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k-MMBP Characterization of the Departure Process of a D-BMAP/Geo/1/*K* Queue Arising in an ATM Network

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ATM 망에서 발생되는 D-BMAP/Geo/1/*K* Queue의 Departure 프로세스의 *k*-MMBP 특성화

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We first obtain the departure process of a D-BMAP/Geo/1/*K* queue. The departure process is then exactly characterized by a *k*-state MMBP in order to capture the burstiness and correlation of the departure process.

ATM 망의 모델링에서 발생되어지는 D-BMAP/Geo/1/*K* queue의 Departure 프로세스를 구하고 burstiness와 correlation에 대하여 연구하였다. 또한 그 프로세스를 *k*-state MMBP로 특성화하는 방법을 제시하였다.

Key words : Discrete-time queue, Departure process, Correlation, MMBP, ATM

I. Introduction

In recent years there has been a lot of interest in the development of high-speed communication networks. The most promising design for high-speed networks is the Asynchronous Transfer Mode(ATM). The need for performance evaluation of ATM networks has given rise to a widespread interest for the analysis of discrete-time queueing systems. Discrete-time single server queues with or without finite capacity have been extensively analyzed. For a review of relevant results see Pujolle and Perros[1]. However, little has been done for the analysis of networks of discrete-time finite capacity queues. A network of discrete-time finite capacity queues can be used to model the queueing within an ATM switch, or the queueing within a network of ATM switches. The external arrival process to the network is assumed to be bursty and correlated. Markov Modulated Poisson Processes(MMPP)[2-3], and Markov Modulated Bernoulli Processes(MMBP) are used to model a bursty arrival stream since they capture the randomly varying arrival rate. The MMPP and MMBP capture the notion of burstiness and correlation of successive interarrival times. In this paper, we assume that the arrival process to the queue is a Discrete-time Batch Markovian Arrival Process (D-BMAP) which belongs to a class of versatile point processes[4-5]. A D-BMAP is the proposed model for a single variable bit rate source. Also, it can be used to model the superposition of several such sources[6]. The MMBP or IBP is a special case of the D-BMAP, with all arrival having a batch of size 1.

In this paper, we consider discrete-time finite capacity queues with cell loss. The service time at the queue is assumed to be geometrically distributed. The choice of the geometric distribution was motivated by ATM networks[7]. In general, a service time represents a transmission time. In an ATM networks the size of a cell is constant, and therefore, the transmission time is constant as well. However, in some ATM switch architectures a cell may be re-transmitted several times due to possible collisions with other cells. In this case, the total transmission time is typically modeled by a geometric distribution.

In general, discrete-time queueing networks as they arise in ATM do not lend themselves to an exact analysis. They can be analyzed, however, approximately using the notion of decomposition. That is, the network is decomposed into individual queues, and each queue is then analyzed separately. The most important aspect of such a decomposition is the characterization of the arrival process to an intermediate queue. In continuous-time queueing networks, typically such as the departure process is characterized approximately by a phased-type distribution, or by a general distribution defined by the mean and squared coefficient of variation. Although there has been some work regarding the departure process[8-13], most of this work bears some limitations which serious undermine their applicability on network-wide traffic analysis. Most of these studies only provide results on the stationary distribution of the interdeparture time. Although this is a very important piece of information, it is by no means sufficient for characterizing the non-renewal departure process: the lengths of successive interdeparture times are highly correlated and such correlation will have significant impact on downstream queueing performance. As a result, details about the dynamic behaviour of the departing stream, e.g., burstiness and correlation, have to be studied. In this paper, the departure process of the D-BMAP/Geo/1/K queue has been studied.

Blondia and Casals[6] showed that the output process of a D-BMAP/G/1/K queue is a D-BMAP. Park and Perros[14-15] derived the generating function of the interdeparture time distribution and correlation of the departure process of an MMBP/Geo/1/K queue. They also obtained an approximation model for characterizing the departure process by an MMBP in order to capture the correlation and burstiness of the departure process of the queue.

