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Job Scheduling Problem Using Fuzzy Numbers and Fuzzy Delphi Method

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Abstract

This paper shows that fuzzy set theory can be useful in modeling and solving job scheduling problems with uncertain processing times. The processing times are considered as fuzzy numbers(fuzzy intervals or time intervals) and the fuzzy Delphi method is used to estimate a reliable time interval of each processing time. Based on these time estimates, we then propose an efficient methodology for calculating the optimal sequence and the fuzzy makespan.

1. Introduction

Some of the well-known scheduling models have assumed the availability of precise data. Frequently, these assumptions are not valid. For example, in the job scheduling models, the processing times are assumed to be known exactly by expert. However, in practical situations, this is seldom the case. The estimate for each processing time is largely subjective evaluation, which is influenced by the estimator and the operating environment.

Occasionally, a manager is challenged by job scheduling problems with which she/he have had no prior experience. In such cases, it is very difficult to forecast the precise estimates of processing times. Therefore, if the uncertainty is due to a lack of experience or information, then each processing time should be estimated as being within a certain interval. This time interval can be naturally represented by a fuzzy number which is described using the concept of an interval of confidence as in Kaufmann and Gupta[10].

In this paper, we present an efficient methodology for solving job scheduling problem when processing times are uncertain due to a lack of information. The processing times

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are considered as fuzzy numbers, and the fuzzy Delphi method[5, 11] is used to estimate a reliable time interval(fuzzy interval) of each processing time.

The main goal of the job scheduling is to order the jobs through the machines in such a way as to optimize certain performance criteria. Many scheduling problem solution methods involve comparison of partial results before moving to the next step. The calculations are mainly based on max-min operation. One of the problems in solving job scheduling problems by considering fuzzy numbers is comparing the fuzzy numbers to obtain a total order. There are two methods in deciding maximum and minimum of fuzzy numbers. One is the composite method and another is the comparison method[4, 17]. In the composite method, the fuzzy maximum and minimum operators are used and these operators do not give the structure of total order because of the combining portions of two fuzzy numbers when performing the fuzzy max-min operation. On the other hand, the comparison method using the discrete maximum and minimum operators is useful for comparing the fuzzy numbers to obtain a total or linear order. Many people have studied the comparison of fuzzy numbers and have proposed many methods. A summary of such methods can be found in Ref.[6]. In this paper, we define the operators $\widetilde{max}$ and $\widetilde{min}$ as the fuzzy maximum and minimum operators, respectively. On the other hand, the operators *max* and *min* represent the discrete maximum and minimum operators, respectively.

In general, the composite method using the fuzzy maximum and fuzzy minimum operators defines the maximum and minimum of fuzzy numbers[7, 10]. If the processing times are considered as fuzzy numbers and the ordinary fuzzy operations on fuzzy numbers are used, then a unique optimal sequence is not identified. The job scheduling problems with fuzzy processing times can be efficiently solved by using the comparison method.

The paper shows that the optimal job sequence can be efficiently determined using the time estimate for processing of each job gained by the fuzzy Delphi method and the comparison method. The three-machine flow shop problem is considered as an example to illustrate the approach.

2. Fuzzy Delphi Method for Forecasting of Fuzzy Processing Times

The traditional Delphi method is one of the effective methods which enables forecasting by converging a possibility value through the feedback mechanism of the results of questionnaires, based on experts' judgments. For estimating a reliable time interval for processing of each job, the fuzzified Delphi method may be effectively used. Occasionally, a manager is challenged by large scale job scheduling problems that have never been attempted before. In such cases, highly qualified experts are interviewed to give their opinions for the possibility of each processing time. In such projects it is very difficult to forecast the precise estimates of processing times because there is a lack of human intelligence and suitable equipment at present. The difficulty concerning time estimations may be relieved by using fuzzy numbers. In this work, Triangular fuzzy numbers(TFNs) which represent the pessimistic, moderate and optimistic estimate are used to represent the opinions of experts for each processing times. A TFN $\tilde{A}$ can be defined by a triplet (a, b, c)[10]. The membership function is defined as

$$\begin{aligned} \mu_{\tilde{A}}(x) &= 0 , & x < a, \\ &= \frac{x-a}{b-a} , & a \leq x \leq b, \\ &= \frac{c-x}{c-b} , & b \leq x \leq c, \\ &= 0 , & x > c, \end{aligned} \qquad (1)$$

