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ON L–FUZZY ALMOST PRECONTINUOUS FUNCTIONS

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1. Introduction

In 1981, R . Badard introduced the notion of fuzzy pretopological spaces and their representation[1]. And in 1992, R. Badard, et al. introduced the L-fuzzy pretopological spaces and studied properties of continuity, open map, closed map,and homeomorphism in L-fuzzy pretopological spaces. In this paper we introduce and study the concepts of almost continuous functions and weakly pre-continuous functions on L-$fpts'$s. The symbol L denote a complete lattice, with infimum o and supremum 1, that L is equipped with an order reversing involution. For a lattice, the De Morgan laws hold for arbitrary indexed suprema and infima. Given such a lattice L and a non-empty set X, the L-fuzzy sets of X[2] are just the elements of L^X,i.e.,the functions from X to L. 0 is the L-fuzzy set defined by $0: X \to L$, $0(x) = o$ for each $x \in X$. 1 is the L-fuzzy set defined by $1: X \to L$, $1(x) = 1$ for each $x \in X$. For $u, v \in L^X$, the intersetion $u \wedge v$ and the union $u \vee v$, respectively, are defined by: $(u \wedge v)(x) = u(x) \wedge v(x), x \in X$, $(u \vee v)(x) = u(x) \vee v(x), x \in X$. Let $u, v \in L^X$. u is included in $v(u \leq v)$ provided that $u(x) \leq v(x)$ holds for every $x \in X$. For any L-fuzzy set u, u' will stand for the complement of u.

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Definition 1.1[2]. An L-fuzzy pretopology on a set X is a function $a\colon L^X \to L^X$ such that

(1) $a(0) = 0$,
(2) $a(u) \geq u$

are satisfied for every $u \in L^X$.

The pair (X, a) is said to be an L-fuzzy pretopological space (for short, L-$fpts$). An L-$fpts$ is said to be of:

(1) Type I if for every $u, v \in L^X$ such that $u \leq v$ we have $a(u) \leq a(v)$.
(2) Type D if for every $u, v \in L^X$ we have $a(u \vee v) = a(u) \vee a(v)$.
(3) Type S if for every $u \in L^X$ we have $a^2(u) = a(u)$.

It is clear that (2) implies (1).

Definition 1.2[1]. Let (X, a) and (Y, b) be $fpts$'s. A function $f\colon (X, a) \to (Y, b)$ is said to be precontinuous if $f(a(u)) \leq b(f(u))$, for every $u \in I^X$.

Definition 1.3[2]. Let (X, a) be an L-$fpts$ and $u \in L^X$. We define the L-fuzzy interior operator $i_a\colon L^X \to L^X$ by $i_a(u) = (a(u'))'$.

Then it is clear that the properties (1) to (5) become, for the operator i_a (see [1]):

(1) $i_a(0) = 0$.
(2) $i_a(u) \leq u$ for each $u \in L^X$.
(3) If (X, a) is of type I, then $u \leq v$ implies $i_a(u) \leq i_a(v)$.
(4) If (X, a) is of type D, then $i_a(u \wedge v) = i_a(u) \wedge i_a(v)$ for each $u, v \in L^X$.
(5) If (X, a) is of type S, then $(i_a)^2(u) = i_a(u)$ for $u \in L^X$.

A more successful denomination would be:

(1) u is L-preclosed iff $a(u) = u$,
(2) u is L-preopen iff $i_a(u) = u$.

It is clear that $u \in L^X$ is preclosed if and only if u' is preopen.

Definition 1.4[2]. Let (X, a) and (Y, b) be L-$fpts$'s. A function $f\colon (X, a) \to (Y, b)$ is to be preopen (resp., preclosed) if for every $u \in L^X$ we have $f(i_a(u)) \le i_b(f(u))$ (resp., $f(a(u)) \ge b(f(u))$).

Throughout this paper, we assume that every L-fuzzy pretopological space is Type I and S.

2. Main Theorems

Definition 2.1. A fuzzy subset u of an L-$fpts(X, a)$ is called a regularly preopen L-fuzzy set if $i_a(a(u)) = u$. An L-fuzzy set whose complement is a regularly preopen L-fuzzy set is called a regularly preclosed L-fuzzy set.

We obtain easily the following lemma by Definition 1.3.

Lemma 2.2. *Let (X, a) be an L-$fpts$ and $u \in L^X$.*

(1) $a(u') = (i_a(u))'$,
(2) $i_a(u') = (a(u))'$.

Definition 2.3. Let (X, a) and (Y, b) be L-$fpts$'s. A fuzzy mapping $f\colon (X, a) \to (Y, b)$ is called an L-fuzzy almost precontinuous mapping if for each preopen L-fuzzy set u in (Y, b), $f^{-1}(u) \le i_a(f^{-1}(i_b(b(u))))$.

