⊠ 연구논문

# Optimal Design of Accelerated Life Tests with Different Censoring Times*

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## Abstract

This paper presents optimal accelerated life test plans with different censoring times for exponential, Weibull, and lognormal lifetime distributions, respectively. For an optimal plan, low stress level, proportion of test units allocated and censoring time at each stress are determined such that the asymptotic variance of the maximum likelihood estimator of a certain quantile at use condition is minimized.

The proposed plans are compared with the corresponding optimal plans with a common censoring time over range of parameter values. Computational results indicate that those plans are statistically optimal ones in terms of accuracy of estimator when total censoring times of two plans are equal.

## 1. Introduction

Most modern products have very high reliability when operating within their use(normal, design) environment. This presents a problem in measuring reliabilities of such products due to long lives of the products and relatively shorter time available for testing purpose.

One solution to problem of obtaining meaningful test data is accelerated life tests (ALT's) to test items at higher-than-usual levels of stress so that we get information quickly on their failure time distribution at use stress level. Generally, information from tests at overstress levels of stress(e.g., use rate, temperature,

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voltage, pressure, etc.) is extrapolated, through a physically reasonable model, to obtain estimates of life at usual level of stress. Nelson(1990) provides a comprehensive source for background material, practical methodology, and examples.

In some cases, stress is increased or otherwise changed during the course of a test (step-stress and progressive stress ALT's) instead of holding the stress at a constant level during testing period (constant-stress ALT's)[Meeker and Escobar, 1993].

Traditional constant-stress test plans generally use equally spaced test stresses each with the same number of test units. Such test plans are usually inefficient for estimating the $q$ th quantile at use stress. Statistical optimal plans assuming a two-parameter life-stress relationship will test units at only two levels of stress. They choose levels of stress and allocations to satisfy a specified optimality criterion (see Bai and Chung(1991) and Yum and Choi(1989) for exponential distribution, Meeker and Nelson(1975, 1978) and Seo and Yum(1991) for Weibull distribution and Kielpinski and Nelson(1975, 1976) for lognormal distribution). Also compromise test plans with 3 or 4 levels of stress have somewhat reduced statistical efficiency, but tend to be more robust to misspecifications of unknown inputs[3, 5, 11]. Statistical optimal plans are useful because they provide a benchmark for best precision that one can achieve for a given model. They also provide important insights and a useful starting point from which to develop more compromise plans.

The most test plans employ a common censoring time at each stress level. This practice does not completely cover field applications. A censoring time that is long enough to yield sufficient failures at low stress is too long at high stress. The accuracy of estimators from a test plan with a common censoring time is less than that from a test plan with different censoring times. This paper devises statistically optimal plans with a longer censoring time at low stress than at high stress. Recently, Yang and Jin(1994) presented the best compromise 3-level constant-stress ALT plans for Weibull distribution with different censoring times and Yang(1994) discussed optimal design of 4-level constant-stress ALT plans with various censoring times to minimize simultaneously the asymptotic variance of the maximum likelihood estimator(MLE) of the mean log life at use stress and total running time for lognormal and Weibull models.

The present investigation is different from the Yang and Lin and Yang works in three directions. First, optimality criterion of design is to minimize asymptotic variance of the MLE of the $q$ th quantile at use stress while above works adopted cost functions considering asymptotic variance of the MLE of mean log time at use stress and total running time. Second, we consider exponential, Weibull and lognormal distributions as models to describe failure mechanism. Third, this paper deals with statistically optimal plans to employ subexperiments at two overstress levels.

Computational experiments are conducted for various combinations of parameters involved and relative efficiency of ALT plan with different censoring times (ALTDC) to the corresponding ALT plan with a common censoring time(ALTCC) is evaluated.

# 2. The Model

## 2.1 Notation

| | |
|---|---|
| $N$ | Total sample size |
| $s_0$, $s_1$, $s_2$ | Standardized use, low and high (transformed) test stresses |
| $n_i$ | Number of test units allocated at $s_i$, $i=1, 2$ |
| $\alpha_i$ | $n_i/N$, $i=1, 2$ |
| $t_{ci}$ | Standardized censoring time at $s_i$, $i=1, 2$ |
| $\tau$ | Total allowable test length at $s_1$ and $s_2$ |
| $t_{c0}$ | Standardized censoring time at use or high stress necessary to guess $P_u$ or $P_h$ |
| $P_u$ | Probability that an item tested at use stress will fail by time $t_{c0}$ |
| $P_h$ | Probability that an item tested at high stress will fail by time $t_{c0}$ |
| $T$ | Lifetime of a test unit (random variable) |
| $Y$ | $\ln T$ |
| $\theta$ | Location parameter of location-scale family of distribution |
| $\sigma$ | Scale parameter of location-scale family of distribution |
| $\beta_0$ | Standardized intercept of log linear relationship between location parameter and stress |
| $\beta_1$ | Standardized slope of log linear relationship between location parameter and stress |
| $t_q$ | $q$ th quantile of the lifetime distribution at use condition |

| | |
|---|---|
| $f(\cdot)$ | Probability density function of lifetime of a test unit |
| $F(\cdot)$ | Cumulative distribution function of lifetime of a test unit |
| $\phi(\cdot)$ | Probability density function of standard normal distribution |
| $\Phi(\cdot)$ | Cumulative distribution function of standard normal distibution |
| $\mathrm{avar}(\cdot)$ | Asymptotic variance |
| $v_0$ | Standardized asymptotic variance |

## 2.2 Assumptions

The following assumptions are made.