This paper is organized as follows. In section II, we give a brief description of the D-BMAP. The generating function of the interdeparture time of a D-BMAP/Geo/1/K queue and the correlation coefficients for the departure process are obtained in section III. In section IV, we present a fitting model for characterizing exactly the departure process as a k-MMBP.

II. The Discrete-time Batch Markov Arrival Process

1. The Generating Function of the Interarrival Time of the D-BMAP

A D-BMAP can be represented by a 2-dimensional discrete-time Markov process $\{(J(k), N(k)) : k \geq 0\}$ on the state space $\{(i, j) : 1 \leq i \leq m, j \geq 0\}$, where i indicates the state of the arrival process, and j indicates the number of arrivals. The transition matrix T of the counting process has the following structure:

$$\mathbf{T} = \begin{bmatrix} \mathbf{P}_0 & \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 & \cdots \\ & \mathbf{P}_0 & \mathbf{P}_1 & \mathbf{P}_2 & \cdots \\ & & \mathbf{P}_0 & \mathbf{P}_1 & \cdots \\ & & & & \ddots \end{bmatrix}$$

where $\mathbf{P}_k$, $k \geq 0$, are $m \times m$ matrices. Let $\mathbf{P} = \sum_{k=0}^{\infty} \mathbf{P}_k$ be the transition matrix of the underlying Markov process. If $J(k)$ represents a phase variable and $N(k)$ a counting variable then the above Markov process defines a batch arrival process where transitions from a state (i, j) to state $(1, j+n)$, corresponding to batch arrivals of size n.

Consider a discrete-time Markov chain with transition probability matrix P. Assume the underlying Markov process is in some state i, $1 \leq i \leq m$ at time k. At the next time instant $k+1$, the process may transit to another state or it may stay in the same state, and a batch arrival may or may not occur. Let $p_{(n,i,j)}$, $n \geq 0$, $1 \leq i, j \leq m$, be the probability that there is a transition to state j from state i with a batch arrival of size n. Then, with probability $p_{(0,i,j)}$, $n \geq 1$, $1 \leq i, j \leq m$, a transition to state j will take place without an arrival, and with probability $p_{(n,i,j)}$, $n \geq 1$, $1 \leq i, j \leq m$, there will be a transition to state j with a batch arrival of size n. We have

$$\sum_{j=1}^{m} p_{(0,i,j)} + \sum_{n=1}^{\infty} \sum_{j=1}^{m} p_{(n,i,j)} = 1.$$

Using this notation, it is clear now that matrices $P_0 = [p_{(0,i,j)}]_{m \times m}$ and $P_k = [p_{(k,i,j)}]_{m \times m}$, govern transitions that correspond to no arrival and arrival of batch of size k where $k \neq 0$, respectively. A D-MAP is a special case of the D-BMAP, with all arrivals having a batch of size 1.

Through this paper, we consider an arrival process to the queue which is a D-BMAP characterized by the transition probability matrix P of the Markov process, Λ, $m \times m$ diagonal matrix with elements $a_1, \cdots, a_m$ and B, defined by

$$\mathbf{P} = \begin{bmatrix} p_{11} & & p_{1m} \\ & \ddots & \\ & p_{m1} & & p_{mm} \end{bmatrix},$$

$$\Lambda = \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_m \end{bmatrix}, \quad \text{and}$$

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots \\ & \ddots & \\ b_{m1} & b_{m2} & \cdots \end{bmatrix}$$

where p_{ij}, $1 \leq i, j \leq m$ is the transition probability that the process changes from state i to state j, $\sum_{j=1}^{m} p_{ij} = 1$, a_i is the probability that a batch arrival occurs when the D-BMAP shifts to state i, and b_{ik} is the probability that the arriving batch size is equal to k, $k \geq 1$, $\sum_{n=1}^{\infty} b_{in} = 1$. The D-BMAP satisfies following equations: $p_{(0,i,j)} = p_{ij}(1 - a_j)$, $p_{(n,i,j)} = p_{ij}\, a_j\, b_{jn}$, and $p_{ij} = \sum_{k=0}^{\infty} p_{(k,i,j)}$ for $n \geq 1$, $1 \leq i, j \leq m$. This process can be also referred to as a Markov Modulated Batch Bernoulli Process (MMBBP). In general, a D-BMAP becomes an MMBP if the following relation are satisfied:

For $1 \leq i, j \leq m$,

$$b_{i1} = 1$$
$$p_{ij} = p_{(0,i,j)} + p_{(1,i,j)}$$
$$p_{ij}\,(1 - a_j) = p_{(0,i,j)} \quad (1)$$
$$p_{ij}\, a_j = p_{(1,i,j)} \quad (2)$$

A D-BMAP has been proposed as a model from single variable bit rate source and its superposition[6]. Therefore, we assume that the batch size of a batch is bounded. Let N be the maximum batch size. Let T be the interarrival time between two successive batch arrivals.

Also let $\vec{\pi}=[\pi_{1,\cdots,}\pi_m]^T$ be the stationary probability vector satisfying $\vec{\pi}=\vec{\pi}\mathbf{P}$, where π_i, $1\le i\le m$, is the probability that the process is in state i. The generating function of batch interarrival time $T(z)$ is

$$\begin{aligned} T(z) &= \vec{\mathbf{p}}_a \vec{\mathbf{T}}(z) \\ &= z\,\vec{\mathbf{p}}_a(\mathbf{I}-z\mathbf{M})^{-1}\mathbf{P}\vec{\lambda} \end{aligned}$$

where

$$\begin{aligned} \vec{\mathbf{p}}_a &= \frac{\vec{\pi}\Lambda}{\vec{\pi}\vec{\lambda}}, \\ \vec{\mathbf{T}}(z) &= z(\mathbf{I}-z\mathbf{M})^{-1}\mathbf{P}\vec{\lambda}, \\ \mathbf{M} &= \mathbf{P}(\mathbf{I}-\Lambda), \\ \vec{\lambda} &= [\alpha_{1,\cdots,}\alpha_m]^T. \end{aligned}$$

The average batch arrival rate ρ_b, the average cell arrival rate ρ_c, and the squared coefficient of variation of the interarrival time between two successive arrival of batch, C_b^2 are as follows:

$$\begin{aligned} \rho_b &= \vec{\pi}\vec{\lambda}, \quad \rho_c = \sum_{i=1}^{N} i\ \vec{\pi}\Lambda\vec{\mathbf{b}}_i, \quad \text{and} \\ C_b^2 &= \frac{T^{(2)}(1)}{[T^{(1)}(1)]^2}+\rho_b-1 \end{aligned}$$

where $\vec{\mathbf{b}}_i=[b_{1i,}\cdots,b_{mi}]^T$ and $T^{(n)}(1)=\left.\frac{d^nT(z)}{dz^n}\right|_{z=1}$.

2. The Autocorrelation of the D-BMAP

In this section, we obtain the autocorrelation of the interarrival time of batches, and the autocorrelation of the number of arrivals per slot. Let t_n be the time interval between the $(n-1)$st and nth arrival of a batch. Also, let t_{ij}^n, $1\le i,\ j\le m$ be the time interval to the moment that the D-BMAP is in state j and nth arrival occurs given that the D-BMAP is in state i, and t_i^n, $1\le i\le m$ be the time interval to the nth arrival given that the D-BMAP is in state i. Define

$$\begin{aligned} \mathbf{A}(z) &= \begin{bmatrix} A_{11}(z) & & A_{1m}(z) \\ & \ddots & \\ A_{m1}(z) & & A_{mm}(z) \end{bmatrix} \quad \text{and} \\ \vec{\mathbf{A}}(z) &= \begin{bmatrix} A_1(z) \\ \vdots \\ A_m(z) \end{bmatrix} \end{aligned}$$

where $A_{ij}(z)$ and $A_i(z)$ are z-transforms of t_{ij}^n and t_i^n, respectively. From the definition of $A_{ij}(z)$ and $A_i(z)$ for $1\le i,j\le m$, we have following equations:

$$\begin{aligned} \mathbf{A}(z) &= z\mathbf{P}\Lambda+z\mathbf{M}\mathbf{A}(z) \quad \text{and} \\ \vec{\mathbf{A}}(z) &= \vec{\mathbf{T}}(z). \end{aligned}$$

