論文97-34S-1-8

동등하중함수를 이용한 μ-최적제어기 설계

(μ-optimal controller design using equivalent weighting function)

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요 약

본 논문에서는 동등하중함수를 이용한 새로운 μ-제어기 설계방법을 제안한다. 제안한 방법은 D-K와 μ-K 반복법과 마찬가지로 최소값의 수렴성은 보장하지 못하지만 구조적 불확실성의 견실성능문제를 동등한 비구조적 불확실성의 H^∞ 최적문제로 전환하여 제어기를 설계한다. 또한 반복적으로 최대특이치와 μ값과의 차이로 나타나는 오차지수 d_ε을 이용하여 μ-최적제어기를 찾을 수 있으며 같은 차수의 스켈링함수를 사용했을 경우 D-K 반복법보다 낮은 차수의 μ-최적제어기를 설계할 수 있다.

Abstract

In this paper, we propose a new μ-controller design method using an equivalent weighting function $W_\mu(s)$. The proposed method is not guaranteed to converge to the minimum as D-K and μ-K iteration method. However, the robust performance problem can be converted into an equivalent H^∞ optimization problem of unstructured uncertainty by using an equivalent weighting function $W_\mu(s)$. Also we can find a μ-optimal controller iteratively using an error index d_ε of difference between maximum singular value and μ-norm. And under the condition of the same order of scaling functions, the proposed method provides the μ-optimal controller with the degree less than that obtained by D-K iteration.

I. Introduction

Maintaining stability in the presence of uncertainty has long been recognized as the crucial requirement for a closed loop feedback system. Classical designers developed the concepts of gain and phase margin to quantify stability-robustness measure. In the modern control era, criteria for maintaining closed loop stability in the presence of a single unstructured uncertainty have been formulated in terms of a singular value frequency domain inequality on the closed loop transfer function[1].

Recently, the issue of multiple modeling uncertainties appearing at different locations in the feedback loop, and the related requirement of performance-robustness, has been addressed[2,3]. Multiple unstructured uncertainty blocks and performance specifications give rise to so-called structured uncertainty. A new analysis framework, based on the structured singular value μ, has been proposed by Doyle[4] to assess the stability and performance robustness of linear time invariant feedback systems in the presence of structured uncertainty. The design of a feedback

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接受日字: 1996年10月24日, 수정완료일: 1997年1月7日

system that exhibits closed loop stability and performance in the face of structured uncertainty is the so-called μ-synthesis problem. The synthesis approach proposed by Doyle is an iterative scheme, referred to as D-K iteration, that involves a sequence of scaled H^∞ based feedback design problems.

Another approach, μ-K iteration is also used in μ-controller design. This iteration scheme, proposed by Lin[5], is motivated from the observation that the μ-optimal controller tends to flatten the μ-curve at least over the bandwidth. And this method is to determine a sequence of controllers which yields a flat structured singular value. This after all is what happens in H^∞ optimization where the H^∞-optimal controller results in a cost function with a flat maximum singular value. However, the result of a μ-optimal curve plot does not always guarantee flat shape.

In this paper, we propose a new μ-controller design method using an equivalent weighting function $W_\mu(s)$. The proposed method does not give an analytic solution, therefore we design a controller using iterative schemes as D-K iteration and μ-K iteration. However, the robust performance problem can be converted into an equivalent H^∞ optimization problem by using a proposed method. Also we can find an μ-optimal controller using an error index d_ε of difference between maximum singular value and μ-norm.

The organization of this paper is as follows: Section II discusses the analysis of μ-synthesis and D-K iteration and section III gives a main results of the proposed method. In section IV, for convenience of comparing with D-K and μ-K iteration, we take the SISO case design example and get good results. Finally, conclusions are given in section V.

II. Preliminaries

The block diagram in Figure 1 is the standard framework for considering the robust feedback design problem. The diagram represents any linear interconnection of inputs, outputs, perturbations, and a compensator. G is the known model that contains the plant to be controlled and K is the compensator to be designed.

