms of the beam intensities are given differently. However, the amplitude of the resulting photorefractive grating will be different for the two cases because of the different dependencies of the amplitude on *m*. Hence Γ should be adjusted to make the Bragg diffraction of the strong reference identical. Since, in reality, Γ is material-dependent and therefore should be independent of *m*, we can use the calculated Γ as a check for the correct dependency of the photorefractive grating on *m*. When *m* is small, the first and the second analytic derivations produce the same Γ for the same Bragg diffraction. However, when m becomes larger, a larger Γ is predicted by the first derivation as can be seen from Fig. 5, which means the photorefractive grating is underestimated at a large *m*, and a smaller Γ by the second derivation, which means the photorefractive grating is overestimated at a large *m*.

In conclusion, the Bragg diffraction efficiency of the strong reference from the steady-state photorefractive grating is measured, for the first time to our knowledge, and the analytic prediction is derived from the coupled wave equations of two-beam energy coupling using the anti-symmetry in the Bragg diffraction of the two writing beams. The conventional diffraction efficiency of a third probe beam is also measured in a four-wave-mixing geometry and is shown to be smaller than that of the reference by a factor of 4 to 6. The analytic prediction for the Bragg diffraction of the reference is also derived when the photorefractive grating is assumed to be proportional to $2m/\left(1+\sqrt{1-|m|^2}\right)$ instead of m. From the relation between the Bragg diffraction of the reference and the two-beam energy coupling it is shown that the photorefractive grating proportional to m is underestimated, and that proportional to $2m/\left(1+\sqrt{1-|m|^2}\right)$ is overestimated for large m.

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