Therefore, we can obtain

$$\begin{aligned} \mathbf{A}(z) &= z(\mathbf{I}-z\mathbf{M})^{-1}\mathbf{P}\Lambda \quad \text{and} \\ \vec{\mathbf{A}}(z) &= z(\mathbf{I}-z\mathbf{M})^{-1}\mathbf{P}\vec{\lambda} \end{aligned} \tag{3}$$

Using equation (3), we have

$$\begin{aligned} G_a(z_1,z_2) &= E\{z_1^{t_n}z_2^{t_{n+k}}\} \\ &= \vec{\mathbf{p}}_a\,\mathbf{A}(z_1)\,\mathbf{T}^{k-1}\,\vec{\mathbf{A}}(z_2) \\ &= \vec{\mathbf{p}}_a\,z_1(\mathbf{I}-z_1\mathbf{M})^{-1}\mathbf{P}\Lambda \\ &\quad \mathbf{T}^{k-1}z_2(\mathbf{I}-z_2\mathbf{M})^{-1}\mathbf{P}\vec{\lambda} \end{aligned} \tag{4}$$

where $\mathbf{T}=[\mathbf{I}-\mathbf{M}]^{-1}\mathbf{P}\Lambda$.

By differentiating equation (4) with respect to z_1 and z_2, we have

$$\begin{aligned} E\{t_nt_{n+k}\} = \vec{\mathbf{p}}_a(\mathbf{I}-\mathbf{M})^{-1}\mathbf{P}\Lambda\mathbf{T}^{k-1} \\ (\mathbf{I}-\mathbf{M})^{-2}\mathbf{P}\vec{\lambda}. \end{aligned}$$

The autocorrelation coefficient of the interarrival time of batches of a D-BMAP for lag k, $\psi_b(k)$, is given by

$$\phi_b(k) = \frac{E\{t_n t_{n+k}\} - E^2\{t_n\}}{Var\{t_n\}} \qquad (5)$$

Let X_n be the random variable representing the number of arrivals at nth slot, where $X_n = 0, 1, \cdots, N$. Then, we have

$$E\{X_n\} = \rho_c,$$

$$E\{X_n^2\} = \sum_{i=1}^{N} i^2 \vec{\pi}\Lambda \vec{\mathbf{b}}_i,$$

$$E\{X_n X_{n+k}\} = \sum_{i,j=1}^{N} i\, j\, \vec{\pi}\Lambda \mathbf{B}_i \mathbf{P}^k \Lambda \vec{\mathbf{b}}_j,$$

$$Var\{X_n\} = E\{X_n^2\} - E^2\{X_n\}$$

where B_i is a diagonal matrix with elements $b_{1i}, \cdots, b_{mi}$.

Of interest is the autocorrelation coefficient of the number of arrival per slot of a D-BMAP for lag k, $\phi_c(k)$, given by

$$\phi_c(k) = \frac{E\{X_n X_{n+k}\} - E^2\{X_n\}}{Var\{X_n\}}\ . \qquad (6)$$

III. The Departure Process of an D-BMAP/Geo/1/K Queue

We consider a D-BMAP/Geo/1/K queue, where the service time is defined over a slotted time axis. A service starts at the beginning of a service slot, and service completion is assumed to take place just before the end of the service slot. The arrival process is also defined over a slotted time axis with the same slot size, and it is assumed to be a D-BMAP. The parameters of the arrival process are: p_{ij}^A, α_i^A, and b_{ij}^A, where p_{ij}^A is the (i,j)th element of the transition probability matrix $\mathbf{P}$, α_i^A is the (i,i)th element of the diagonal matrix Λ, and b_{ij}^A is the (i,j)th element of the matrix $\mathbf{B}$. We define the state of the queue by the variable (s,n). Variable s represents the state of the arrival process at the end of a slot and it takes the values: i, $1 \le i \le m$, if the arrival process is in the state i. Variable n indicates the number of cells in the system at the end of a slot. We have $n=0,1,\cdots,K$, where K is the capacity of the system including the cell in service. Let P_d be the transition probability matrix of the queue. Define P_{wd} and P_{wod} as follows:

$$\mathbf{P}_{wd} = (1-\sigma)\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & & & & \\ \mathbf{M} & \mathbf{M}_1 & \mathbf{M}_2 & \mathbf{M}_3 & \mathbf{M}_4 & & & & \\ & \mathbf{M} & \mathbf{M}_1 & \mathbf{M}_2 & \mathbf{M}_3 & & & & \\ & & \mathbf{M} & \mathbf{M}_1 & \mathbf{M}_2 & & & & \\ & & & \mathbf{M} & \mathbf{M}_1 & & & & \\ & & & & \ddots & & & & \\ & & & & \mathbf{M}_1 & \mathbf{M}_2 & \mathbf{M}_3 & \overline{\mathbf{M}}_3 & \mathbf{0} \\ & & & & \mathbf{M} & \mathbf{M}_1 & \mathbf{M}_2 & \overline{\mathbf{M}}_2 & \mathbf{0} \\ & & & & & \mathbf{M} & \mathbf{M}_1 & \overline{\mathbf{M}}_1 & \mathbf{0} \\ & & & & & & \mathbf{M} & \mathbf{P}\Lambda & \mathbf{0} \\ & & & & & & & \mathbf{P} & \mathbf{0} \end{bmatrix}$$

and

$$\mathbf{P}_{wod} = \begin{bmatrix} \mathbf{M} & \mathbf{M}_1 & \mathbf{M}_2 & \mathbf{M}_3 & \mathbf{M}_4 & & & \\ & \mathbf{M}\sigma & \mathbf{M}_1\sigma & \mathbf{M}_2\sigma & \mathbf{M}_3\sigma & & & \\ & & \mathbf{M}\sigma & \mathbf{M}_1\sigma & \mathbf{M}_2\sigma & & & \\ & & & \mathbf{M}\sigma & \mathbf{M}_1\sigma & & & \\ & & & & \mathbf{M}\sigma & & & \\ & & & & \ddots & & & \\ & & & & \mathbf{M}_1\sigma & \mathbf{M}_2\sigma & \mathbf{M}_3\sigma & \overline{\mathbf{M}}_3\sigma \\ & & & & \mathbf{M}\sigma & \mathbf{M}_1\sigma & \mathbf{M}_2\sigma & \overline{\mathbf{M}}_2\sigma \\ & & & & & \mathbf{M}\sigma & \mathbf{M}_1\sigma & \overline{\mathbf{M}}_1\sigma \\ & & & & & & \mathbf{M}\sigma & \mathbf{P}\Lambda\sigma \\ & & & & & & & \mathbf{P}\sigma \end{bmatrix}$$

where

$$\mathbf{M}_i = \mathbf{P}\Lambda \mathbf{B}_i, \quad \overline{\mathbf{B}}_i = \sum_{n=i+1}^{m} \mathbf{B}_n, \text{ and}$$

$$\overline{\mathbf{M}}_i = \mathbf{P}\Lambda \overline{\mathbf{B}}_i.$$

We can see that the transition probability matrix, P_d can be decomposed into two matrices, P_{wd} and P_{wod}, where P_{wd}, P_{wod} is a matrix that contains transitions with a departure respectively without a departure. Therefore, $P_d = P_{wd} + P_{wod}$. We compute the generating function of the probability distribution of the interdeparture time, and then we obtain the autocorrelation of the interdeparture time and the autocorrelation of the number departure per slot.

