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A CHARACTERIZATION OF SOME REAL HYPERSURFACES IN A COMPLEX HYPERBOLIC SPACE

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0. Introduction

We denote by $M_n(c)$ a complete and simply connected complex n-dimensional Kählerian manifold of constant holomorphic sectional curvature $4c$, which is called a *complex space form*. Such an $M_n(c)$ is bi-holomorphically isometric to a complex projective space $P_n\mathbb{C}$, a complex Euclidean space $\mathbb{C}^n$ or a complex hyperbolic space $H_n\mathbb{C}$, according as $c > 0$, $c = 0$ or $c < 0$.

In this paper, we consider a real hypersurface M in $M_n(c)$. Typical examples of M in $P_n\mathbb{C}$ are the six model spaces of type A_1, A_2, B, C, D and E , and the ones of M in $H_n\mathbb{C}$ are the four model spaces of type A_0, A_1, A_2 and B (cf. Theorem A in §1), which are all given as orbits under certain Lie subgroups of the group consisting of all isometries of $P_n\mathbb{C}$ or $H_n\mathbb{C}$. Denote by (ϕ, ξ, η, g) the *almost contact metric structure* of M induced from the almost complex structure of $M_n(c)$, and by A the shape operator of M. The structure vector ξ is said to be *principal* if $A\xi = \alpha\xi$, where $\alpha = \eta(A\xi)$. Many differential geometers have studied M from various points of view. Berndt [1] and Takagi [14] investigated the homogeneity of M. According to Takagi's classification theorem and Berndt's one, the principal curvatures and their multiplicities of a homogeneous real hypersurface in $M_n(c)$ are given. Moreover, it is very interesting to give a characterization of homogeneous real hypersurfaces of $M_n(c)$. Let $\mathcal{L}_\xi$ be the Lie derivative in the direction of ξ. Then Okumura [13] and Montiel-Romero [12] proved the fact in $P_n\mathbb{C}$ and $H_n\mathbb{C}$, respectively that M is locally congruent to one of homogeneous ones of type A if and only if ξ is an infinitesimal isometry, that is, $\mathcal{L}_\xi g = 0$, where *type* A means type A_1 or A_2 in $P_n\mathbb{C}$ and type A_0, A_1

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or A_2 in $H_n\mathbb{C}$. Motivated by these results, Maeda-Udagawa [11] studied the condition "$\mathcal{L}_\xi \phi = 0$" and Ki-Kim-Lee [3] investigated the condition "$\mathcal{L}_\xi A = 0$". Recently, Kimura and Maeda [10] completly classified M in $P_n\mathbb{C}$ satisfying $\mathcal{L}_\xi S = 0$, where S denotes the Ricci tensor of M.

The purpose of the present paper is to investigate M of $H_n\mathbb{C}$ which satisfies $\mathcal{L}_\xi S = 0$ under the condition that $A\xi$ is principal.

1. Preliminaries

We begin with recalling the basic properties of real hypersurfaces of a complex space form. Let N be a unit normal vector field on a neighborhood of a point p in M and J the almost complex structure of $M_n(c)$. For a local vector field X on a neighborhood of p, the images of X and N under the transformation J can be represented as

$$JX = \phi X + \eta(X)N \quad , \quad JN = -\xi,$$

where ϕ defines a skew-symmetric transformation on the tangent bundle TM of M, while η and ξ denote a 1-form and a vector field on the neighborhood of p , respectively. Moreover, it is seen that $g(\xi, X) = \eta(X)$, where g denotes the induced Riemannian metric on M. By the properties of the almost complex structure J, the set (ϕ, ξ, η, g) of tensors satisfies

$$\phi^2 = -I + \eta \otimes \xi, \quad \phi\xi = 0, \quad \eta(\phi X) = 0, \quad \eta(\xi) = 1, \tag{1.1}$$

where I denotes the identity transformation. Accordingly, this set (ϕ, ξ, η, g) defines the *almost contact metric structure* on M. Furthermore, the covariant derivatives of the structure tensors are given by