where $\mu_{\tilde{A}}(x)$ is the degree of membership or membership function value of x in $\tilde{A}$. The TFN which is a special kind of fuzzy number can represent the estimated processing time naturally. For example, an expert may say that the processing time of job A is generally b minutes. But, due to other factors which cannot be controlled, the processing time may be occasionally as slow as c minutes or as fast as a minutes. This result is naturally a TFN. An alternative definition of the fuzzy number can be obtained by the concept of the interval of confidence. An interval of confidence is one way of reducing the uncertainty of using lower and upper bounds. The interval of confidence at presumption level α of the TFN $\tilde{A} = (a, b, c)$ is characterized as

$$A_{\alpha} = [a_L^{(\alpha)}, a_U^{(\alpha)}] = [(b-a)\alpha + a, (b-c)\alpha + c], \quad \alpha \in [0, 1], \qquad (2)$$

where $a_L^{(\alpha)}$ and $a_U^{(\alpha)}$ are lower boundary and upper boundary respectively.

The fuzzy Delphi method is a methodology in which subjective data of experts are transformed into quasiobjective data using the statistical analysis and fuzzy operations. The process of the fuzzy Delphi method is continued until the process converge to a reasonable stable solution. The fuzzy Delphi method is characterized by the following steps.

[Step 1] A group of n experts is requested to give a possible processing time (the optimistic time, the moderate time, the pessimistic time) of each job using a TFN $(a_1^{(l)}, b_1^{(l)}, c_1^{(l)})^{ji}$, where l indicates the index attached to the expert, 1 indicates that this is the first phase of the forecasting process, and ji indicates the index attached to the job i on machine j. For example, a TFN $(a_1^{(2)}, b_1^{(2)}, c_1^{(2)})^{1B} = (2, 4, 8)$ is the fuzzy opinion of expert 2 for job B on machine 1 in the first phase.

[Step 2] These responses from n experts form a sheaf $(a_1^{(l)}, b_1^{(l)}, c_1^{(l)})^{ji}$, $l = 1, 2, \cdots, n$, for each processing time. The mean of this TFN sheaf is then computed $(a_1^m, b_1^m, c_1^m)^{ji}$ for each job, and for each expert the divergence is computed $(a_1^m - a_1^{(l)}, b_1^m - b_1^{(l)}, c_1^m - c_1^{(l)})^{ji}$. This information is then sent to each individual expert.

[Step 3] Each expert now gives a new TFN $(a_1^{(l)}, b_2^{(l)}, c_2^{(l)})^{ji}$ for each processing time and the process, starting with phase 2, is repeated as in step 2.

[Step 4] When the mean TFN of each processing time becomes sufficiently stable, the process is stopped.

If the mean TFN of each processing time that satisfies a given convergence criterion is found, this process has been completed and the corresponding mean TFN becomes a time estimate for processing of each job. The dissemblance index which exists between two fuzzy numbers[10] can be used as a criterion for the stable solution. Let's define the intervals of the confidence at presumption level α of the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ as $A_\alpha = [a_L^{(\alpha)}, a_U^{(\alpha)}]$ and $B_\alpha = [b_L^{(\alpha)}, b_U^{(\alpha)}]$, where $a_L^{(\alpha)}$ and $b_L^{(\alpha)}$ are their respective lower boundaries and $a_U^{(\alpha)}$ and $b_L^{(\alpha)}$ are their respective upper boundaries. The distance between two fuzzy numbers is given by

$$\delta(\tilde{A}, \tilde{B}) = \int_{\alpha=0}^{1} \delta\,[A_\alpha, B_\alpha]\,d\alpha$$

$$= \frac{1}{2}(\beta_2 - \beta_1)\int_{\alpha=0}^{1}(|a_L^{(\alpha)} - b_L^{(\alpha)}|)$$

$$+ (|a_U^{(\alpha)} - b_U^{(\alpha)}|)d\alpha, \qquad (3)$$

where $0 \leq \delta(\tilde{A}, \tilde{B}) \leq 1$ and β_1, β_2 are given any convenient values in order to surround both $A_{\alpha=0}$ and $B_{\alpha=0}$. It is also called the dissemblance index of $\tilde{A}$ and $\tilde{B}$. Figure 1 illustrates the concept of dissemblance index of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$. If all the distances between TFNs of experts and mean TFN for each processing time satisfy a given value, the corresponding mean TFN becomes a time estimate for processing of each job.