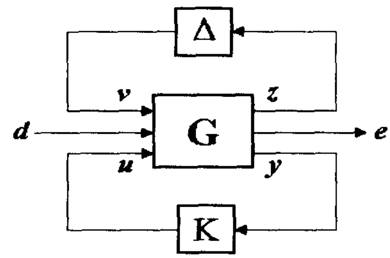


그림 1. 일반적인 견실궤환 설계문제에 대한 구조
Fig. 1. General framework for the robust feedback design problem.

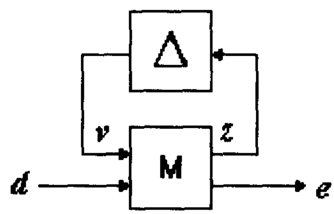


그림 2. 분석된 블럭 선도
Fig. 2. Analyzed block diagram.

The synthesis objective is to find a K to achieve nominal stability and performance of the feedback loop and to provide robustness with respect to the modeling error. In Figure 2, M represents the lower linear fractional transformation of G closed by K,

$$M = F_l(G, K) = [G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}]. \tag{1}$$

Δ is an $n \times n$ block diagonal matrix of perturbations representing uncertainty except for one block which is used to characterize performance. Mathematically Δ is an element of the set

$$\Delta := \{ \mathrm{diag}[\delta_1 I_{r_1}, \cdots, \delta_s I_{r_s}, \Delta_1, \cdots, \Delta_f], \delta_i \in C, \Delta_j \in C^{m_j \times m_j} \} \tag{2}$$

where s is the number of repeated scalar blocks, f is the number of full blocks, $\mathbb{C}$ is the set of complex and

$$\sum_{i=1}^{s} r_i + \sum_{j=1}^{f} m_j = n \; . \tag{3}$$

Definition 1 [2].

The structured singular value of M, $\mu_\Delta(M)$, is defined such that $\mu_\Delta^{-1}(M)$ is equal to the smallest $\bar{\sigma}(\Delta)$ needed to make $(I+M\Delta)$ singular, i.e.,

$$\mu_\Delta^{-1}(M) = \min_{\Delta \in B\Delta} \{\bar{\sigma}(\Delta) \mid \det(I+M\Delta)=0\}, \tag{4}$$

where $B\Delta$ is a norm bounded subsets of Δ, defined as

$$B\Delta = \{\Delta \in \Delta : \bar{\sigma}(\Delta) \le 1\}. \tag{5}$$

If no $\Delta \in B\Delta$ exists such that $\det(I+M\Delta)=0$, then $\mu_\Delta(M)=0$.

Obviously, μ is a function of M which also depends on the structure of Δ. However, definition 1 is not typically useful in computing μ since the implied by it does not appear to be easily solvable. Fortunately, several properties of μ can be proven which make it a powerful tool for applications. For better understanding, some important properties of μ are described here without proofs.

Lemma 1 [2].

(1) $$\mu_\Delta(kM) = |k|\mu_\Delta(M) \tag{6}$$

(2) $$\rho(M) \le \mu_\Delta(M) \le \bar{\sigma}(M) \tag{7}$$

(3) $$\max_{U \in U} \rho(UM) \le \mu_\Delta(M) \le \inf_{D \in D} \bar{\sigma}(DMD^{-1}) \tag{8}$$

where $U = \{U \in \Delta : U^* U = I_n\}$
$D = \{\text{diag}[D_1, \cdots, D_s, d_1 I_{m_1}, \cdots, d_f I_{m_f}] : D_i \in \mathbb{C}^{r_i \times r_i}, D_i = D_i^* > 0, d_j \in \mathbb{R}, d_j > 0\}$ and ρ is a spectral radius, $\bar{\sigma}$ is a maximum singular value, and $\mathbb{R}$ is the set of real numbers.