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi, \tag{1.2}$$

$$\nabla_X \xi = \phi AX, \tag{1.3}$$

where ∇ is the Riemannian connection of g. Since the ambient space is of constant holomorphic sectional curvature $4c$, the equations of Gauss and Codazzi are respectively given as follows :

$$\begin{aligned} R(X,Y)Z = c\{&g(Y,Z)X - g(X,Z)Y + g(\phi Y, Z)\phi X \\ &- g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z\} \\ &+ g(AY,Z)AX - g(AX,Z)AY \end{aligned} \tag{1.4}$$

$$(1.5)\qquad \begin{aligned}&(\nabla_X A)Y - (\nabla_Y A)X\\ &= c\{\eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi\},\end{aligned}$$

where R denotes the Riemannian curvature tensor of M. The Ricci tensor S' of M is the tensor of type $(0,2)$ given by $S'(X,Y) = tr\{Z \to R(Z,X)Y\}$. But it may be also regarded as a tensor of type $(1,1)$ and denoted by $S : TM \to TM$; it satisfies $S'(X,Y) = g(SX,Y)$. From the Gauss equation and (1.1), the Ricci tensor S is given by

$$(1.6)\qquad S = c\{(2n+1)I - 3\eta \otimes \xi\} + hA - A^2,$$

where h is the trace of A. Moreover, using (1.3), we get

$$(1.7)\qquad \begin{aligned}(\nabla_X S)Y = &-3c\{g(\phi AX, Y)\xi + \eta(Y)\phi AX\} + (Xh)AY\\ &+ (Xh)AY + (hI - A)(\nabla_X A)Y - (\nabla_X A)AY.\end{aligned}$$

Now we quote the following in order to prove our results.

THEOREM A [1]. *Let M be a real hypersurface of $H_n\mathbb{C}$. Then M has constant principal curvatures and ξ is principal if and only if M is locally congruent to one of the following:*

A_0. *a horosphere in $H_n\mathbb{C}$,*

A_1. *a geodesic hypersphere $H_0\mathbb{C}$ or a tube over a hyperplane $H_{n-1}\mathbb{C}$,*

A_2. *a tube over a totally geodesic $H_k\mathbb{C}$ $(1 \le k \le n-2)$,*

B. *a tube over a totally real hyperbolic space $H_n\mathbb{R}$.*

THEOREM B [4]. Let M be a real hypersurface of $H_n\mathbb{C}(n \ge 3)$. *If ξ is principal and M satisfies $\mathcal{L}_\xi S = 0$, then M is locally congruent to type A.*

2. Real hypersurfaces in $M_n(c)$ satisfying $\mathcal{L}_\xi S = 0$

We denote by $M_n(c)$ a complex space form with the metric of constant holomorphic sectional curvature $4c$ and M a real hypersurface in $M_n(c), c \neq 0$. In this section, we suppose that the Ricci tensor S

satisfies the condition $\mathcal{L}_\xi S = 0$. The following discussion in the case where $c > 0$ is indebted to Kimura and maeda [10]:

From (1.3), for any $X \in TM$ we have

$$\begin{aligned}(\mathcal{L}_\xi S)X &= [\xi, SX] - S[\xi, X] \\ &= (\nabla_\xi S)X - \nabla_{SX}\xi + S\nabla_X \xi \\ &= (\nabla_\xi S)X - \phi ASX + S\phi AX.\end{aligned}$$

Then we see that "$\mathcal{L}_\xi S = 0$" is equivalent to

$$\nabla_\xi S = \phi AS - S\phi A. \tag{2.1}$$

Since $g((\nabla_S)X, Y) = g((\nabla_S)Y, X)$ for any $X, Y \in TM$, the equation (2.1) shows

$$(\phi A - A\phi)S = S(\phi A - A\phi). \tag{2.2}$$

From (1.6) it follows that

$$\phi S - S\phi = h(\phi A - A\phi) - (\phi A^2 - A^2\phi). \tag{2.3}$$

Here we hope to calculate $\|\phi S - S\phi\|^2$, which is equivalent to $tr(\phi S - S\phi)^2$ because $\phi S - S\phi$ is symmetric. From (2.3), we get

$$\begin{aligned}tr(\phi S - S\phi)^2 &= htr(\phi A - A\phi)(\phi S - S\phi) \\ &\quad - tr(\phi A^2 - A^2\phi)(\phi S - S\phi).\end{aligned} \tag{2.4}$$

In general, we get

$$tr(\phi A - A\phi)(\phi S - S\phi) = 2tr\phi A\phi S - trA\phi^2 S - tr\phi AS\phi. \tag{2.5}$$