1. The log times of test units to failure are distributed with a specified location-scale family of distribution $F(y:\theta,\sigma)$ where $\theta$ and $\sigma$ are location and scale parameters, respectively. That is, the lifetime distribution is exponential or Weibull or lognormal for any constant stress.
2. The location parameter is a log linear function of stress, i.e.

$$\ln\theta_i = \beta_0 + \beta_1 s_i . \tag{1}$$

3. The scale parameter does not depend on the stress level.
4. The lifetimes of test units are statistically independent.

The inverse power law and Arrhenius models are special cases of the simple life-stress relationship (1) for a single accelerating stress. The most commonly used distributions for $F(\cdot)$ are smallest extreme value distribution (corresponding to logs of Weibull and exponential data) and normal distribution (corresponding to logs of lognormal data).

## 2.3 The test method

It is assumed that

1. The test uses only two stresses, $s_1$ and $s_2$.
2. The high stress $s_2$ is specified.
3. $N\alpha_1$ test units randomly chosen among $N$ test units are allocated to low stress $s_1$ and the remaining items are allocated to high stress $s_2$.
4. The total allowable test length at $s_1$ and $s_2$ is given, i.e., $\tau = t_{c_1} + t_{c_2}$ . The units allocated to $s_i$ are tested simultaneously until the censoring time( $t_{c_i}$ ) at $s_i$

## 2.4 The estimation method

The ML estimation method is used since the method provides asymptotically minimum variance estimates and generally compares well with other estimates for small sample sizes. Asymptotic (large-sample) variance of the $q$th quantile of the lifetime distribution at use condition is adopted as a criterion for determining optimal plans.

Parameters are standardized such that use condition and high stress level as well as total censoring time( $\tau$ ) at two overstress levels become 0, 1, and 2, respectively. Such standardization does not alter the nature of the problem[Seo and Yum, 1991].

## 2.5 Asymptotic variance of MLE of $q$th quantile at use stress

2.5.1 The case of exponential distribution

Assume that lifetimes( $T$ ) of test units at stress level $s$ are independent and identically distributed with probability density function :

$$f(t) = (1/\theta)\exp(-t/\theta), \quad t>0. \tag{2}$$

Of particular interest is the logarithm of the mean lifetime at use condition which is defined by

$$\mu_0 \equiv \ln\theta_0 = \beta_0 + \beta_1 s_0. \tag{3}$$

Note that $t_q$, the $q$th quantile of the exponential distribution at use condition, is related to $\mu_0$ as follows :

$$y_q = \ln t_q = \mu_0 + \ln\{-\ln(1-q)\}. \tag{4}$$

Let $\hat\beta_0$ and $\hat\beta_1$ be ML estimates of $\beta_0$ and $\beta_1$, respectively. Then, ML estimates of $\mu_0$ and $y_q$ is

$$\hat\mu_0 = \widehat{\ln\theta_0} = \hat\beta_0 + \hat\beta_1 s_0$$

$$\hat y_q = \hat\mu_0 + \ln\{-\ln(1-q)\}.$$

The Fisher information matrix is given by

$$F_E = N(f_{jk}),\ j,\ k=0,\ 1 \tag{5}$$

$$f_{00} = \sum_{i=1}^{2} \alpha_i Q_i$$

$$f_{01} = f_{10} = \sum_{i=1}^{2} \alpha_i s_i Q_i$$

$$f_{11} = \sum_{i=1}^{2} \alpha_i s_i^2 Q_i$$

where $Q_i = 1-\exp(-t_{ci}/\theta_i)$ [Yum and Choi, 1989].

Thus, asymptotic variance(avar) of $\hat{\mu}_0$ and $\hat{y}_q$ is obtained by

$$avar(\hat{y}_q) = avar(\hat{\mu}_0) = avar(\hat{\beta}_0+\hat{\beta}_1 s_0)$$

$$= \frac{N^{-1}\{s_2^2 Q_2 + (s_1^2 Q_1 - s_2^2 Q_2)\alpha_1\}}{Q_1 Q_2 (s_1-s_2)^2(-\alpha_1^2+\alpha_1)} \tag{6}$$

We also define

$$v_o = \frac{N}{\sigma^2} avar(\hat{y}_q)$$

for later use.