The μ-synthesis problem is to find controllers K which stabilize the general plant G and minimizes the worst case value of $\mu_\Delta(F_l(G,K)(j\omega))$, i.e.,

$$\inf_{K} \sup_{\omega \in \mathbb{R}} \mu_\Delta(F_l(G,K)(j\omega)). \tag{9}$$

Unfortunately, due to the inherent non-convexity of the problem, the complete solutions to this problem are still not available. However D-K iteration method provides itself with an efficient way of synthesizing robust performance controller. This numerical approach, based on the upper bound of μ in (8), is proposed by Doyle, which replaced (9) with the following two-parameter optimization problem.

$$\inf_{K} \sup_{\omega \in \mathbb{R}} \inf_{D \in D} \bar{\sigma}(D(j\omega)(F_l(G,K)(j\omega))D^{-1}(j\omega)) \tag{10}$$

One method to solve the above problem is to alternatively minimize the norm $\|DF_l(G,K)D^{-1}\|_\infty$ for either K or D while holding the other constant. For fixed D, this reduces to an H^∞ optimization problem and can be solved by various methods. When K is fixed, the optimal scaling D can be derived since the function is convex in D. A curve fitting routine can be used to construct a stable, minimum phase rational D when the scalings at each frequency are available.

Theorem 1 [4]. (Robust performance condition)

The robust performance is achieved if and only if $\mu_\Delta(M)$ with respect to the structured uncertainty $\Delta \in B\Delta$, satisfies

$$\mu_\Delta(M) < 1. \tag{11}$$

Theorem 1 may be interpreted as a 'generalized small gain theorem' which also takes into account the structure of Δ.

Ⅲ. Main results

The proposed method is originated from the following observations. For any non-singular constant matrix $N \in \mathbb{C}^{n \times n}$, with a structured uncertainty, one can always find a corresponding scaling factor $k>0$, such that

$$\mu_\Delta(N) = k\bar{\sigma}(N) = \mu_{\Delta_S}(kN) \qquad (12)$$

where k can be chosen as $k = \mu_\Delta(N)/\mu_{\Delta_S}(N)$, $\Delta_S \in \Delta$ is a full block uncertainty with same dimensions of Δ. The above equation implies the facts that; for any non -singular constant matrix N, we can find an equivalent unstructured uncertainty $\Delta_E = k\Delta_S$, such that

$$\mu_\Delta(N) = \mu_{\Delta_E}(N). \qquad (13)$$

Equation (13) can be also be applied to any H^∞ norm bounded transfer function matrix $M(s)$ by proper choice of an equivalent weighting function $W_\mu(s)$ to derive the following results,

$$\mu_\Delta(M(j\omega_0)) = \mu_{\Delta_E}(M(j\omega_0)) \qquad (14)$$

where we have used the $N = M(j\omega_0)$ and $\Delta_E = |W_\mu(j\omega_0)|\Delta_S$. Therefore, for any norm bounded transfer function $M(s) \in RH^\infty$, we can find a stable minimum-phase real scalar rational transfer function $W_\mu(s) \in RH^\infty$, such that

$$\mu_\Delta(M(j\omega)) = |W_\mu(j\omega)|\bar{\sigma}(M(j\omega)) = \bar{\sigma}(W_\mu(j\omega)M(j\omega)) \quad (15)$$

holds for every frequency points of interests. $W_\mu(s)$ can be found by fitting the magnitude curve of

$$|W_\mu(j\omega)| = \frac{\mu_\Delta(M(j\omega))}{\bar{\sigma}(M(j\omega))} \qquad (16)$$

by a stable, minimum-phase, real rational transfer function. Assume that K_{opt} represents the optimal controller of μ-synthesis problem, then by the above discussions, we can find an equivalent weighting function $W_{\mu opt}(G, K_{opt})$, such that

$$|W_{\mu opt}(G, K_{opt})(j\omega)| = \frac{\mu_\Delta(F_l(G, K_{opt}))(j\omega)}{\bar{\sigma}(F_l(G, K_{opt}))(j\omega)} \qquad (17)$$

or equivalently

$$\mu_\Delta(F_l(G, K_{opt}))(j\omega) = \bar{\sigma}(W_{\mu opt}(G, K_{opt})F_l(G, K_{opt}))(j\omega) \quad (18)$$

at every frequency points of interests. According to the equation (15), we can rewrite equation (9) as

$$\inf_K \sup_{\omega\in R} \mu_\Delta(F_l(G, K))(j\omega) = \inf_K \sup_{\omega\in R} \bar{\sigma}(W_\mu F_l(G, K))(j\omega) \qquad (19)$$