Taking the trace of (2.2), we find

$$tr\phi^2 AS - 2tr\phi S\phi A + tr\phi^2 SA = 0. \tag{2.6}$$

Combining (2.5) with (2.6), we obtain

$$tr(\phi A - A\phi)(\phi S - S\phi) = 0. \tag{2.7}$$

On the other hand, we find

$$(2.8)\quad tr(\phi A^2 - A^2\phi)(\phi S - S\phi) = 2tr\phi A^2\phi S - trA^2\phi^2 S - tr\phi A^2 S\phi.$$

From (2.2) it follows that

$$\phi A\{(\phi A - A\phi)S - S(\phi A - A\phi)\} = 0,$$

which implies

$$(2.9)\qquad tr\phi ASA\phi = tr\phi A^2\phi S.$$

Then combining (2.8) with (2.9) we have

$$(2.10)\quad tr(\phi A^2 - A^2\phi)(\phi S - S\phi) = 2tr\phi^2 ASA - tr\phi^2 SA^2 - tr\phi^2 A^2 S.$$

Thus substituting (2.7) and (2.10) into (2.4) and using (1.1) and (1.6), we can see that

$$(2.11)\qquad tr(\phi S - S\phi)^2 = -\frac{3}{2}c(\beta - \alpha^2),$$

where we have put $\beta = \eta(A^2\xi)$ and $\alpha = \eta(A\xi)$. Taking account of (1.1), we find

$$(2.12)\qquad \|\phi A\xi\|^2 = \beta - \alpha^2.$$

Hence from (2.11) and (2.12), we have

$$tr(\phi S - S\phi)^2 = -\frac{3}{2}c\|\phi A\xi\|^2$$

or

$$\|\phi S - S\phi\|^2 + \frac{3}{2}c\|\phi A\xi\|^2 = 0.$$

Consequently, the condition "$\mathcal{L}_\xi S = 0$" implies the fact that $\phi S = S\phi$ and ξ is principal in the case where c > 0 and that $\phi S = S\phi$ if and only if ξ is principal in the case where $c < 0$. Here we note that Kimura and Maeda [10] proved a local classification theorem for real hypersurfaces in $P_n\mathbb{C}$ which satisfy $\mathcal{L}_\xi S = 0$. Thus because of Theorem B, it is seems to be interested to consider real hypersurfaces in $H_n\mathbb{C}$ satisfying $\mathcal{L}_\xi S = 0$ under the weaker condition than one that ξ is principal.

3. Real hypersurfaces in $H_n\mathbb{C}$ satisfying $\mathcal{L}_\xi S = 0$

Let M be a real hypersurface in a complex hyperbolic space $H_n\mathbb{C}$ endowed with the Bergmann metric of constant holomorphic sectional curvature -4. In this section, we assume that M satisfies $\mathcal{L}_\xi S = 0$ and $A\xi$ is principal. The second assumption means

$$A^2\xi = \lambda A\xi, \tag{3.1}$$

where $\lambda = \eta(A^3\xi)$. For simplicity we put $U = \nabla_\xi \xi$. Then we have $U = \phi A\xi$, which together with (1.1) implies

$$\phi U = -A\xi + \alpha\xi \tag{3.2}$$

and so $g(\phi U, \xi) = 0$. Thus we define ϕU by $\phi U = -\mu W$, where W is a unit vector field orthogonal to ξ and μ is a smooth function on M. Namely, we have

$$A\xi = \alpha\xi + \mu W. \tag{3.3}$$

Here we note that this and $U = \mu\phi W$ give $g(U, W) = 0$. Moreover, it follows from (1.6) and (3.1) that

$$S\xi = -2(n-1)\xi + (h-\lambda)A\xi, \tag{3.4}$$

$$SU = -(2n+1)U + hAU - A^2U. \tag{3.5}$$

From (3.1) and (3.3) we find

$$AW = \gamma A\xi, \tag{3.6}$$

where $\gamma\mu = \lambda - \alpha$. Thus (1.6) combined with (3.1) and (3.6) gives us

$$SW = -(2n+1)W + \gamma(h-\lambda)A\xi. \tag{3.7}$$

From (2.2), we find

$$(\phi A - A\phi)S\xi = S(\phi A - A\phi)\xi,$$

which, together with (1.1), (3.4), (3.5) and the definition of U, yields

$$A^2U = (2h - \lambda)AU + (\lambda^2 - \lambda h - 3)U. \tag{3.8}$$

Also, from (2.2) we get

$$(\phi A - A\phi)SW = S(\phi A - A\phi)W,$$

which, together with (1.1), (1.6), (3.5) $\sim$ (3.8) and the definition of W, leads to

$$\{2(\lambda - h)^2 - 3\}AU = \{\lambda(\lambda - h)^2 + 3(h - 2\lambda + \alpha)\}U. \tag{3.9}$$