2.5.2 The case of Weibull distribution

The lifetimes of test units at stress level $s$ follow a Weibull distribution with probability density function :

$$f(t) = (\delta/\theta)(t/\theta)^{\delta-1}\exp\{-(t/\theta)^{\delta}\},\quad t>0. \tag{7}$$

It is further assumed that scale parameter $\theta$ and stress $s$ are related as $\theta = \exp(\beta_0+\beta_1 s)$. Since lifetime $T$ at a stress level $s$ follows a Weilbull distribution, $Y = \ln T$ has the smallest extreme value distribution with the cumulative distribution function :

$$G(y) = 1-\exp[-\exp\{(y-\mu)/\sigma\}],\ -\infty<y<\infty$$

where $\mu = \ln\theta = \beta_0+\beta_1 s,\quad \sigma = \dfrac{1}{\delta}$ .

In terms of the extreme value distribution, we want to estimate

$$y_q = \ln t_q = (\beta_0 + \beta_1 s_0) + \sigma \ln\{-\ln(1-q)\} . \tag{8}$$

Then, Fisher information matrix for $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2 = \hat{\sigma}$ is given by

$$F_W = N\,(f_{jk}),\ j,\ k=0,\ 1,\ 2 \tag{9}$$

$$f_{00} = \sigma^{-2}\sum_{i=1}^{2}\alpha_i H_i^{(1)}$$

$$f_{01} = f_{10} = \sigma^{-2}\sum_{i=1}^{2}\alpha_i s_i H_i^{(1)}$$

$$f_{02} = f_{20} = \sigma^{-2}\sum_{i=1}^{2}\alpha_i H_i^{(2)}$$

$$f_{11} = \sigma^{-2}\sum_{i=1}^{2}\alpha_i s_i^2 H_i^{(1)}$$

$$f_{12} = f_{21} = \sigma^{-2}\sum_{i=1}^{2}\alpha_i s_i H_i^{(2)}$$

$$f_{22} = \sigma^{-2}\sum_{i=1}^{2}\alpha_i H_i^{(3)}$$

where

$$H_i^{(1)} = \int_{-\infty}^{z_{ci}} g(z)dz = G(z_{ci})$$

$$H_i^{(2)} = \int_{-\infty}^{z_{ci}} (1+z)g(z)dz$$

$$H_i^{(3)} = \int_{-\infty}^{z_{ci}} (1+z)^2 g(z)dz$$

$z_{ci} = (\ln t_{ci} - \beta_0 - \beta_1 s_i)/\sigma$ [Nelson and Meeker, 1978].

Thus, asymptotic variance of $\hat{y}_q$ is obtained as

$$avar(\hat{y}_q) = a' F^{-1} a \tag{10}$$

where $a' = (1,\ s_0,\ \ln\{-\ln(1-q)\})$ .

The standardized asymptotic variance is also given by

$$v_0 = (N/\sigma^2)\, avar(\hat{y}_q) .$$

2.5.3 The case of lognormal distribution

The lifetimes $(T)$ of test units at stress level $s$ follow a lognormal distribution with probability density function :

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma t}\exp\left\{-\frac{(\ln t-\theta)^2}{\sigma^2}\right\},\quad t>0 \tag{11}$$

Also, it is assumed that scale parameter $\theta$ and stress $s$ are related as equation (1) and we want to estimate

$$y_q = \ln t_q = (\beta_0+\beta_1 s_0)+\sigma z_q \tag{12}$$

where $z_q$ is the $q$th quantile of standard normal distribution.

The Fisher information matrix for $\hat{\beta}_0$, $\hat{\beta}_1$ and $\hat{\beta}_2=\hat{\sigma}$ is given by

$$\begin{aligned}
F_L &= N(f_{jk}),\quad j,\ k = 0,\ 1,\ 2\\
f_{00} &= \sigma^{-2}\sum_{i=1}^{2}A_i\\
f_{01} &= f_{10} = \sigma^{-2}\sum_{i=1}^{2}s_iA_i\\
f_{11} &= \sigma^{-2}\sum_{i=1}^{2}s_i^2A_i\\
f_{02} &= f_{20} = \sigma^{-2}\sum_{i=1}^{2}s_i^2B_i\\
f_{12} &= f_{21} = \sigma^{-2}\sum_{i=1}^{2}s_iB_i\\
f_{22} &= \sigma^{-2}\sum_{i=1}^{2}C_i
\end{aligned} \tag{13}$$

where

$$A_i = \Phi(z_{ci})-\phi(z_{ci})\left[s_i-\frac{\phi(z_{ci})}{1-\Phi(z_{ci})}\right]$$

$$B_i = -\phi(z_{ci})\left[1+s_i\left\{s_i-\frac{\phi(z_{ci})}{1-\Phi(z_{ci})}\right\}\right]$$

$$C_i = 2\Phi(z_{ci})-s_i\phi(z_{ci})\left[1+s_i^2-\frac{s_i\phi(z_{ci})}{1-\Phi(z_{ci})}\right]$$

[Nelson and Kielpinski, 1976].