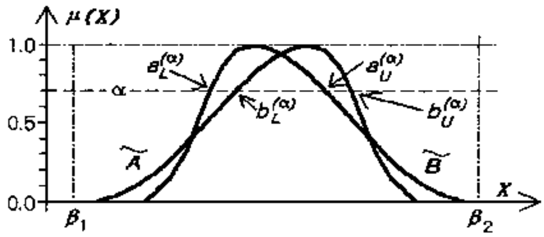

Figure 1. Concept of dissemblance index of two fuzzy numbers $\tilde{A}$ and $\tilde{B}$

The traditional Delphi method, in essence, attempts to eliminate ambiguity in a statistical and procedural manner. On the other hand, the fuzzy Delphi method adopts the approach where fuzziness is preserved by discarding clarified judgments. Fuzziness(time interval), at the stage of questionnaire surveys, is preserved. Since human judgments, in general, are considered variable and movable within a certain interval, rather than converging into a single point, it is worth exploring the time interval for processing of each job. The merits of this method are summarized as follows:

(1) It enables processing fuzziness in relation to the information contents of the respondents.

(2) Individual attributes of the expert are elucidated because the time interval is preserved and utilized.

3. A Ranking method for Solving Job Sequencing Problem

The main goal of the job scheduling is to order the jobs through the machines in such a way as to optimize certain performance criteria. One of the problems in solving job sequencing problems by considering fuzzy numbers is comparing the fuzzy numbers to obtain a total order. The job sequencing problems with fuzzy processing times can be solved by using the comparison method. The comparison method is an act of comparing fuzzy numbers to yield a totally ordered set and the discrete *max* and *min* are used in comparison method. These operators can be performed by using any fuzzy number ranking method[2, 6, 12, 13]. By the use of these ranking methods, the job sequencing problems with fuzzy processing times can be solved effectively and various methods have been proposed[3, 4, 7, 8, 14, 15, 16, 18]. In this paper, Kaufmann and Gupta's ranking method is used to order fuzzy numbers. Kaufmann and Gupta's ranking method can be easier to use without computerized calculations if the fuzzy numbers are triangluar. Let us consider a sheaf G composed of TFNs $\tilde{A}_i = (a_i, b_i, c_i)$, $i=1, 2, \cdots, m$. Kaufmann and Gupta's ranking method[10] is accomplished using the following three criteria.

C1: The greatest associated ordinary number

$$O[\tilde{A}_i] = \left(\frac{a_i + 2b_i + c_i}{4} \right). \qquad (4)$$

C2: If C1 does not separate the two TFNs, those which have the best maximum presumption (the best mode) will be chosen.

C3: If the C1 and C2 do not separate the TFNs, the divergence (the distance between two end points) will be used as the third criterion.

When the first criterion dealing with the associated ordinary number is not sufficient, then the second criterion is applied, and if it is also not sufficient, the third criterion is applied. Using the stable mean TFN of each processing time gained by the fuzzy Delphi method and the comparison method, the complex and large scale job scheduling problems which a manager has not experienced before can be efficiently solved.

4. Three-Machine Flow Shop Problem with TFN Processing Times

We will assume that all jobs are available for processing immediately. The optimal sequence is defined as the sequence which minimizes the makespan. The length of time required to complete all jobs is called the makespan. The three-machine flow shop problem with the objective of minimizing makespan can be solved more efficiently by Ignall and

Schrage's branch & bound algorithm[1, 9]. Also, the algorithm should be modified with fuzzy components to accept fuzzy processing times. For a given partial sequence σ, let $\widetilde{LC}_1(\sigma)$, $\widetilde{LC}_2(\sigma)$ and $\widetilde{LC}_3(\sigma)$ be the fuzzy completion time of the last job on machine 1, 2 and 3, respectively, among jobs in σ. The lower bounds based on the processing required of machine 1, 2 and 3 respectively, are

$$\widetilde{lb}_1 = \widetilde{LC}_1(\sigma) \oplus \sum_{i\in \bar{\delta}} \tilde{p}_{1i} \oplus \min_{i\in \bar{\delta}}\{\tilde{p}_{2i} \oplus \tilde{p}_{3i}\}, \quad (5)$$

$$\widetilde{lb}_2 = \widetilde{LC}_2(\sigma) \oplus \sum_{i\in \bar{\delta}} \tilde{p}_{2i} \oplus \min_{i\in \bar{\delta}}\{\tilde{p}_{3i}\}, \quad (6)$$

$$\widetilde{lb}_3 = \widetilde{LC}_3(\sigma) \oplus \sum_{i\in \bar{\delta}} \tilde{p}_{3i}, \quad (7)$$

where $\tilde{p}_{ji}$ is the fuzzy processing time of job i on machine j, the symbol $\oplus$ is the fuzzy addition and $\bar{\delta}$ denote the set of jobs that are not contained in the partial sequence σ.