On the other hand, differentiating (3.2) covariantly in the direction of X and making use of (1.1), (1.2) and (1.3), we obtain

$$g(AX, U)\xi - \phi(\nabla_X A)\xi + A\phi AX - d\alpha(X)\xi - \alpha\phi AX.$$

Taking the inner product of this and ξ and using (1.1) and (1.3), we have

$$g((\nabla_X A)\xi, \xi) = 2g(AU, X) + d\alpha(X). \tag{3.10}$$

Moreover, differentiating (3.1) covariantly in the direction of X, we get

$$\begin{aligned}(\nabla_X A)A\xi + A(\nabla_X A)\xi + A^2\phi AX \\ = d\lambda(X)A\xi + \lambda(\nabla_X A)\xi + \lambda A\phi AX.\end{aligned} \tag{3.11}$$

If we take the inner product of this and ξ and make use of (3.1), (3.10) and the fact that $g((\nabla_X A)\xi, Y) = g((\nabla_X A)Y, \xi)$ for any $X, Y \in TM$, then we find

$$g((\nabla_X A)\xi, A\xi) = \frac{1}{2}d(\lambda\alpha)(X) + \lambda g(AU, X). \tag{3.12}$$

From (3.11), replacing X by ξ and taking the inner product of this result and ξ, we have

$$\begin{aligned}\frac{1}{2}d(\lambda\alpha)(X) + g(U, X) + 3g(A^2U, X) + d\alpha(AX) \\ = d\lambda(\xi)g(A\xi, X) + 2\lambda g(AU, X) + \lambda d\alpha(X),\end{aligned} \tag{3.13}$$

where we have used (1.5), (3.10) and (3.12).

Let M_0 be the set of consisting of points x in M such that $(\lambda - h)(x) = 0$. On the subset M_0, from (3.9) it is seen that

$$AU = (\lambda - \alpha)U \quad \text{on} \quad M_0. \tag{3.14}$$

Then, by using (3.14) the equation (3.8) turns out to be $\{\alpha(\lambda - \alpha) - 3\}U = 0$ on M_0.

LEMMA 3.1. *Let M be a real hypersurface of $H_n\mathbb{C}$. Assume that it satisfies $\mathcal{L}_\xi S = 0$ and $A\xi$ is principal. If $U \neq 0$, then $Int(M_0) = \emptyset$, where $Int(M_0)$ denotes the interior of $M_0 = \{x \in M \mid (\lambda - h)(x) = 0\}$.*

Proof. We assume that the interior of M_0 is not empty. Since we have supposed that $U \neq 0$, from the above equation it follows that $\alpha(\lambda - \alpha) = 3$. By means of (3.1), it is clear that $\beta = \lambda\alpha$, which together with (2.12) gives us $g(U,U) = 3$. Thus, using (3.8) and (3.14), the equation (3.13) is reformed as

$$\alpha d\lambda(\xi)g(A\xi, X) = (3\lambda - 8\alpha)g(U,X) - 3d\alpha(X) + \alpha d\alpha(\dot{A}X).$$

Replacing X by U into this equation and making use of (3.14), we obtain $3(3\lambda - 8\alpha) = 0$, which yields $3\lambda = 8\alpha$. Thus we can see that $\alpha = 3/\sqrt{5}$, $\lambda = 8/\sqrt{5}$ and $\mu = \sqrt{3}$.

On the other hand, since $g((\phi S - S\phi)U, W) = -g(SU, \phi W) - g(SW, \phi U)$, using (3.5), (3.7) and (3.8), we get $g((\phi S - S\phi)U, W) = 3g(\phi U, W)$, which together with (3.2) implies $g((\phi S - S\phi)U, W) = -3\sqrt{3}$. Then it is clear that $\|\phi S - S\phi + \sqrt{3}(W \otimes U + U \otimes W)\|^2 = 0$, where we have used (2.11) and (2.12). Thus we can see that $\phi S - S\phi = -\sqrt{3}(W \otimes U + U \otimes W)$. Since $\phi U = -\sqrt{3}W$, we obtain $\phi S - S\phi = \phi U \otimes U + U \otimes \phi U$. Combining this with (2.3), we have

$$\lambda\phi A - \phi A^2 - \lambda A\phi + A^2\phi = \phi U \otimes U + U \otimes \phi U, \tag{3.15}$$

which, together with (3.1), (3.2) and (3.14), shows that

$$\begin{aligned} &\lambda\phi A^2 - \phi A^3 - \lambda A\phi A + A^2\phi A \\ &= (\alpha - \lambda)\{A\xi \otimes U + U \otimes A\xi\} + 3U \otimes \xi. \end{aligned} \tag{3.16}$$