Thus, asymptotic variance of $\hat{y}_q$ has the same form as equation (10) and the standardized asymptotic variance is defined by

$$v_0 = \frac{N}{\sigma^2}avar(\hat{y}_q).$$

# 3. Optimal Plans

The problem of optimally designing an ALT with different censoring times can now be stated as; given $N$, $s_0$, $s_2$, and $q$, determine $s_1$ and $\alpha_i$, $t_{ci}$, $i=1, 2$ such that $v_0$ is minimized. We also standardize a censoring time necessary to guess $P_u$ and $P_h$ as 1. The optimization procedure is initiated by first providing guesstimates of the following quantities

$$P_u = P_r\{\text{a test unit fails at use condition in } (0, t_{c0}=1]\}, \tag{14}$$

$$P_h = P_r\{\text{a test unit fails at high stress level in } (0, t_{c0}=1]\}. \tag{15}$$

Then, $\beta_0/\sigma$ and $\beta_1/\sigma$ ( $\sigma=1$ for the exponential case) can be easily determined.

Optimal plans with a common censoring time at $s_1$ and $s_2$ are usually inefficient. In such test plans, a censoring time that is long to yield sufficient failures at $s_1$ is too long at $s_2$. Our developed plans choose a longer censoring time at $s_1$ than one at $s_2$ in order to share with total allowable testing time efficiently.

Therefore a common optimization model for designing optimal test plans for any distribution is given by :

$$\min_{s_1, \alpha_1, t_{c1}} \; v_0 \tag{16}$$

$$\text{s.t.} \quad 0 < s_1 < 1,$$

$$0 < \alpha_1 < 1$$

$$0 < t_{c1} < 2 \; .$$

For cases of Weibull and lognormal distributions, test plans are determined by using the Powell(1964) algorithm for finding the minimum of a function without calculating derivatives with respect to $s_1$, $\alpha_1$, and $t_{c1}$. In case of exponential distribution, the following two step procedure is adopted to minimize $v_0$ with respect to $s_1$, $\alpha_1$, and $t_{c1}$.

First, we optimize $\alpha_1$ for given $s_2$ and $t_{c1}$ by equation(17)[Yum and Choi, 1989] :

$$\alpha_1^* = (-s_2^2 Q_2 + \sqrt{s_1^2 s_2^2 Q_1 Q_2}\,) \,/\, (s_1^2 Q_1 - s_2^2 Q_2) \; . \tag{17}$$

The second stage of optimization employs the Powell's method with respect to $s_1$ and $t_{c1}$.

Optimal values of $s_1$, $\alpha_1$, $t_{c1}$, and $v_0$ are tabulated in Tables 1, 2, and 3 for cases of exponential, Weibull and lognormal distributions, respectively. For plans with a common censoring time ( $t_{c1}=t_{c2}=1$), the corresponding optimal $s_1$ and $\alpha_1$ are determined using the methods described in Nelson, Meeker and other authors[2, 6, 8, 9, 14].

In the Tables, the following quantities are also included

$$R = \frac{v_1 \text{ for ALTCC}}{v_0 \text{ for ALTDC}} . \tag{18}$$

Thus, $R$ represents the relative efficiency of ALTDC plan with respect to the corresponding ALTCC one.

We illustrate the developed plans with an example.

< Example >

Suppose that a certain type of electrical capacities is known to have a Weibull distribution. An ALT is to be conducted using temperature as a stress. The use condition is 30℃ and number of available tested items are 300. The experimenter assures that relationship (1) is valid up to 70 ℃ (high stress level) and total allowed test duration is 2000hrs.

The experimenter's guesstimates of $P_u$ and $P_h$ are 0.001, and 0.9, respectively, and the 1th percentile of the lifetime distribution at use condition is of interest.

From <Table 2>, the selected plan is characterized in terms of the original scale denoted with the prime by

$$(s_1', s_2') = (53, 70)$$
$$(n_1, n_2) = (236, 64)$$
$$(t_{c1}', t_{c2}') = (1602, 398) .$$

# 4. Discussion

Base upon the computational results of Tables 1~3, we observe the following.

1) For three lifetime distributions, $v_0$ of ALTDC is quite smaller than $v_0$ of the corresponding ALTCC.
2) For exponential distribution, $R$ increases as $P_u$ increases.
3) For Weibull distribution, $R$ increases as $q$ increases and/or $P_h$ decresases
4) For lognormal distribution, $R$ increases as $q$ increases and/or $P_h$ decreases
5) For three lifetime distributions, $s_1$ of ALTDC is lower than $s_1$ of the corresponding ALTCC. Thus, ALTDC plans reduce the hazards of extraploation in stress (see Meeker and Hahn(1985))
6) For all the cases of each distribution, $t_{c1}$ of ALTDC is longer than $t_{c2}$. The statistcially optimal plans to minimize a given criterion should use a longer censoring time at low stress than at high stress.