The equivalent weighting function $W_\mu(s)$ depends on the general plant G and the controller K. The exact solution of equation (19) cannot be obtained analytically, so we propose an iteration method using an error index with a wide enough range $\omega_1 \sim \omega_2$ as follows.

$$d_\varepsilon = \int_{\omega_1}^{\omega_2} |\bar{\sigma}(M(j\omega)) - \mu_\Delta(M(j\omega))| d\omega \qquad (20)$$

In many examples, it can be observed that the optimal $\mu_\Delta(M)$ curve tends converging to the optimal $\bar{\sigma}(M)$ curve at interested frequency range. Therefore, difference between $\bar{\sigma}(M(j\omega))$ and $\mu_\Delta(M(j\omega))$ are minimized in the optimization procedure. If d_ε does not decrease, then designed μ-controller is optimal, equivalently $|W_\mu(j\omega)| \approx 1$. In applying to proposed method, we can finish the iteration from error index d_ε.

Based on the above discussion, we propose the following iteration method.

Step 1. Design a stabilizing H^∞ optimal controller.

$$K_1 = \inf_K \| F_l(G, K)\|_\infty \qquad (21)$$

Step 2. Find a μ-curve over a wide enough range ($\omega_1 \sim \omega_2$).

$$\mu_1(j\omega) = \mu_\Delta(F_l(G, K_1))(j\omega) \qquad (22)$$

Step 3. Find a maximum singular value curve over the same range.

$$\gamma_1(j\omega) = \bar{\sigma}(F_l(G, K_1))(j\omega) \qquad (23)$$

Step 4. Calculate the difference between $\gamma_1(j\omega)$ and $\mu_1(j\omega)$.

$$d_{\varepsilon 1} = \int_{\omega_1}^{\omega_2} |\bar{\sigma}(F_l(G, K_1))(j\omega) - \mu_\Delta(F_l(G, K_1))(j\omega)| d\omega \quad (24)$$

Step 5. Compute an approximate $|W_{\mu 1}(j\omega)|$ curve.

$$|W_{\mu 1}(j\omega)| = \frac{\mu_1(j\omega)}{\gamma_1(j\omega)} \qquad (25)$$

Step 6. Find a stable minimum-phase scalar rational function $W_{\mu 1}(s)$ by step 5.

Step 7. Design a stabilizing H^∞optimal controller.

$$K_2 = \inf_K \| W_{\mu 1}(s) F_l(G, K) \|_\infty \quad (26)$$

Step 8. Continue doing step 2~6 to n-th iteration as follows.

$$K_i = \inf_K \| W_{\mu i-1} W_{\mu i-2} \cdots W_{\mu 2} W_{\mu 1} F_l(G, K) \|_\infty \quad (27)$$

$$\mu_i(j\omega) = \mu_\Delta(F_l(G, K_i))(j\omega) \quad (28)$$

$$\gamma_i(j\omega) = \bar{\sigma}(W_{\mu i-1} W_{\mu i-2} \cdots W_{\mu 2} W_{\mu 1} F_l(G, K_i))(j\omega) \quad (29)$$

$$d_{\varepsilon i} = \int_{\omega_1}^{\omega_2} | \gamma_i(j\omega) - \mu_i(j\omega) | dw \quad (30)$$

Step 9. If $d_{\varepsilon i}$ does not decrease, then stop the iteration.

The convergence of this method relies on the accuracy of curve fitting $W_\mu(s)$ and on the calculation of structured singular values for wide enough range frequency.

Ⅳ. Example

This section presents a simple design example to illustrate the proposed method. To simplify simulation, we take the single input single output model from reference [5].

$$A = \begin{bmatrix} -2.5 & -1 \\ 1 & 0 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ C = [-0.5 \quad -0.5],\ D = [0] \quad (31)$$

The objective of this design problem is to synthesize controller such that the output disturbance rejection requirements is satisfied in the presence of input multiplicative uncertainty (Fig. 3), by which means, we have to solve robust performance problem.