Substituting X by ξ into (1.7), we get

$$\begin{aligned} (\nabla_\xi S)X =& 3g(U,X)\xi + 3\eta(X)U + \lambda(\nabla_\xi A)X \\ &- A(\nabla_\xi A)X - (\nabla_\xi A)AX, \end{aligned}$$

which, together with (1.6) and (2.1), leads to

$$\begin{aligned} &\lambda\phi A^2X - \phi A^3X - \lambda A\phi AX + A^2\phi AX \\ &= 3g(U,X)\xi + \lambda(\nabla_\xi A)X - A(\nabla_\xi A)X - (\nabla_\xi A)AX. \end{aligned} \tag{3.17}$$

From (3.16) and (3.17), it is seen that

$$
\begin{aligned}
&(\alpha-\lambda)\{A\xi\otimes U+U\otimes A\xi\}(X)+3U\otimes\xi(X)\\
&=3g(U,X)\xi+\lambda(\nabla_\xi A)X-A(\nabla_\xi A)X-(\nabla_\xi A)AX.
\end{aligned}
\tag{3.18}
$$

By using the Codazzi equation (1.5), the equation (3.17) is reformed as

$$
\begin{aligned}
&\lambda\phi A^2X-\phi A^3X-\lambda A\phi AX+A^2\phi AX\\
&=3g(U,X)\xi+\lambda(\nabla_X A)\xi-\lambda\phi X-A(\nabla_X A)\xi+A\phi X-(\nabla_\xi A)AX,
\end{aligned}
$$

which together with (3.11) yields

$$
\begin{aligned}
&(\nabla_X A)A\xi-(\nabla_\xi A)AX\\
&=\lambda\phi A^2X-\phi A^3X-3g(X,U)\xi+\lambda\phi X-A\phi X.
\end{aligned}
$$

Transforming this by ϕ and taking account of (1.1), we get

$$
\begin{aligned}
&\phi\{(\nabla_X A)A\xi-(\nabla_\xi A)AX\}\\
&=A^3X-\lambda A^2X-\lambda X+\lambda\eta(X)\xi-\phi A\phi X.
\end{aligned}
\tag{3.19}
$$

Differentiating (3.15) covariantly along M_0 and making use of (3.1), we obtain

$$
\begin{aligned}
&(\nabla_X\phi)(\lambda A-A^2)Y+\phi(\lambda(\nabla_X A)Y-(\nabla_X A)AY-A(\nabla_X A)Y)\\
&-(\lambda(\nabla_X A)-(\nabla_X A)A-A(\nabla_X A))\phi Y-(\lambda A^- A^2)(\nabla_X\phi)Y\\
&=\{\phi\nabla_X U\otimes U+\phi U\otimes\nabla_X U+\nabla_X U\otimes\phi U+U\otimes\phi\nabla_X U\\
&+(\lambda-\alpha)g(U,X)(\xi\otimes U+U\otimes\xi)\}(Y).
\end{aligned}
$$

Taking the skew symmetric part for X and Y of this and then replacing X by ξ into the obtained result, then we get

$$
\begin{aligned}
&\lambda\{X-\eta(X)\xi\}-\phi(\nabla_\xi A)AX+\phi(\nabla_X A)A\xi+\phi A\phi X\\
&-\lambda(\nabla_\xi A)\phi X+(\nabla_\xi A)A\phi X+A(\nabla_\xi A)\phi X+\lambda A^2X-A^3X\\
&=g(U,X)\phi\nabla_\xi U+g(\nabla_\xi U,X)\phi U-g(\nabla_X U,\xi)\phi U\\
&+g(\phi U,X)\nabla_\xi U+g(\phi\nabla_\xi U,X)U-(\lambda-\alpha)g(U,X)U,
\end{aligned}
$$