The optimal ALT plan requires the knowledge of $P_u$ and $P_h$ (or, equivalently $\beta_0/\sigma$ and $\beta_1/\sigma$ ). This situation is termed "locally optimal design" and sensitivity analysis is needed[Meeker, 1984]. Let $\widetilde{P_u}$ and $\widetilde{P_u}$ be the guessed values of $P_u$ and $P_h$, respectively. For the guessed values, false optimal $s_1$, $\alpha_1$, and $t_{c1}$ can be determined using $\widetilde{P_u}$ and $\widetilde{P_h}$. Then, the sensitivity is defined as the ratio of $v_0$ with false optimal $s_1$, $\alpha_1$ and $t_{c1}$ to $v_0$ with true optimal $s_1$, $\alpha_1$ and $t_{c1}$. These ratios are calculated for ALTDC and ALTCC in cases of exponential Weibull and lognormal distributions, respectively when $\widetilde{P_u} = 0.001$, $\widetilde{P_h} = 0.9$ (and $q = 0.01$) as shown in Tables 4~6. ALTCC is less sensitive to departures from true values of $P_u$ and $P_h$ than ALTDC in most cases but the difference of sensitivites of two plans is trivial little.

In conclusion, this paper presents statistically optimal plans with different censoring times to employ subexperiments at two stress levels under the assumption of exponential, Weibull, and lognormal distributions, respectively Compared with the statistically optimal plans with a common censoring time ALTDC plan provides higher efficiency than the corresponding ALTCC plan when total censoring times of two plans are equal. Accordingly, the proposed ALTDC plan may be virtually optimal ALT one to minimize the asymptotic variance of the MLE of a given quantile of failure time distribution at use condition.

< Table 1 > Optimal ALT Plans : the Exponential Case

| $P_u$ | $P_h$ | $\beta_0$ | $\beta_1$ | ALTDC | | | | ALTCC | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $s_1^*$ | $\alpha_1^*$ | $t^*_{c1}$ | $\upsilon_0$ | $s_1^*$ | $\alpha_1^*$ | $R$ |
| 0.0001 | 0.99 | 9.2103 | -10.7375 | 0.6770 | 0.7351 | 1.6896 | 82.35 | 0.7066 | 0.7690 | 1.333 |
| | 0.9 | | -10.0443 | 0.6799 | 0.7494 | 1.6284 | 124.50 | 0.7108 | 0.7950 | 1.283 |
| | 0.5 | | -8.8438 | 0.6621 | 0.7677 | 1.5776 | 277.99 | 0.6976 | 0.8243 | 1.240 |
| | 0.1 | | -6.9599 | 0.5793 | 0.7906 | 1.5867 | 1015.10 | 0.6299 | 0.8489 | 1.250 |
| | 0.01 | | -4.6101 | 0.3403 | 0.8497 | 1.6995 | 3902.53 | 0.4450 | 0.8896 | 1.352 |
| 0.001 | 0.99 | 6.9073 | -8.4344 | 0.5870 | 0.7595 | 1.7089 | 47.34 | 0.6264 | 0.7897 | 1.356 |
| | 0.9 | | -7.7413 | 0.5812 | 0.7737 | 1.6565 | 68.78 | 0.6247 | 0.8152 | 1.312 |
| | 0.5 | | -6.5407 | 0.5353 | 0.7960 | 1.6252 | 139.27 | 0.5911 | 0.8470 | 1.283 |
| | 0.1 | | -4.6569 | 0.3446 | 0.8481 | 1.6992 | 384.16 | 0.4469 | 0.8879 | 1.352 |
| 0.01 | 0.99 | 4.6001 | -6.1273 | 0.4255 | 0.8083 | 1.7501 | 21.85 | 0.4858 | 0.8288 | 1.408 |
| | 0.9 | | -5.4342 | 0.3915 | 0.8268 | 1.7223 | 29.21 | 0.4654 | 0.8555 | 1.382 |
| | 0.5 | | -4.2336 | 0.2393 | 0.8798 | 1.7711 | 46.67 | 0.3683 | 0.8989 | 1.424 |
| 0.1 | 0.99 | 2.2504 | -3.7775 | 0.0000 | 0.9999 | 2.0000 | 5.26 | 0.1659 | 0.9341 | 1.749 |

