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Enhancement in quantum noise correlation between the two outputs of a nondegenerate optical amplifier with a non-vacuum state idler input

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The theoretical limit of the noise correlation between the signal and idler outputs of a nondegenerate optical parametric amplifier (NOPA) with a coherent state signal and vacuum state idler input can be enhanced if a non-vacuum coherent state idler input is employed. By choosing a balanced signal and idler input, the noise correlation is $1/(\sqrt{g}+\sqrt{g-1})^2$, where g is the intensity gain of the NOPA, and that is superior to the prediced outputs with single signal input by approximately 3dB. The result is applicable to all the schemes that use the NOPA to produce a sub-shot noise light generation such as feed-back or feed-forward control.

A nondegenerate optical parametric amplifier (NOPA) is known to produce quantum noise correlated signal and idler beams, that have been used for the generation of nonclassical lights such as squeezed light[1] or twin beams of light[2]. The squeezed light, of which one quadrature noise is less than the vacuum fluctuation noise, is generated by interfering the signal output with the idler output of the NOPA when both signal and idler inputs are in the vacuum states. The twin beams are the two output beams of NOPA operating in a coherent state signal and vacuum state idler inputs and the noise correlation between the two channels is proportional to the intensity gain of the NOPA[3]. Using the feed back or feed forward scheme in which one of the correlated photon beams is detected and fed back or forward to control the other beam, an amplitude-squeezed light or sub-Poissonian light can be generated from the twin beams[4]. A few realizations of such schemes have been reported[5] and showed that the feed-forward scheme has advantages of wide bandwidth and tunability over the feed-back schemes, and the merit of simplicity is an advantage if the twin beams are generated from a NOPA. In such schemes, the theoretical limit of noise reduction using the feed forward scheme on the NOPA twin beam is $1/(2g-1)$ below the shot noise level, where g is the intensity gain of the NOPA[6]. In this paper, it is shown that the theoretical limit of noise reduction can be enhanced to $1/(\sqrt{g}+\sqrt{g-1})^2$ when a suitable non-vacuum state idler input to the NOPA is chosen. The physical interpretation of such improvement is also presented with the relation between NOPA's and the degenerate optical parametric amplifiers (DOPA's).

Let us first consider a NOPA configuration shown schematically in Fig. 1-a. The input signal and idler beams to the NOPA are described by the modal annihilation operators $\hat{a}_s$ and $\hat{a}_l$ respectively. We assume the single input, that the input state is

$$|\psi\rangle = |\alpha_s\rangle_{a_s} \otimes |\alpha_i\rangle_{a_i} \equiv |\alpha_s, \alpha_i\rangle_a = |\alpha, 0\rangle_a \tag{1}$$

so that $\hat{a}_s|\psi\rangle=\alpha|\psi\rangle$ and $\hat{a}_i|\psi\rangle=0$. This means that the input signal beam is in a coherent state while the input idler is in vacuum. The NOPA produces two quantum-correlated output signal and idler beams with annihilation operators $\hat{b}_s$ and $\hat{b}_i$ respectively. The input and output annihilation operators of the NOPA are related via[7]

$$\hat{b}_s = \mu\hat{a}_s + \nu\hat{a}_i^\dagger, \tag{2}$$
$$\hat{b}_i = \mu\hat{a}_i + \nu\hat{a}_s^\dagger, \tag{3}$$

where $[\hat{b}_s, \hat{b}_i] = [\hat{b}_s, \hat{b}_i^\dagger] = O$, $[\hat{b}_j, \hat{b}_j^\dagger] = 1$ for $j = s, i$ and $|\mu|^2 - |\nu|^2 = 1$. Fig. 1-b describes the twin beams measurement scheme. The direct detection of each output is measuring the photon numbers $\hat{n}_s \equiv \hat{b}_s^\dagger\hat{b}_s$, $\hat{n}_i \equiv \hat{b}_i^\dagger\hat{b}_i$ and the statistics of the photon numbers are transferred to the photocurrent of the detectors. The gain characteristics are defined by the input-output mean photon number relations and obtained by

$$\langle n_s\rangle = g|\alpha|^2 + g - 1, \tag{4}$$
$$\langle n_i\rangle = (g-1)|\alpha|^2 + g - 1. \tag{5}$$

where $|\mu|^2 = g$, $|\nu|^2 = g-1$, g is the intensity gain of the NOPA.