1. The Generating Function of the Interdeparture Time Distribution

Let t_n be the time interval between the $(n\text{-}1)$st and the nth departure. Also, let t^n_{ij}, $1 \le i,\ j \le L$ where $L=m(K+1)$, be the time interval to the moment that the state of the queue is j and the nth departure occurs given the queue is in state i, and t^n_i, $1 \le i \le L$, be the time interval to the nth departure given that the queue is in state i. Define

$$\mathbf{D}(z) = \begin{bmatrix} D_{1,1}(z) & & D_{1,L}(z) \\ & \ddots & \\ D_{L,1}(z) & & D_{L,L}(z) \end{bmatrix} \quad \text{and}$$
$$\vec{\mathbf{D}}(z) = \begin{bmatrix} D_1(z) \\ \vdots \\ D_L(z) \end{bmatrix}$$

where $D_{ij}(z)$ and $D_i(z)$ are the z-transforms of t^n_{ij} and t^n_i, respectively. Also, let $p^+(s,n)$ be the probability that immediately after a departure the system is in state (s,n). From the definition of $D_{ij}(z)$ and $D_i(z)$, we have following equations:

$$\mathbf{D}(z) = z(\mathbf{I} - z\mathbf{P}_{wod})^{-1}\mathbf{P}_{wd} \quad \text{and}$$
$$\vec{\mathbf{D}}(z) = z(\mathbf{I} - z\mathbf{P}_{wod})^{-1}\mathbf{P}_{wd}\,\vec{\mathbf{e}}.$$

Then, the generating function of the interdeparture time distribution $D(z)$can be obtained from as follows:

$$\begin{aligned} D(z) &= \vec{\mathbf{p}}^{+}\,\vec{\mathbf{D}}(z) \\ &= z\,\vec{\mathbf{p}}^{+}(\mathbf{I} - z\,\mathbf{P}_{wod})^{-1}\,\mathbf{P}_{wd}\,\vec{\mathbf{e}} \end{aligned}$$

where

$$\begin{aligned} \vec{\mathbf{p}}^{+} &= [p^+(1,0),\cdots,p^+(m,0),p^+(1,1),p^+(2,1),\cdots,p^+(m,K)] \\ &= \frac{\vec{\mathbf{x}}\,\mathbf{P}_{wd}}{\vec{\mathbf{x}}\,\vec{\lambda}_d}. \end{aligned}$$

From the generating function, we can obtain the moments of the time between successive departures, the squared coefficient of variation of the interdeparture time C^2_d, and throughput ρ_d.

2. The Autocorrelation of the Departure Process

In this section, we obtain the autocorrelation of the interdeparture time, and the autocorrelation of the number of departure per slot. In order to obtain the autocorrelation of the interdeparture time, we have

$$\begin{aligned} G_d(z_1 z_2) & \\ &= E\{z_1^{t_n} z_2^{t_{n+k}}\} \\ &= \vec{\mathbf{p}}^{+}\,\mathbf{D}(z_1)\,\mathbf{R}^{k-1}\,\vec{\mathbf{D}}(z_2) \\ &= \vec{\mathbf{p}}^{+} z_1(\mathbf{I} - z_1\,\mathbf{P}_{wod})^{-1}\mathbf{P}_{wd} \\ &\quad \mathbf{R}^{k-1} z_2(\mathbf{I} - z_2\,\mathbf{P}_{wod})^{-1}\,\mathbf{P}_{wd}\,\vec{\mathbf{e}} \end{aligned} \tag{7}$$

where $\mathbf{R} = (\mathbf{I} - \mathbf{P}_{wod})^{-1}\,\mathbf{P}_{wd}$.

By differentiating equation (7) with respect to z_1 and z_2 and substituting $z_1 = 1$ and $z_2 = 1$ into equation (7), we have

$$\begin{aligned} E\{t_n t_{n+k}\} &= \vec{\mathbf{p}}^{+}(\mathbf{I} - \mathbf{P}_{wod})^{-2}\,\mathbf{P}_{wd} \\ &\quad \mathbf{R}^{k-1}(\mathbf{I} - \mathbf{P}_{wod})^{-2}\,\mathbf{P}_{wd}\,\vec{\mathbf{e}}. \end{aligned}$$

The autocorrelation coefficient of the interdeparture time of an D-BMAP/Geo/1/K queue for lag k, $\psi_d(k)$, can now we obtained using expression (5)