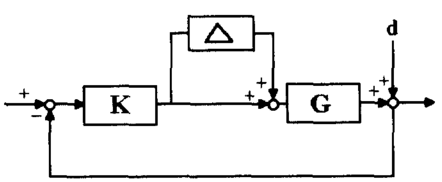
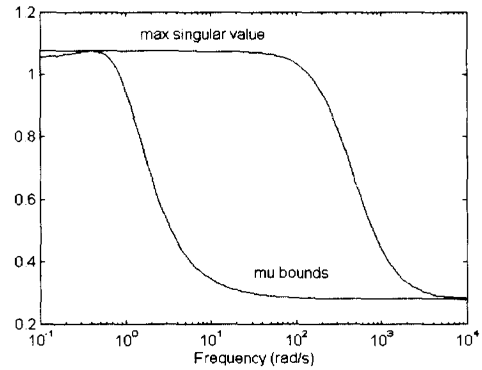
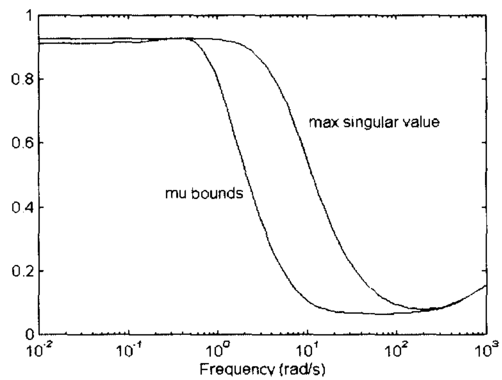


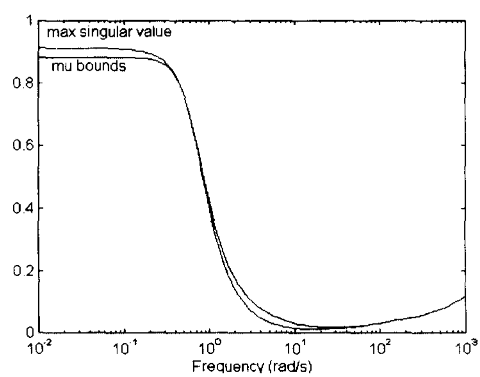
그림 3. 단일 입·출력 견실성능문제
Fig. 3. A SISO robust performance problem.



(a) 반복 전
(a) Before iteration

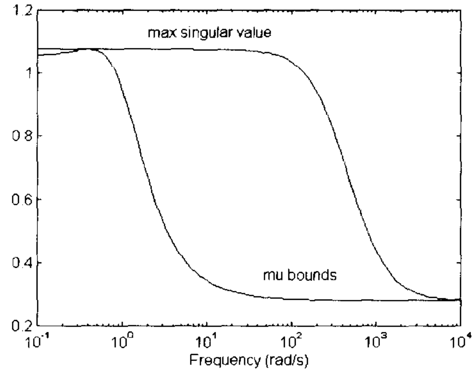


(b) 1회 반복 후
(b) After first iteration

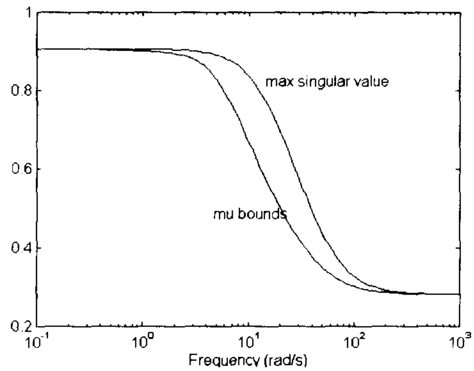


(c) 2회 반복 후
(c) After second iteration

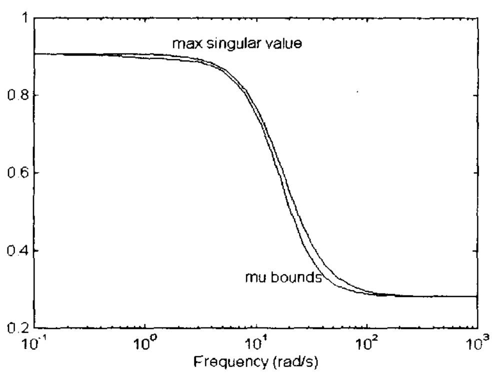
그림 4. 제안한 방법의 최대특이치와 μ-plot
Fig. 4. Maximum singular value and μ-plot of proposed method.