The outputs are independent of the relative phase between gain parameters (μ, ν) and the input beams, so the NOPA in this configuration is a Phase Insensitive Amplifier (PIA)[10]. The photon number fluc-

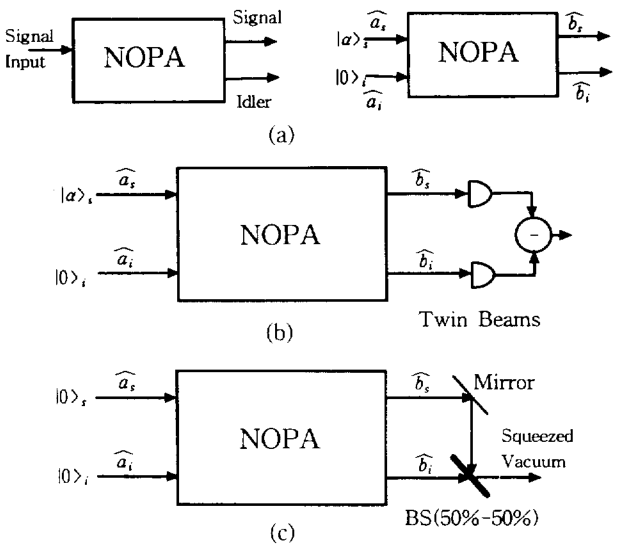

Fig 1. The schematic diagrams of NOPA and its inputs and outputs. (a) NOPA outputs with single input. The signal input is in a coherent state while there is no input to the idler input; the idler input is regarded as a vacuum state input. (b) Twin beam measurement setup. The individual signal and idler outputs are directly measured by the photodetectors. The quantum efficiency of the detectors are assumed to be unity. The subtracted current of the detectors reveals the noise correlation between the two channels. (c) Quadrature squeezed state generation via a NOPA. Both inputs are in vacuum states and the two outputs are arranged to interfere with each other via a 50%-50% beam splitter. The sub-shot noise can be measured by the balanced homodyne detection of the output with a suitable choice of a LO phase.

tuations can be written as

$$<\Delta^2 n_s> = g(2g-1)|\alpha|^2 + g(g-1). \quad (6)$$
$$<\Delta^2 n_i> = (g-1)(2g-1)|\alpha|^2 + g(g-1). \quad (7)$$

and these fluctuations are transferred to the photocurrent of the two detectors measuring the signal and idler output beams, respectively. The twin beam measurement is the observation of the subtracted current between the two detectors and thus depends on the statistics of $\hat{n}_T \equiv \hat{n}_s - \hat{n}_i$; that is, given by

$$<\hat{n}_T> = <\Delta^2 \hat{n}_T> = |\alpha|^2. \quad (8)$$

If the two outputs are statistically independent, the fluctuation should be added so that

$$<\Delta^2 \hat{n}_T> = <\Delta^2 \hat{n}_s> + <\Delta^2 \hat{n}_i> = (2g-1)|\alpha|^2. \quad (9)$$

Thus the two outputs of the NOPA are correlated by the factor of $2g$ - 1, which is the theoretical limitation of noise reduction for the application of the feed-forward or feed-back schemes on the twin beams[5][6].

On the other hand, a squeezed vacuum state can be

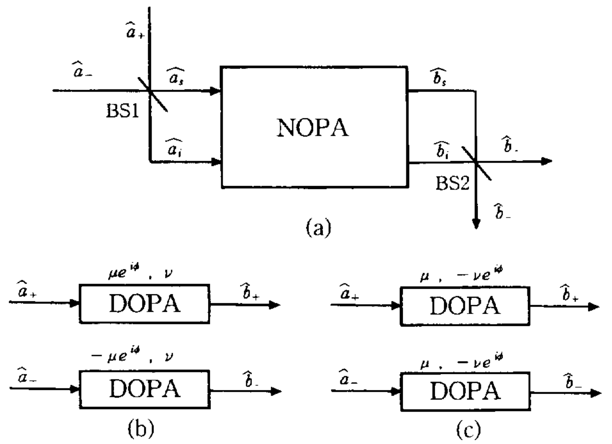

Fig 2. The equivalence of a NOPA with two independent DOPA's in parallel. (a) The NOPA is surrounded by two 50%-50% beam splitters. (b),(c) The equivalent DOPA's with a suitable choice of the phase parameters.