Let X_n be the random variable representing the number of departures in the nth slot, where X_n = 0, 1. We have

$$\begin{aligned} E\{X_n\} &= E\{X_n^2\} = \rho_d \quad \text{and} \\ E\{X_n X_{n+k}\} &= \vec{\mathbf{x}}\,\mathbf{P}_{wd}\,\mathbf{P}_d^{k-1}\,\vec{\lambda}_d \end{aligned}$$

where $\vec{\lambda}_d = [0,\cdots,0,1-\sigma,\cdots,1-\sigma]^T$ and $\vec{\mathbf{x}}$ is the steady-state probability vector satisfying $\vec{\mathbf{x}}\,\mathbf{P}_d = \vec{\mathbf{x}}$. The autocorrelation coefficient of the number departures of the queue for lag k, $\phi_d(k)$ can now be obtained from.[6)]

Let us consider the autocorrelation of the interdeparture time of the queue. One of the most interesting facts that we have observed is that the autocorrelation coefficients of the interdeparture time (correlogram) may fluctuate quite a lot[16)]. We note that this behaviour is barely seen in the departure process of an MMBP/Geo/1/K queue. This oscillation seems to be due to the variability of the number of arrivals per slot within the same state of the arrival process. Let us consider the case where

$$\mathbf{P} = \begin{bmatrix} 0.98 & 0.01 & 0.01 \\ 0.01 & 0.98 & 0.01 \\ 0.01 & 0.01 & 0.98 \end{bmatrix},$$
$$\boldsymbol{\Lambda} = \begin{bmatrix} 0.9 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.1 \end{bmatrix}, \quad \text{and}$$
$$\mathbf{B}_B = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

When the arrival process is in state 3, the rate of arrivals a_3 is very low. Also, b_{33} is quite large in relation to b_{31} and b_{32}. When the arrival process is in state 3, there may be long interarrival periods and the queue may empty out between successive batch arrivals. In this case, the pattern of the interdeparture times consists of one long interval followed by small intervals. This pattern causes the autocorrelation of the interdeparture time to fluctuate.

IV. Characterization of the Departure Process

In this section, we obtain a model for characterizing the departure process by a k-MMBP. This model exactly captures the correlation and burstiness of the departure process of the queue. It can be shown that the output process of a D-BMAP/G/1/K queue is a D-MAP[6)] and the MMBP is a special case of the D-BMAP. Note that the fitted k-MMBP is characterized by the transition probability matrix $\mathbf{P}_{est}$ of the Markov process and $\boldsymbol{\Lambda}_{est}$ given by

$$\mathbf{P}_{est} = \begin{bmatrix} p_{11}^{est} & & p_{1k}^{est} \\ & \ddots & \\ p_{k1}^{est} & & p_{kk}^{est} \end{bmatrix} \quad \text{and}$$
$$\boldsymbol{\Lambda}_{est} = \begin{bmatrix} a_1^{est} & & 0 \\ & \ddots & \\ 0 & & a_k^{est} \end{bmatrix}$$

where p_{ij}^{est}, $1 \le i,\ j \le k$, is the transition probability that the fitted MMBP changes from state i to state j, $\sum_{j=1}^{k} p_{ij}^{est} = 1$ for $1 \le i,\ j \le k$ and a_i^{est}, $1 \le i \le k$, is the probability that a slot contains a cell during the time that the MMBP is in state i. Therefore, a k-MMBP is characterized by k^2 parameters. It is practically impossible to obtain these parameters using the method of moments, particularly when k is large. Other fitting techniques, such as minimum distance estimation and least squared estimation, can be used, but they are time consuming. Also, $\mathbf{P}_{est}$ and $\boldsymbol{\Lambda}_{est}$ can be approximated from grouping of states as in previous work[14)]. The parameters of the fitted MMBP, p_{ij}^{est} and a_i^{est}, $1 \le i,\ j \le k$, can be calculated as follows:

$$p_{ij}^{est} = \frac{\sum_{(s,n)\in S_i} P(s,n) \left[\sum_{\bar{s},\bar{n}\in S_j} t[(s,n) \to (\bar{s},\bar{n})] \right]}{\sum_{(s,n)\in S_i} P(s,n)},$$