Then, the lower bound on the completion of the corresponding partial sequence σ is

$$\widetilde{LB}(\sigma) = \max\{\widetilde{lb}_1, \widetilde{lb}_2, \widetilde{lb}_3\}. \quad (8)$$

Fuzzy makespan of the schedule produced by the algorithm can be expressed as

$$\begin{aligned} \tilde{m}_s &= \widetilde{max}\ \tilde{c}_{3i(k)} \\ &= \widetilde{max} \sum_{j=1}^{3} [\, \tilde{q}_{j(k)} \oplus \tilde{p}_{ji(k)} \,], \end{aligned} \quad (9)$$

where $\tilde{C}_{3i(k)}$ is the fuzzy completion time of the kth job i in order on machine 3, $\tilde{q}_{j(k)}$ is the fuzzy waiting time of the kth job between machines j-1 and j and $\tilde{p}_{ji(k)}$ is the fuzzy processing time of the kth job i in order on machine j. The operators *max* and *min* are the discrete maximum and minimum operators performed by the comparison method respectively. The operator $\widetilde{max}$ is the ordinary fuzzy maximum operator performed by the composite method. The fuzzy waiting time of the kth job between machines 2 and 3 is

$$\begin{aligned} \tilde{q}_{3(k)} &= \tilde{c}_{3i(k-1)} \ominus \tilde{c}_{2i(k)}, & k \geq 2, \\ &= 0, & k = 1, \end{aligned} \quad (10)$$

where the symbol $\ominus$ is the fuzzy subtraction and $\tilde{c}_{2i(k)}$ is the fuzzy completion time of the kth job i in order on machine 2 or

$$\tilde{c}_{2i(k)} = \sum_{j=1}^{2} (\tilde{q}_{j(k)} \oplus \tilde{p}_{ji(k)}). \quad (11)$$

The fuzzy waiting time between machines 1 and 2 is

$$\begin{aligned} \tilde{q}_{2(k)} &= \tilde{c}_{2i(k-1)} \ominus \tilde{c}_{1i(k)}, & k \geq 2, \\ &= 0, & k = 1, \end{aligned} \quad (12)$$

where $\tilde{c}_{1i(k)}$ is the fuzzy completion time of the kth job i in order on machine 1 or

$$\tilde{c}_{1i(k)} = \tilde{q}_{1(k)} \oplus \tilde{p}_{1(k)}. \quad (13)$$

The fuzzy waiting time for machine 1 is

$$\begin{aligned} \tilde{q}_{1(k)} &= \tilde{c}_{1i(k-1)}, & k \geq 2, \\ &= 0, & k = 1. \end{aligned} \quad (14)$$

Then, the optimal sequence and the fuzzy makespan can be constructed by the following fuzzified branch & bound algorithm.

[Step 1] Let $\tilde{p}_{ji}$ = processing time of job i on machine j and r = number of jobs in σ.

[Step 2] Set $r=1$.

[Step 3] Calculate $\widetilde{LB}(\sigma)$ for σ respectively.

[Step 4] Find a partial sequence node with the least lower bound $\widetilde{LB}(\sigma)$. If this is a partial sequence node with least lower bound, go to Step 5. Otherwise, go to Step 5 with new partial sequence node with least lower bound $\widetilde{LB}(\sigma)$.

[Step 5] If $r = n$, stop. Otherwise, go to Step 6.

[Step 6] Branch from this node to node with $r=r+1$ and return to Step 3.

5. Numerical Example

To illustrate the proposed approach, a four-job three-machine flow shop problem is presented as follows: For each processing time, a group of four experts was requested to express separate and subjective estimation using TFNs. Table 1 gives the TFN estimates for each processing time.