generated via the setup shown in the Fig. 1-c. In this setup, both of the inputs to the NOPA are in vacuum states and two outputs are mixed via a 50%-50% beam splitter. The squeezed vacuum state should be measured by the balanced homodyne detector[8], and the noise reduction factor is $1/(\sqrt{g}+\sqrt{g-1})^2$ when the squeezed state is generated from a NOPA of the intensity gain of g. The noise reduction is about 3dB superior to the noise reduction via twin beams measurement scheme, despite the use of same NOPA, is used in the two schemes. The difference in theoretical noise reduction limit between the two schemes can be explained by investigating the NOPA theory along with the homodyne detection theory. First, A NOPA can be regarded as two DOPA's in parallel[9]. Let us consider a non-degenerate optical parametric amplifier (NOPA) configuration shown in Fig. 2-a . The signal and idler input beams to the NOPA are obtained by linearly superimposing two incident beams on the surface of a 50%-50% beam splitter (BS1). Let $\hat{a}_+$ and $\hat{a}_-$ denote the annihilation operators for the two incident beams. The input-output properties of a linear 50%-50% beam splitter then imply that

$$\begin{pmatrix} \hat{a}_s \\ \hat{a}_i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\phi_1} \\ -e^{-i\phi_1} & 1 \end{pmatrix} \begin{pmatrix} \hat{a}_+ \\ \hat{a}_- \end{pmatrix} \quad (10)$$

where $\hat{a}_s$ and $\hat{a}_i$, respectively, are the annihilation operators for the input signal and idler beams to the NOPA and ϕ_1 is an arbitrary phase. The NOPA output beams are superimposed on a second 50%-50% beam splitter (BS2) to yield two new beams which are represented by annihilation operators $\hat{b}_+$ and $\hat{b}_-$. Once again, the input-output properties of a 50%-50% beam splitter imply that

$$\begin{pmatrix} \hat{b}_+ \\ \hat{b}_- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & e^{i\phi_2} \\ -e^{-i\phi_2} & 1 \end{pmatrix} \begin{pmatrix} \hat{b}_s \\ \hat{b}_i \end{pmatrix} \tag{11}$$

where ϕ_2 is another arbitrary phase. Substituting Eq. (10) and NOPA equations (Eq. (2)-(3)) into Eq. (11) and choosing $\phi_1 = \phi_2 = \phi$ gives

$$\hat{b}_+ = \mu e^{i\phi}\hat{a}_- + \nu\hat{a}_-^\dagger , \tag{12}$$

$$\hat{b}_- = -\mu e^{i\phi}\hat{a}_+ + \nu\hat{a}_+^\dagger . \tag{13}$$

Equations (12) and (13) imply that the configuration of the NOPA shown in Fig. 2-a , i.e., a NOPA surrounded by two linear 50%-50% beam splitters, is equivalent to two DOPA's that are in parallel. This equivalence is shown schematically in Fig. 2-b . If $\phi_1 = \phi$ and $\phi_2 = \phi_1 + \pi$ are chosen, then