where we have used (1.1), (1.2), (1.5) and (3.1). Combining this and (3.19) and taking account of the fact that $g(\nabla_X U, \xi) = -g(\nabla_X \xi, U) = (\alpha - \lambda) g(A\xi, X)$, we get

$$
\begin{aligned}
&3g(U, \phi X)\xi + (\lambda - \alpha) g(U, \phi X) A\xi \\
(3.20) \qquad &= g(U, X)\phi\nabla_\xi U + g(\nabla_\xi U, X)\phi U + (\lambda - \alpha) g(A\xi, X)\phi U \\
&\quad + g(\phi U, X)\nabla_\xi U + g(\phi\nabla_\xi U, X) U.
\end{aligned}
$$

On the other hand, differentiating U covariantly in the direction of X and making use of (1.2), (1.3) and (3.1), we get

$$\nabla_X U = \alpha AX - \lambda g(X, A\xi)\xi + \phi(\nabla_X A)\xi + \phi A\phi AX,$$

which implies that

$$\nabla_\xi U = \alpha A\xi - \lambda\alpha\xi + \phi(\nabla_\xi A)\xi + \phi AU.$$

Then, by means of (3.14), we have

$$\phi\nabla_\xi U = (2\alpha - \lambda)U - (\nabla_\xi A)\xi.$$

Substituting the last two equations into (3.20) and taking the inner product of this result and U, we can see that

$$(6\alpha - 4\lambda) g(U, X) = 0,$$

where we have used (3.14). Thus, we obtain $3(6\alpha - 4\lambda) = 0$. Hence $3\alpha = 2\lambda$. Since $3\lambda = 8\alpha$ on the subset M_0, we get $\alpha = 0$. This contradicts the fact that $\alpha = 3/\sqrt{5}$. Consequently, we conclude that $Int(M_0) = \emptyset$.

The following is immediate from Lemma 3.1.

LEMMA 3.2. *Let M be a real hypersurface of $H_n\mathbb{C}$. If it satisfies $\mathcal{L}_\xi S = 0$ and $A\xi$ is principal such that $\eta(A^3\xi) = trA$, then ξ is principal.*

REMARK 1. In general, "ξ is principal" implies "$A\xi$ is principal". But the converse is not true.

REMARK 2. The structure vector ξ is *principal with respect to* S if the Ricci tensor S satisfies $S\xi = \sigma\xi$ for some function σ on M. Under the same assumption as Lemma 3.2, we have ξ is principal with respect to S. In fact, since $\lambda = \eta(A^3\xi) = trA = h$, taking account of (3.4) we have $S\xi = -2(n-1)\xi$.

REMARK 3. A ruled real hypersurface does not satisfy the condition that $A\xi$ is principal. In fact, let M be a ruled real hypersurface in a complex space form $M_n(c)$. Then M satisfies

$$\begin{aligned} &A\xi = \alpha\xi + \beta V(\beta \neq 0), \\ &AV = \beta\xi, \\ &AX = 0 \end{aligned}$$

for any vector X orthogonal to ξ and V, where V is a unit orthogonal to ξ, and α and β are smooth functions on M. Assume that M satisfies the condition that $A\xi$ is principal, that is, $A^2\xi = \lambda A\xi$. Then using the above properties of M, we get $A^2\xi = A(\alpha\xi + \beta V) = (\alpha^2 + \beta^2)\xi + \alpha\beta V$ and $A^2\xi = \lambda(A\xi) = \alpha\lambda\xi + \beta\lambda V$. Thus comparing to these two equations, we have $\alpha = \lambda$ and $\beta = 0$. This contradicts the fact that $\beta \neq 0$.

From Lemma 3.2 and Theorem B we have the following.

THEOREM 3.3. *Let M be a real hypersurface of $H_n\mathbb{C}(n \geq 3)$. If $A\xi$ is principal such that $\eta(A^3\xi) = trA$ and M satisfies $\mathcal{L}_\xi S = 0$, then M is locally congruent to type A.*

For a real hypersurface of $H_n\mathbb{C}$ satisfying the condition "$\mathcal{L}_\xi S = 0$", we see that $\phi S = S\phi$ if and only if ξ is principal. Thus we get the following.

THEOREM 3.4. *Let M be a real hypersurface of $H_n\mathbb{C}(n \geq 3)$. If M satisfies $\mathcal{L}_\xi S = 0$ and $\phi S = S\phi$, then M is locally congruent to type A.*

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