For each job in Table 1, row 1 gives the TFN of expert 1 and row 2 gives the TFN of expert 2, etc. For processing of each job, the manager applied the fuzzy Delphi method to converge the conflicting opinion among experts to a stable point and the manager is interested in the mean TFN such that all the distances between TFNs of experts and mean TFN for each processing time are $\delta \leq 0.13$. For example, the computation from TFNs of experts for processing time of job A on machine 1 gives the mean TFN

$$(a_1^m, b_1^m, c_1^m)^{1A} = (3, 6, 12). \tag{15}$$

If manager set $\beta_1 = 2$ and $\beta_2 = 16$, the distance between TFN of expert 1 and mean TFN for processing time of job A on machine 1 is 0.06. The process of the fuzzy Delphi method is continued until the criterion $\delta \leq 0.13$ for each processing time is reached. As a result of the process of the fuzzy Delphi method, the mean TFNs for processing times became

Table 1. The expert's TFN for each processing time

Job(i)	Machine 1	Machine 2	Machine 3
	Expert's TFN($\tilde{p}_{1i}$)	Expert's TFN($\tilde{p}_{2i}$)	Expert's TFN($\tilde{p}_{3i}$)
A	(2, 5, 13)	(7, 15, 17)	(15, 21, 27)
	(3, 5, 9)	(2, 11, 12)	(13, 16, 18)
	(4, 6, 10)	(1, 10, 15)	(6, 9, 15)
	(3, 8, 16)	(4, 8, 15)	(17, 25, 30)
B	(5, 7, 10)	(9, 9, 10)	(4, 6, 10)
	(4, 8, 11)	(6, 11, 21)	(8, 8, 10)
	(2, 5, 7)	(5, 10, 22)	(5, 5, 7)
	(3, 6, 8)	(11, 15, 19)	(3, 6, 9)
C	(7, 14, 17)	(4, 4, 4)	(10, 13, 16)
	(5, 12, 13)	(4, 6, 7)	(7, 9, 11)
	(6, 11, 14)	(5, 5, 5)	(8, 10, 15)
	(4, 15, 19)	(3, 7, 8)	(8, 9, 13)
D	(9, 12, 17)	(10, 12, 17)	(4, 7, 9)
	(6, 10, 14)	(9, 13, 15)	(6, 6, 6)
	(3, 8, 10)	(7, 9, 12)	(5, 5, 5)
	(7, 9, 11)	(13, 16, 19)	(5, 7, 8)

Table 2. Mean TFNs for each processing time after round 5

Job(i)	Machine 1	Machine 2	Machine 3
	$\tilde{p}_{1i}=(a_5^m,b_5^m,c_5^m)^{1i}$	$\tilde{p}_{2i}=(a_5^m,b_5^m,c_5^m)^{2i}$	$\tilde{p}_{3i}=(a_5^m,b_5^m,c_5^m)^{3i}$
A	(4, 8, 13)	(3, 13, 13)	(12, 20, 24)
B	(3, 8, 8)	(8, 13, 17)	(5, 7, 9)
C	(6, 13, 15)	(5, 6, 7)	(8, 11, 14)
D	(6, 12, 13)	(11, 13, 14)	(7, 7, 7)

sufficiently stable as TFNs listed in Table 2.

Using these mean TFNs in Table 2, when $r=1$, we obtain the result shown in Figure 2. The node xxxx indicates the start node with no job sequenced and an 'x' in the processing sequence indicates that no job has yet been assigned that position.

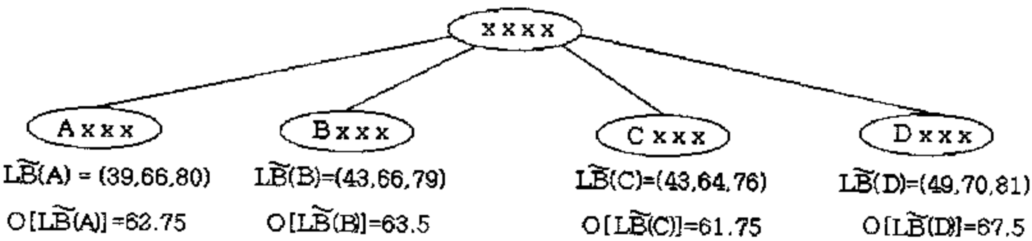

Figure 2. The first branching tree

For partial sequence node Axxx, the lower bound calculations are

$$\tilde{lb}_1 = \tilde{LC}_1(A) + \{\tilde{p}_{1B} + \tilde{p}_{1C} + \tilde{p}_{1D}\}$$
$$+ \min\{\tilde{p}_{2B} + \tilde{p}_{3B},\ \tilde{p}_{2C} + \tilde{p}_{3C},\ \tilde{p}_{2D} + \tilde{p}_{3D}\}$$
$$= (32,\ 58,\ 70)$$