$$\hat{b}_+ = \mu\hat{a} + \nu e^{i\phi}\hat{a}_+^\dagger , \tag{14}$$

$$\hat{b}_- = \mu\hat{a}_- - \nu e^{i\phi}\hat{a}_-^\dagger . \tag{15}$$

Fig. 2-c shows the equivalence for this case. In a type-II phase-matched NOPA, in which the signal and idler beams have orthogonal polarizations, the beam splitters BS1 and BS2 can be replaced by beam-splitting polarizers. The two DOPA's in this case are orthogonally polarized. Next, regarding a NOPA as two independent DOPA's with two beam splitters, it can be shown that the direct detection scheme of the twin beam measurements is equivalent to the homodyne detection of one DOPA output using the other DOPA output as a local oscillator(LO) (Fig. 3). If the NOPA is in PIA configuration, i.e. the input state to the NOPA is $|\alpha,0\rangle_{s,i}$, and the corresponding input state to the equivalent DOPA's is $|\alpha/\sqrt{2}, \alpha/\sqrt{2}\rangle_{+,-}$, then the generated outputs are two statistically independent squeezed states with the same non-zero mean field. If one of the output is considered as LO and the other as the input to the homodyne detector (HD), the LO intensity is same as that of the input to the HD and thus the noise reduction is $1/(2g-1) \approx 1/2g$, while $1/(\sqrt{g}+\sqrt{g-1})^2 \approx 1/4g$ of the noise reduction is predicted when LO intensity is much higher than the HD input intensity. The physical interpretation of this difference is as follows: In usual homodyne detection, the power of the LO is assumed to be much higher than that of the signal field to be detected. This assumption guarantees the fluctuation of the LO is negligible compared to the fluctuation of the signal so that the noise of the detected photocurrent is mostly dependent on the statistics of the signal field. In the twin beam detection scheme,

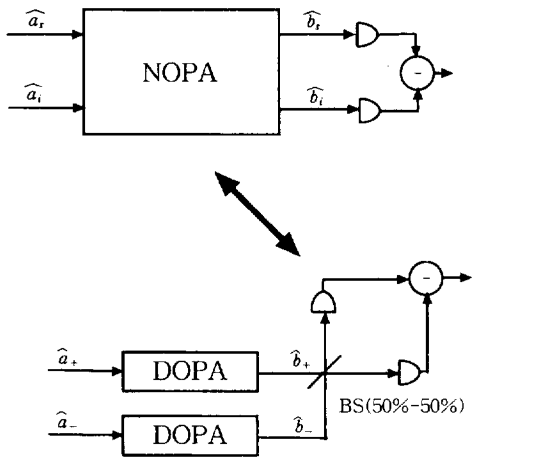

Fig 3. Interpretation of the twin beam measurement using the equivalence relation between a NOPA and two DOPA's.

however, the power of the signal and LO is same, and here the assumption for the usual homodyne detection is no longer valid; the LO fluctuation contributes to the detector noise by the same amount as the signal does, which results in the degradation of noise reduction factor by 3dB.

The investigation of the twin beams with homodyne detection theory gives us the insight that if the input states are chosen such that two outputs of the equivalent DOPA`s have different intensities, then the output of higher intensity can be regarded as an LO and the output noise is minimized as the intensity difference between the two outputs is maximized. This scheme is realized when a non-vacuum state is applied to the idler input of the NOPA. The noise minimization can be achieved if the input signal and idler beams are balanced to make the equivalent DOPA input state in $|0,\alpha/\sqrt{2}\rangle_{+,-}$. If the balanced signal and idler input is put into the NOPA, one of the two equivalent DOPA's produces a squeezed vacuum state and the output of the other DOPA, interpreted as a LO, is in a squeezed coherent state. In this case, the characteristics of the homodyne detector output current in Fig. 3 are proportional to the photon number difference $\hat{n}_T$ defined in Eq. (8) as

$$\langle\hat{n}_T\rangle = 0 , \tag{16}$$

$$\langle\Delta^2\hat{n}_T\rangle = \frac{1}{(\sqrt{g}+\sqrt{g-1})^2} , \tag{17}$$

which verifies the enhanced noise reduction factor predicted. In terms of the equivalent DOPA and homodyne detection schemes, this occurs when HD input intensity is negligible compared to the LO intensity, and the LO phase is orthogonal to the squeezed quadrature of the squeezed vacuum state output of the other equivalent DOPA, so that the squeezed quadrature is detected. As a price for the noise reduction enhancement, however, the NOPA outputs with balanced signal and idler inputs are phase sensitive, so that the output intensity and noise are dependent upon the relative phase between the two inputs. For this reason, the practical implementation of the balanced input NOPA configuration may require a phase locking mechanism to maintain the relative phase difference of the two balanced inputs for the stable operation.

In summary, we investigated the quantum noise correlation between two outputs of the non-degenerate optical parametric amplifier with generalized signal and idler inputs. By use of the balanced signal and idler inputs, the noise correlation can be enhanced by about 3dB over the theoretical limit predicted from the NOPA output with a single signal input.

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