< Table 2 > Optimal ALT Plans : the Weibull Case

| $q$ | $P_u$ | $P_h$ | $\beta_0$ | $\beta_1$ | ALTDC | | | | ALTCC | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $s_1^*$ | $\alpha_1^*$ | $t^*_{c1}$ | $\upsilon_0$ | $s_1^*$ | $\alpha_1^*$ | $R$ |
| .01 | .0001 | .99 | 9.2103 | -10.7375 | .6765 | .7364 | 1.6864 | 82.3505 | .7268 | .7344 | 1.415 |
| | | .9 | | -10.0443 | .6843 | .7346 | 1.6664 | 125.2059 | .7380 | .7354 | 1.465 |
| | | .5 | | -8.8438 | .6746 | .7247 | 1.6728 | 287.2296 | .7329 | .7279 | 1.550 |
| | | .1 | | -6.9599 | .6264 | .6942 | 1.7282 | 1145.0206 | .6867 | .6991 | 1.679 |
| | .001 | .99 | 6.9073 | -8.4344 | .5752 | .7807 | 1.6204 | 49.0368 | .6350 | .7803 | 1.321 |
| | | .9 | | -7.7413 | .5759 | .7881 | 1.6023 | 69.4317 | .6444 | .7868 | 1.371 |
| | | .5 | | -6.5407 | .5365 | .7928 | 1.6339 | 139.3060 | .6251 | .7868 | 1.484 |
| | | .1 | | -4.6569 | .4051 | .7806 | 1.7638 | 408.2821 | .5251 | .7651 | 1.753 |
| | .01 | .99 | 4.6001 | -6.1273 | .4321 | .7866 | 1.4384 | 29.3565 | .4731 | .8216 | 1.119 |
| | | .9 | | -5.4342 | .4183 | .8228 | 1.3778 | 36.6686 | .4608 | .8541 | 1.121 |
| | | .5 | | -4.2336 | .3089 | .8727 | 1.3823 | 57.8838 | .3677 | .8993 | 1.149 |
| | | .1 | | -2.3498 | .0000 | 1.0000 | 1.9999 | 73.9273 | .0000 | 1.0000 | 1.353 |
| .1 | .0001 | .99 | 9.2103 | -10.7375 | .7024 | .6775 | 1.7775 | 88.2454 | .7503 | .6798 | 1.554 |
| | | .9 | | -10.0443 | .7115 | .6686 | 1.7559 | 138.8025 | .7579 | .6757 | 1.598 |
| | | .5 | | -8.8438 | .7105 | .6485 | 1.7470 | 335.0069 | .7505 | .6596 | 1.657 |
| | .001 | .99 | 6.9073 | -8.4344 | .6111 | .7168 | 1.7737 | 49.2318 | .6761 | .7118 | 1.560 |
| | | .9 | | -7.7413 | .6161 | .7067 | 1.7584 | 74.2162 | .6820 | .7061 | 1.616 |
| | | .5 | | -6.5407 | .6028 | .6827 | 1.7620 | 164.4635 | .6620 | .6863 | 1.703 |
| | .01 | .99 | 4.6001 | -6.1273 | .4352 | .7964 | 1.7681 | 21.9243 | .5368 | .7747 | 1.553 |
| | | .9 | | -5.4342 | .4213 | .7901 | 1.7701 | 29.9316 | .5343 | .7709 | 1.644 |
| | | .5 | | -4.2336 | .3638 | .7699 | 1.8098 | 53.4459 | .4781 | .7507 | 1.834 |
| | .1 | .99 | 2.2504 | -3.7775 | .0000 | 1.0000 | 1.9999 | 7.3115 | .1489 | .9408 | 1.271 |
| | | .9 | | -3.0844 | .0000 | 1.0000 | 1.9999 | 7.3115 | .0577 | .9795 | 1.357 |
| | | .5 | | -1.8839 | .0000 | 1.0000 | 1.9999 | 7.3115 | .0000 | 1.0000 | 1.369 |

