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Retro-self-focusing and pinholing effect in a refractive index grating

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In this paper we will show theoretically that a refractive index grating exhibits a retro-self-focusing effect and an accompanying pinholing effect under the Gaussian intensity distribution of an incident optical field. Those effects result from an effective wave number change of the medium due to the intense optical field.

I. INTRODUCTION

In[1], we showed both theoretical and experimental details of a retro-self-focusing and a pinholing effect in a cholesteric liquid crystal with a helical molecular structure. In addition to this medium, those nonlinear effects can be also observed in a refractive index grating. Those effects are approximately the converse of self-focusing in a medium with quadratic index dependence. In the case of a refractive index grating, the periodicity of the medium causes the beam to be reflected and the quadratic dependence of the refractive index causes the beam to focus; This is a retro-self-focusing effect. In this paper, we will use the plane wave approximation to show a retro-self-focusing and a pinholing effect in a refractive index grating.

II. PROBLEM DESCRIPTION AND IMPLICIT SOLUTIONS

Let us consider a lossless isotropic medium of length L whose refractive index is given by

$$n(z) = n_0 + n_1 \cos(2\beta_0 z + \phi)$$

where $n_1 \ll n_0$ and ϕ is constant phase. If the refractive indices is modified in the presence of intense optical field, we can define the intensity-dependent refractive index by

$$n(z) = n_0 + n_1 \cos(2\beta_0 z + \phi) + n_2 \langle I \rangle \tag{1}$$

where the bracket means time average, and n_2 is the nonlinear refractive index. Near the Bragg frequency, one can write the field in-medium as a summation of forward-propagating wave and counter-propagating wave[2]

$$E(z) = E_+(z)\exp(i\beta z) + E_-(z)\exp(-i\beta z)$$

where $\beta = n_0\omega/c$. In the slowly varying approximation, Maxwell's equation in the medium of Eq. 1 can be rewritten as follows:

$$-i\frac{dE_+}{dz} = \kappa E_- \exp[-i(2\Delta\beta z - \phi)] + \alpha(|E_+|^2 + 2|E_-|^2)E_+$$
$$i\frac{dE_-}{dz} = \kappa E_+ \exp[i(2\Delta\beta z - \phi)] + \alpha(2|E_+|^2 + |E_-|^2)E_- \tag{2}$$

where $\alpha = n_2\omega/2c$, $\Delta\beta = \beta - \beta_0$ and the coupling coefficient $\kappa = \beta_0 n_1/2n_0$. Let $E_+ = |E_+| e^{i\phi_+}$, $E_- = |E_-| e^{i\phi_-}$ and substituting them in Eq. 2, we have

$$\frac{d|E_+|}{dz} = \kappa|E_-|\sin\chi$$
$$\frac{d|E_-|}{dz} = \kappa|E\pm|\sin\chi$$
$$\frac{d\chi}{dz} = 2\Delta\beta z + \kappa\left[\left|\frac{E_-}{E_+}\right| + \left|\frac{E_+}{E_-}\right|\right]\cos\chi + 3\alpha[|E_+|^2 + |E_-|^2] \tag{3}$$

where $\chi = 2\Delta\beta z + \phi_+ - \phi_- - \phi$. Then boundary conditions are introduced as follows:

$$|E_+(L)| = |E_T|$$
$$|E_-(L)| = 0. \tag{4}$$

One can solve the coupled amplitude equations in Eq. 3 under boundary conditions Eq. 4:

$$\sqrt{uv}\sin\chi = -\left[(u-J)\left\{u-(u-J)\left(\frac{\Delta\beta}{\kappa} + \frac{3\alpha}{2\kappa}|E_C|^2 u\right)\right\}\right]^{\frac{1}{2}} \tag{5}$$

where $u = |E_+|^2/|E_C|^2$, $J = |E_T|^2/|E_C|^2$ and $|E_C|^2 = 4n_0\lambda/3\pi n_2 L$. Multiplying Eq. 3-1 by $|E_+|$ and substituting Eq. 5, we have

$$\frac{du}{dz} = -2\kappa\sqrt{Q(u)} \tag{6}$$

where $Q(u) = (u - J)\left\{u - (u - J)\left(\frac{\Delta\beta L}{\kappa L} + \frac{2}{\kappa L}u\right)^{\frac{1}{2}}\right\}$.

When the roots of Q(u) are real and $u_1 > u_2 > u_3 > u_4$, Eq. 6 may be solved in terms of a Jacobian elliptic function

$$u(z) = u_3 + \frac{u_2 - u_3}{1 - (u_1 - u_2)(u_1 - u_3)^{-1} Sn^2[2\kappa(z-L)/g, k]}$$

where Sn is a Jacobian elliptic function with $g = 2/[(u_1 - eu_3)(u_2 - u_4)]^{1/2}$ and $\kappa = [(u_1 - u_2)(u_3 - u_4)]\}^{1/2} \cdot g/2$. When the two roots of Q(u) are real and $u_1 \geq u \geq u_2$ and the two roots u_3 and u_4 are complex, the solution of Eq. 6 becomes

$$u(z) = \frac{Bu_1 + Au_2 + (Au_2 - Bu_1)\, Cn[2\kappa(z-L)/g, k]}{A + B + (A - B)\, Cn[2\kappa(z-L)/g, k]}$$

where $A^2 = (u_1 - b_1)^2 + b_2^2$, $B^2 = (u_2 - b_1)^2 + b_2^2$, $b_1 = Re[u_3]$, $b_2 = Im[u_3]$, and Cn is a Jacobian elliptic function with $g = 1/\sqrt{AB}$, $b_2 = (u_1 - u_2)^2 - \{(A - B)^2\}/4AB$.