$$a_i^{est} = \frac{(1-\sigma)\left[\sum_{(s,n>0)\in S_i} P(s,n) \right]}{\sum_{(s,n)\in S_i} P(s,n)}$$

where $0 < p_{ij}^{est} < 1$ and $0 \le a_i^{est} \le 1$ for $1 \le i,\ j \le k$. However, unlike the case of the m-MMBP/Geo/1/K queue, we can see that the autocorrelation of coefficients of the interdeparture time of the D-BMAP/Geo/1/K queue can fluctuate[16]. Due to the characteristic of the departure process, the model proposed in the previous works[14-15] is not suitable for characterizing the departure process of a D-BMAP/Geo/1/K queue. The method estimates poorly the autocorrelation coefficients and the interdeparture time distribution. In this section, we present a simple method for fitting a k-MMBP to the departure process of a D-BMAP/Geo/1/K queue. We note that we do not address the problem of how many stages the fitted MMBP should consist of. In the previous works[14-15], the departure process was assumed to have as many stages as the MMBP describing the arrival process to the queue.

The departure process of a queue is governed by the states of the queue. Therefore, we can obtain valuable information regarding the departure process from the states of the queue. By letting each state (s,n) be a separate state in the departure process, we can easily characterize the departure process as a D-MAP with P_0 = P_{wod} and P_1 = P_{wd}. Note that this D-MAP does not satisfy equations (1) and (2), and therefore, it is not an MMBP. However, we can have an exact MMBP characterization of the departure process of the m-MMBP/Geo/1/K queue only when $\sigma = 0$. In order to characterize the departure process by an MMBP, we have to obtain P_{ij}^{est} and α_i^{est} for $1 \le i,j \le k$ so that they satisfy equations (1) and (2). Given a state, then in the next slot a transition will occur with a departure or without a departure. Let $(s,n)_{wod}$ and $(s,n)_{wd}$ be the two states of the queue representing that the system shifted to (s,n) without a departure and with a departure, respectively. Then, we can separate all states of the queue (s,n) into $(s,n)_{wod}$ and $(s,n)_{wd}$. Note that $P(s,n)$ = $P(s,n)_{wod}$ + $P(s,n)_{wd}$ and $P(s,K)_{wd}$ = 0 for all s. We can now consider states $(s,n)_{wod}$ and $(s,n)_{wd}$ for $1 \le s \le m,\ 0 \le n \le K$ as a separate state of the fitted MMBP. The total number of states of the fitted MMBP is $k=2m(K+1)$. Then, the departure process of the queue can be exactly characterized by the k-MMBP with matrices

$$\mathbf{P}_{est} = \mathbf{P}_0 + \mathbf{P}_1 \quad \text{and} \quad \Lambda_{est} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

where

$$\mathbf{P}_0 = \begin{bmatrix} \mathbf{P}_{wod} & \mathbf{0} \\ \mathbf{P}_{wod} & \mathbf{0} \end{bmatrix}, \quad \mathbf{P}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{P}_{wd} \\ \mathbf{0} & \mathbf{P}_{wd} \end{bmatrix},$$

and $\mathbf{I}$ is a $m(K+1) \times m(K+1)$ identity matrix.

We can see that the number of states of the fitted MMBP is very large when m and K is large. That is, the computational complexity is directly proportional to the buffer capacity and the number of states of the Markov chain of the arrival process. We can significantly reduce the number of states by simply aggregating the states of the fitted MMBP[14-15]. By only matching the interdeparture time distribution, the number of states of the fitted MMBP can be reduced to 2 states[15]. In this case, however, we will ignore the autocorrelation which has a significant impact on the accuracy of the fitted MMBP. There is a trade-off between the number of states of the fitted MMBP and the accuracy of the estimated autocorrelation of the interdeparture time.

V. Conclusion

In this paper, we obtained the generating function of the interdeparture time distribution and the autocorrelation of the departure process of a D-BMAP/Geo/1/K queue. The fitting model for characterizing the departure process of this queue exactly by a k-MMBP is proposed in order to capture both the stationary distribution and the autocorrelation coefficients of the interdeparture time

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