< Table 3 > Optimal ALT Plans : the Lognormal Case

| $q$ | $P_u$ | $P_h$ | $\beta_0$ | $\beta_1$ | ALTDC | | | | ALTCC | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $s_1^*$ | $\alpha_1^*$ | $t^*_{c1}$ | $\nu_0$ | $s_1^*$ | $\alpha_1^*$ | $R$ |
| .01 | .0001 | .99 | 3.7190 | -6.0454 | .3226 | .8138 | 1.7783 | 8.5472 | .4247 | .7860 | 1.397 |
| | | .9 | | -5.0006 | .3556 | .8146 | 1.7191 | 11.6235 | .4764 | .7760 | 1.548 |
| | | .5 | | -3.7190 | .3770 | .8031 | 1.6647 | 21.9264 | .5314 | .7542 | 1.861 |
| | | .1 | | -2.4375 | .3593 | .7419 | 1.6436 | 63.9529 | .5522 | .7079 | 2.491 |
| | .001 | .99 | 3.0902 | -5.4166 | .2452 | .8287 | 1.7277 | 6.9540 | .3481 | .8171 | 1.310 |
| | | .9 | | -4.3718 | .2628 | .8473 | 1.6599 | 8.7245 | .3900 | .8175 | 1.440 |
| | | .5 | | -3.0902 | .2224 | .8790 | 1.6529 | 13.8103 | .4261 | .8082 | 1.780 |
| | | .1 | | -1.8087 | .0000 | 1.0000 | 1.9989 | 17.2813 | .3963 | .7717 | 4.269 |
| | .01 | .99 | 2.3263 | -4.6527 | .1480 | .8020 | 1.6244 | 5.7107 | .2325 | .8419 | 1.161 |
| | | .9 | | -3.6079 | .1562 | .8367 | 1.4961 | 6.5544 | .2439 | .8704 | 1.195 |
| | | .5 | | -2.3263 | .0970 | .8921 | 1.3979 | 8.7066 | .2004 | .9165 | 1.272 |
| | | .1 | | -1.0448 | .0000 | 1.0000 | 1.9990 | 9.7995 | .0000 | 1.0000 | 1.422 |
| .1 | .0001 | .99 | 3.7190 | -6.0454 | .3608 | .7774 | 1.9005 | 7.9542 | .4655 | .7343 | 1.644 |
| | | .9 | | -5.0006 | .4011 | .7484 | 1.8488 | 11.7341 | .5117 | .7098 | 1.812 |
| | | .5 | | -3.7190 | .4547 | .6921 | 1.7593 | 25.0850 | .5605 | .6684 | 2.083 |
| | .001 | .99 | 3.0902 | -5.4166 | .2788 | .8210 | 1.9035 | 5.7708 | .3999 | .7660 | 1.672 |
| | | .9 | | -4.3718 | .3076 | .7943 | 1.8585 | 7.9705 | .4403 | .7411 | 1.869 |
| | | .5 | | -3.0902 | .3433 | .7373 | 1.7807 | 15.1223 | .4745 | .6957 | 2.207 |
| | .01 | .99 | 2.3263 | -4.6527 | .1411 | .9014 | 1.9170 | 3.6053 | .2938 | .8217 | 1.719 |
| | | .9 | | -3.6079 | .1411 | .8878 | 1.8906 | 4.3525 | .3191 | .8008 | 1.996 |
| | | .5 | | -2.3263 | .1260 | .8556 | 1.8467 | 6.2535 | .3141 | .7557 | 2.580 |
| | .1 | .99 | 1.2816 | -3.6079 | .0000 | 1.0000 | 1.9993 | 2.0175 | .0546 | .9650 | 1.367 |
| | | .9 | | -2.5631 | .0000 | 1.0000 | 1.9998 | 2.0175 | .0211 | .9851 | 1.435 |
| | | .5 | | -1.2816 | .0000 | 1.0000 | 1.9999 | 2.0175 | .0000 | 1.0000 | 1.445 |

< Table 4 > Sensitivities of $\nu_0$ When $\widetilde{P_u} = 0.001$ and $\widetilde{P_h} = 0.9$ : the Exponential Case

| $P_u$ \ $P_h$ | 0.7 | 0.8 | 0.9 | 0.95 | 0.99 |
|---|---|---|---|---|---|
| 0.0003 | 1.0247[1)] | 1.0343 | 1.0461 | 1.0540 | 1.0647 |
| | 1.0251[2)] | 1.0322 | 1.0386 | 1.0409 | 1.0400 |
| 0.0005 | 1.0052 | 1.0093 | 1.0157 | 1.0206 | 1.0287 |
| | 1.0057 | 1.0094 | 1.0131 | 1.0146 | 1.0142 |
| 0.001 | 1.0057 | 1.0016 | 1 | 1.0007 | 1.0050 |
| | 1.0018 | 1.0004 | 1 | 1.0001 | 1.0000 |
| 0.002 | 1.0441 | 1.0293 | 1.0178 | 1.0134 | 1.0131 |
| | 1.0286 | 1.0205 | 1.0148 | 1.0128 | 1.0127 |
| 0.003 | 1.0895 | 1.0662 | 1.0471 | 1.0389 | 1.0353 |
| | 1.0623 | 1.0490 | 1.0391 | 1.0355 | 1.0349 |

Note 1) Under ALTDC
2) Under ALTCC

< Table 5 > Sensitivities of $\nu_0$ When $\widetilde{P_u}=0.001$, $\widetilde{P_h}=0.9$ and $q=0.01$
: the Weibull Case

| $P_u$ \ $P_h$ | 0.7 | 0.8 | 0.9 | 0.95 | 0.99 |
|---|---|---|---|---|---|
| 0.0003 | 1.0893[1] | 1.1008 | 1.1110 | 1.1141 | 1.1085 |
| | 1.0439[2] | 1.0505 | 1.0549 | 1.0540 | 1.0432 |
| 0.0005 | 1.0245 | 1.0312 | 1.0378 | 1.0401 | 1.0377 |
| | 1.0139 | 1.0174 | 1.0194 | 1.0182 | 1.0112 |
| 0.001 | 1.0028 | 1.0006 | 1 | 1.0001 | 1.0007 |
| | 1.0004 | 1.0000 | 1 | 1.0001 | 1.0028 |
| 0.002 | 1.0705 | 1.0534 | 1.0399 | 1.0341 | 1.0322 |
| | 1.0304 | 1.0253 | 1.0237 | 1.0266 | 1.0427 |
| 0.003 | 1.1560 | 1.1260 | 1.1004 | 1.0883 | 1.0805 |
| | 1.0761 | 1.0665 | 1.0631 | 1.0675 | 1.0917 |