$$\tilde{lb}_2 = \tilde{LC}_2(A) + \{\tilde{p}_{2B} + \tilde{p}_{2C} + \tilde{p}_{2D}\}$$
$$+ \min\{\tilde{p}_{3B},\ \tilde{p}_{3C},\ \tilde{p}_{3D}\}$$
$$= \tilde{LC}_1(A) + \tilde{p}_{2A} + \{\tilde{p}_{2B} + \tilde{p}_{2C} + \tilde{p}_{2D}\}$$
$$+ \min\{\tilde{p}_{3B},\ \tilde{p}_{3C},\ \tilde{p}_{3D}\}$$
$$= (31,\ 60,\ 71)$$

$$\tilde{lb}_3 = \tilde{LC}_3(A) + \{\tilde{p}_{3B} + \tilde{p}_{3C} + \tilde{p}_{3D}\}$$
$$= \tilde{LC}_2(A) + \tilde{p}_{A1} + \{\tilde{p}_{3B} + \tilde{p}_{3C} + \tilde{p}_{3D}\}$$
$$= (31,\ 66,\ 80)$$

$$\tilde{LB}(A) = \max\{\tilde{lb}_1, \tilde{lb}_2, \tilde{lb}_3\} = (39, 66, 80)$$
$$O[\tilde{LB}(A)] = (39 + 132 + 80)/4 = 62.75$$

The node Cxxx has the lowest lower bound, because it has the smallest associated ordinary number of the lower bound. Therefore, the node Cxxx branches to the nodes CAxx, CBxx and CDxx. Finally, the fuzzified branch & bound algorithm yields optimal job sequence of A-C-B-D. The entire branching tree for the optimal solution is shown in Figure 3. The fuzzy lower bound on the completion of the optimal sequence is (39, 66, 80). The fuzzy parameters for the optimal sequence are summarized in Table 3. Since a negative waiting time is unrealistic, the negative portion of this TFN is deleted. However, this leaves a non-triangular fuzzy number. This will make subsequent waiting time calculations unwieldy. Therefore, the non-triangular fuzzy waiting time is modified to a trianglar fuzzy waiting time. Figure 4 illustrates the calculation of the $\tilde{q}_{3(2)}$. Using the composite method, the fuzzy makespan of the optimal sequence is

$$\tilde{m}_s = \widetilde{\max}\ \tilde{c}_{3i(k)} = \tilde{c}_{34(4)} = (37, 66, 291). \qquad (16)$$