(a) 반복 전
(a) Before iteration

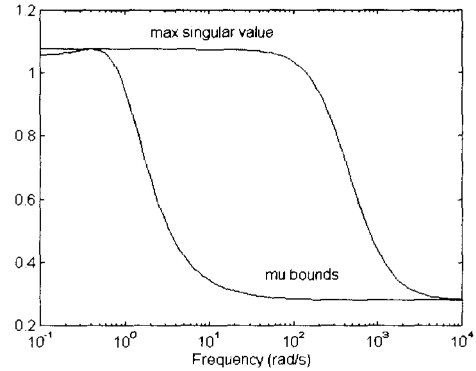


(b) 1회 반복 후
(b) After first iteration

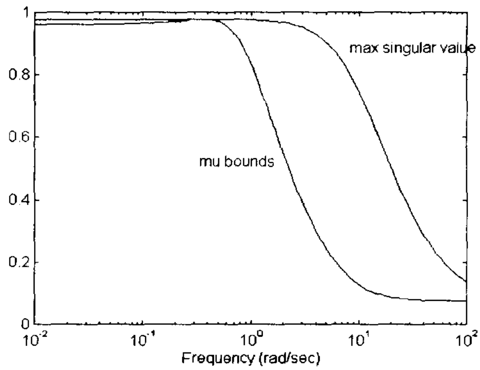


(c) 2회 반복 후
(c) After second iteration

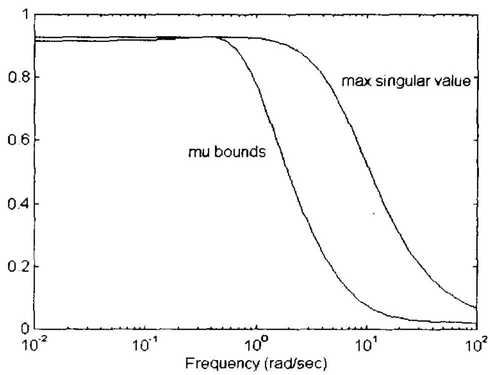
그림 5. D-K 반복법의 최대특이치와 μ-plot
Fig. 5. Maximum singular value and μ-plot of D-K iteration.



(a) 반복 전
(a) Before iteration



(b) 1회 반복 후
(b) After first iteration



(c) 2회 반복 후
(c) After second iteration

그림 6. μ-K 반복법의 최대특이치와 μ-plot
Fig. 6. Maximum singular value and μ-plot of μ-K iteration.

The simulation is implemented by Matlab and μ-Toolbox[6]. The proposed method was carried out as described in section III, and the simulation results are shown in Figure 4. Figure 5 and Figure 6 represent a maximum singular value and μ-plot in results using D-K iteration and μ-K iteration, respectively.

From Fig. 4, Fig. 5, and Fig. 6, we know that the closed loop system satisfies robust performance condition, therefore three methods give good results in μ-synthesis of robust performance problem. Of course the proposed method is less systematic than D-K iteration due to the accuracy of curve fitting $W_\mu(s)$. However, if we find an accurate $W_\mu(s)$ in a single step, we can design a μ-optimal controller without iteration. Also, under the condition of the same order of scaling functions, the proposed method provides the optimal controller with the degree less than that obtained by D-K iteration. Conclusively, we can transform a μ-optimal controller design problem into an H^∞-optimal controller design problem by using equivalent weighting function $W_\mu(s)$, and the effectiveness of this fact shows in error index d_ε.

V. Conclusions

This paper proposes a new method of robust performance problem in feedback systems with a structured uncertainty. For this, we motivate the use of the equivalent weighting function $W_\mu(s)$ using the ratio of $\frac{\mu_\Delta(M(j\omega))}{\bar{\sigma}(M(j\omega))}$, and transform the robust performance problem into the equivalent H^∞ optimization problem. To find a μ-optimal controller, we use an error index d_ε in the procedure of iteration. The accuracy of proposed method depends on curve fitting of the equivalent weighting function $W_\mu(s)$.

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