Note 1) Under AI TDC
2) Under AI TCC

< Table 6 > Sensitivities of $\nu_0$ When $\widetilde{P_u}=0.001$, $\widetilde{P_h}=0.9$ and $q=0.01$
: the Lognormal Case

| $P_u$ \ $P_h$ | 0.7 | 0.8 | 0.9 | 0.95 | 0.99 |
|---|---|---|---|---|---|
| 0.0003 | 1.0760[1] | 1.0798 | 1.0751 | 1.0647 | 1.0408 |
| | 1.0895[2] | 1.0705 | 1.0412 | 1.0195 | 1.0017 |
| 0.0005 | 1.0191 | 1.0237 | 1.0243 | 1.0212 | 1.0150 |
| | 1.0451 | 1.0324 | 1.0142 | 1.0034 | 1.0082 |
| 0.001 | 1.0042 | 1.0012 | 1 | 1.0008 | 1.0100 |
| | 1.0078 | 1.0034 | 1 | 1.0037 | 1.0367 |
| 0.002 | 1.0545 | 1.0348 | 1.0220 | 1.0202 | 1.0341 |
| | 1.0032 | 1.0061 | 1.0161 | 1.0328 | 1.0890 |
| 0.003 | 1.0080 | 1.0759 | 1.0527 | 1.0466 | 1.0587 |
| | 1.0198 | 1.0256 | 1.0420 | 1.0648 | 1.1310 |

Note 1) Under AI TDC
2) Under AI TCC

# References

[1] Bai, D.S. and Chung, W.(1991), "An Optimal Design of Accelerated Life Test for Exponential Distribution," *Reliability Engineering and System Safety*, Vol. 31, pp. 57-64.

[2] Kielpinski, T.J. and Nelson, W.(1975), "Optimum Censored Accelerated Life Tests for Normal and Lognormal Life Distributions," *IEEE Transactions on Reliability*, R-24, pp. 310-320.

[3] Meeker, W.Q.(1984), "A Comparison of Accelerated Life Test Plans for Weibull and Lognormal Distributions and Type I Censoring," *Technometrics*, 26, pp. 157-171.

[4] Meeker, W.Q. and Escobar, L.A.(1993), "A Review of Recent and Current Issues in Accelerated Testing," *International Statistical Review*, 61, pp. 147-168.

[5] Meeker, W.Q. and Hahn, G.J.(1985), *How to Plan an Accelerated Life Test - Some Practical Guidelines*, ASQC Quality Press, Milwaukee.

[6] Meeker, W.Q. and Nelson, W.(1975), "Optimum Accelerated Life Tests for the Weibull and Extreme Value Distributions," *IEEE Transactions on Reliability*, R-24, pp. 321-332.

[7] Nelson, W.(1990), *Accelerated Testing: Statistical Models, Test Plans, and Data Analysis*, John Wiley & Sons, New York

[8] Nelson, W. and Kielpinski, T.J.(1976), "Theory for Optimum Censored Accelerated Life Tests for Normal and Lognormal Life Distributions," *Technometrics*, Vol. 18, pp. 105-114.

[9] Nelson, W. and Meeker, W.Q.(1978), "Theory for Optimum Accelerated Censored Life Tests for Weibull and Extreme Value Distributions," *Technometrics*, Vol. 20, pp. 171-177.

[10] Powell, M.J.D.(1964), "An Efficient Method for Finding the Minimum of a Function of Several Variables without Calculating Derivatives," *Computer Journal*, Vol. 7, pp. 155-162.

[11] Seo, S.K. and Yum, B.J.(1991), "Accelerated Life Test Plans under Intermittent Inspection and Type I censoring : the Case of Weibull Failure Distribution," *Naval Research Logistics*, Vol. 38, pp. 1-22

[12] Yang, G.B.(1994), "Optimum Constant-Stress Accelerated Life Test Plans," *IEEE Transactions on Reliability*, R-43, pp. 575-581.

[13] Yang, G.B. & Jin, L.(1994), "Best Compromise Test Plans for Weibull Distributions with Different Censoring Times," *Quality and Reliability Engineering International*, Vol. 10, pp. 411-415.

[14] Yum, B.J. and Choi, S.C.(1989), "Optimal Design of Accelerated Life Tests under Periodic Inspection," *Naval Research Logistics*, Vol. 36, pp. 779-793.