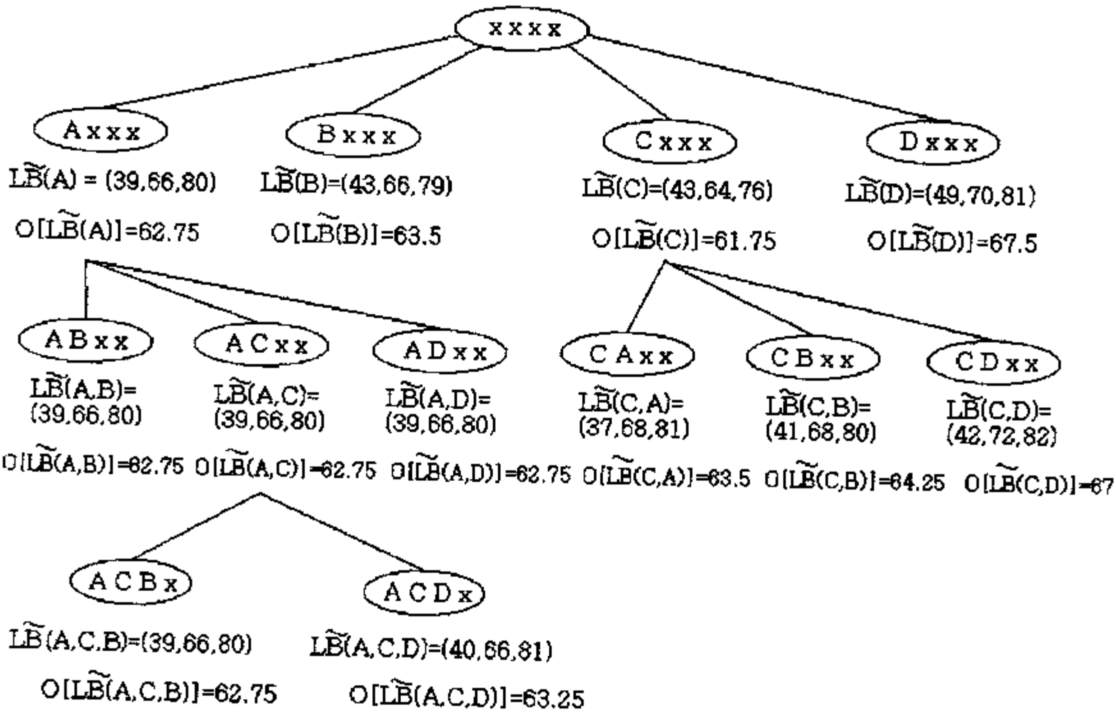

Figure 3. The entire branching tree for the optimal solution

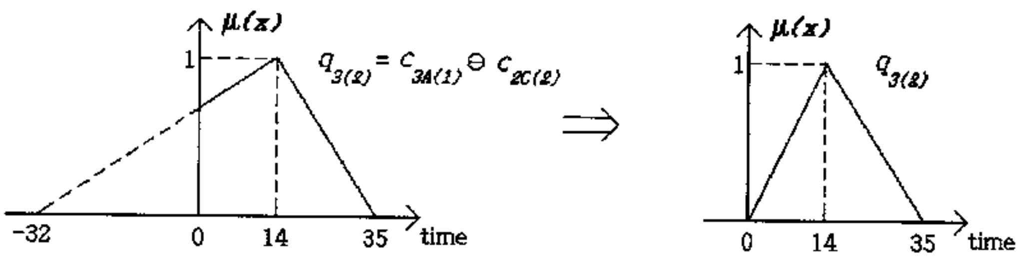

Figure 4. The calculation of the $\tilde{q}_{3(2)}$

Table 3. Fuzzy parameters for the optimal sequence

i	k	$\tilde{q}_{1(k)}$	$\tilde{p}_{1i(k)}$	$\tilde{c}_{1i(k)}$	$\tilde{q}_{2(k)}$	$\tilde{p}_{2i(k)}$	$\tilde{c}_{2i(k)}$	$\tilde{q}_{3(k)}$	$\tilde{p}_{3i(k)}$	$\tilde{c}_{3i(k)}$
A	1	0	(4, 8,13)	(4, 8,13)	0	(3,13,13)	(7,21, 26)	0	(12,20,24)	(19,41, 50)
C	2	(4, 8,13)	(6,13,15)	(10,21,28)	(0, 0,16)	(5, 6, 7)	(15,27, 51)	(0,14, 35)	(8,11,14)	(23,52,100)
B	3	(10,21,28)	(3, 8, 8)	(13,29,36)	(0, 0,38)	(8,13,17)	(21,42, 91)	(0,10, 79)	(5, 7, 9)	(26,59,179)
D	4	(13,29,36)	(6,12,13)	(19,41,49)	(0, 1,72)	(11,13,14)	(30,55,135)	(0, 4,149)	(7, 7, 7)	(37,66,291)

What is gained by using the fuzzy numbers is to yield fuzzy answers(fuzzy parameters), which is intuitively appealing when processing times are themselves fuzzy. Since the answers are fuzzy, a spread or range of answers is provided for the manager to perform subsequent goal analyses, whereas it was assumed away and lost in the deterministic simplification.

6. Conclusions

In this paper, an efficient approach based on fuzzy set theory was developed to solve the job scheduling problem under fuzzy environment. The direct use of fuzzy numbers to modeling imprecision for processing times was emphasized in this paper. In particular, TFNs were used to reflect the reality of human judgment process in prediction. The fuzzy Delphi method was applied to estimate a reliable time interval of each processing time. Although the processing times can be described using many types of shapes, because of the increase in computational effort and the difficulty in estimating a general fuzzy number representation, for scheduling problems it may be the best choice to use triangular shapes to describe the valuation data with certain levels of presumption.

The methodology described in this paper is not limited to only the three-machine flow shop problem. Many models in job scheduling can be constructed using the proposed methodology. This methodology may be useful for solving large scale job scheduling problems that have never been attempted before.

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96년 3월 최초 접수, 96년 8월 최종 수정