Now let us calculate the phase $\phi_{(z)}$ of the reflected field at z = 0. From Eq. 5,

$$\phi_-(0) = \phi_+(0) - \phi - \cos^{-1}\left[-\sqrt{1 - J/I}\left\{\frac{\Delta\beta}{\kappa} + \frac{3\alpha}{2\kappa}\left|E_C\right|^2\right\}\right] \quad (7)$$

where I = u(0). Eq. 7 shows explicitly the intensity dependence of the phase shift on the reflected field through the medium. Fig. 1 shows the phase of the reflected field as a function of the normalized input intensity I and the normalized detuning parameter $\Delta\beta/\kappa$ when $\phi_-(0)$ - = 0. The physical origin of this intensity dependence of the phase of a reflected field is as follows: In a weak field regime ($n_2\langle I\rangle \simeq 0$), the

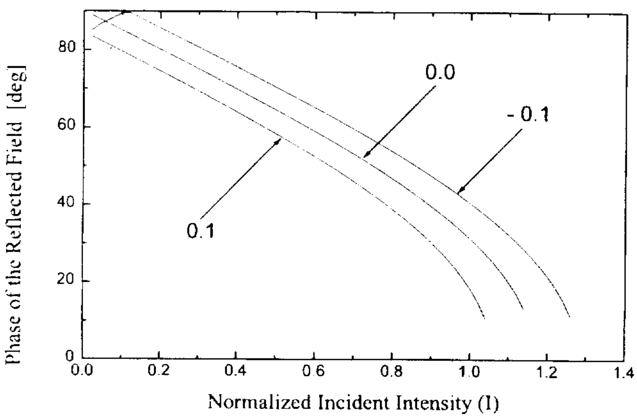

Fig. 1. Incident field intensity versus the phase of the reflected field for various values of $\Delta k/\kappa$ = 0, $\pm$0.1 when the phase of the reflected field $\phi_+(0)$ = 0.

incident field will see the periodic structure with a constant wave number β_0 through the medium. However, in a strong field regime, the third term in Eq. 1 introduces a nonuniform biased refractive index through the medium. This effectively gives rise to the wave number change of the medium. In other words, the incident field will see a different periodicity.

A very interesting condition results from considering a plane wave with intensity distribution $I_p(x, y) = |E(x, y)|^2$ incident on the refractive index grating:

$$I_p(r) = I_O \exp\left[-\frac{2r^2}{w^2}\right]$$

where w is the spot size of the beam. Rewriting this as a normalized intensity with $|E_c|^2$ and introducing a radial dependence, one finds

$$u(0) = I = \frac{|E(r=0)|^2}{|E_C|^2} = \left|\frac{E_O}{E_C}\right|^2 \exp\left[-\frac{2r^2}{w^2}\right]$$

For $I \gg J$ and $\phi_-(0) \leq \pi/4$, Eq. 7 reduces to

$$\cos\phi_-(0) \simeq 1 - \frac{\phi_-^2(0)}{2} = \left\{\frac{\Delta\beta}{\kappa} + \frac{3\alpha}{2\kappa}|E_C|^2\right\}$$

Under the Bragg condition / = 0, Eq. 17 becomes

$$\phi_-(0) \simeq \sqrt{2}\left(1 + \sqrt{\frac{3\alpha}{2\kappa}}|E_O|^2\right) - \sqrt{\frac{3\alpha}{\kappa}}\frac{|E_O|^2 r^2}{w^2} + \sqrt{\frac{3\alpha}{\kappa}}\frac{|E_O|^2 r^4}{w^4} \quad (8)$$

The first term of Eq. 8 corresponds to the constant phase change. The second term corresponds to the quadratic phase change and the third term corresponds to a spherical aberration (r^4 dependence). Note that $\phi_-(0)$ is almost constant when $|r| > w/\sqrt{2}$. Therefore any retro-self-focusing effect can only occur within $|r| \leq w/\sqrt{2}$. This can be interpreted as a form of pinholing or apodizing effect where the aperture possesses a soft edge. By analogy with the quadratic phase term of a Gaussian beam, we can calculate the intensity dependent radius of curvature of the reflected field as

$$R = \frac{\pi w^2}{\lambda}\frac{1}{\sqrt{\frac{3\alpha}{\kappa}}|E_O|^2} \quad (9)$$

Eq. 9 indicated a retro-self-focusing effect because the radius of curvature is negative and inversely proportional to the intensity as we expect.

III. CONCLUSION

We have shown that in the presence of intense optical field with a Gaussian beam profile the refractive index grating can give rise to a retro-self-focusing and pinholing effect.

REFERENCES

[1] J. C. Lee, S. D. Jacobs, and A. Schmid, "Retro-Self-Focusing and Pinholing Effect in a Cholesteric Liquid Crystal," Mol. Cryst. Liq. Cryst. **150b**, 617-629 (1987).

[2] H. G. Winful, J. H. Marburger, and E. Garmire, "Theory of Bistability in Nonlinear Distributed Feedback Structures," Appl. Phys. Lett. **35**, 379-381 (1979).