Theorem 2.4. *Let (X, a) and (Y, b) be L-$fpts$'s. A fuzzy mapping $f\colon (X, a) \to (Y, b)$ is an L-fuzzy almost precontinuous mapping if and only if for each regular preopen L-fuzzy set u in Y, $f^{-1}(u)$ is a preopen L-fuzzy set.*

proof. Assume that $f\colon (X, a) \to (Y, b)$ is an L-fuzzy almost precontinuous mapping. And let u be a regular preopen L-fuzzy set in Y. Then $f^{-1}(u) \le i_a(f^{-1}(i_b(b(u))))$

and by the definition of regular preopen L-fuzzy set, we obtain $f^{-1}(u) \leq i_a(f^{-1}(u))$. Thus $f^{-1}(u)$ is a preopen L-fuzzy set.
For the converse, let u be a preopen L-fuzzy set. Then $i_b(b(u))$ is a regular preopen L-fuzzy set, and $f^{-1}(i_b(b(u))) = i_a(f^{-1}(i_b(b(u))))$. This means $f^{-1}(u) \leq i_a(f^{-1}(i_b(b(u))))$, since $u = i_b(u) \leq i_b(b(u))$. Consequently, f is an L-fuzzy almost precontinuous mapping.

Theorem 2.5. *Let (X,a) and (Y,b) be L-fpts's. A fuzzy mapping $f\colon (X,a) \to (Y,b)$ is an L-fuzzy almost precontinuous mapping if and only if for each preclosed L-fuzzy set u in Y, $a(f^{-1}(b(i_b(u)))) \leq f^{-1}(u)$.*

Proof. Let u be a preclosed L-fuzzy set in Y. Since u' is a preopen L-fuzzy set in Y, $f^{-1}(u') \leq i_a(f^{-1}(i_b(b(u'))))$. By the definition 1.3 and lemma 2.2, we obtain

$$\begin{aligned} f^{-1}(u) &\geq a(f^{-1}(i_b(b(u'))))' \\ &= a(f^{-1}(b(b(u'))'))\\ &= a(f^{-1}(b(i_b(u))). \end{aligned}$$

Thus $a(f^{-1}(b(i_b(u)))) \leq f^{-1}(u)$.

The converse is obvious.

Definition 2.6. Let (X,a) and (Y,b) be L-*fpts*'s. A fuzzy mapping $f\colon (X,a) \to (Y,b)$ is called L-fuzzy weakly precontinuous if for each preopen L-fuzzy set u of Y, $f^{-1}(u) \leq i_a(f^{-1}(b(u)))$.

Theorem 2.7. *The following properties are equivalent*

(1) *f is L-fuzzy weakly precontinuous in L-fpts.*
(2) *$f^{-1}(u) \geq a(f^{-1}(i_b(u)))$ for each preclosed L-fuzzy set u in Y.*
(3) *$a(f^{-1}(u)) \leq f^{-1}(b(u))$ for each pre-open L-fuzzy set u in Y.*

Proof. (1)⇒(2). Let u be preclosed L-fuzzy set in Y. Then u' is a preopen L-fuzzy set in Y and $f^{-1}(u') \leq i_a(f^{-1}(b(u')))$. This implies $f^{-1}(u) \geq a(f^{-1}(b(u')))'$. By Lemma 2.2, $a(f^{-1}(i_b(u))) \leq f^{-1}(u)$.

(2) ⇒(3). Let u be a preopen L-fuzzy set in Y. Since $b(u)$is a preclosed L-fuzzy set in Y, then $f^{-1}(b(u)) \geq a(f^{-1}(i_b(b(u))))$ and $f^{-1}(i_b(b(u)))) \geq f^{-1}(u)$. Therefore $f^{-1}(b(u)) \geq a(f^{-1}(u))$.

(3) ⇒(1). Let u be a preopen L-fuzzy set in Y. Since $b(u)'$ is a preopen L-fuzzy set, $a(f^{-1}(b(u))') \leq f^{-1}(b(b(u))')$. By Lemma 2.2, $(i_a(f^{-1}(b(u))))' \leq f^{-1}(i_b(b(u)))'$. This means that $f^{-1}(i_b(b(u))) \leq i_a(f^{-1}(b(u)))$. Therefore $f^{-1}(u) \leq i_a(f^{-1}(b(u)))$.

Theorem 2.8. *Let (X, a) and (Y, b) be L-fpts's. If $f: (X, a) \to (Y, b)$ is an L-fuzzy weakly precontinuous, onto and fuzzy preopen mapping, then f is fuzzy almost precontinuous.*

Proof. Since f is L-fuzzy weakly precontinuous, for each preopen L-fuzzy set u in Y, we have $f^{-1}(u) \leq i_a(f^{-1}(b(u)))$. And we have $i_a(f^{-1}(b(u)) \leq f^{-1}(i_b(b(u)))$, since f is a fuzzy preopen, onto mapping. Consequently, $f^{-1}(u) \leq i_a(f^{-1}(b)(i_b(b(u))))$.

Theorem 2.9. *Let (X, a), (Y, b) and (Z, c) be L-fpts's. If $f: (X, a) \to (Y, b)$ is a fuzzy preopen, onto, and L-fuzzy precontinuous mapping and $g: (Y, b) \to (Z, c)$ is an L- fuzzy mapping. Then $(g \circ f)$ is L-fuzzy almost precontinuous if and only if g is L-fuzzy almost precontinuous.*

Proof. Assume that $(g \circ f)$ be L-fuzzy almost precontinuous and let u be a preopen L-fuzzy set in Z. Since $(g \circ f)$ is L-fuzzy almost precontinuous, we have $(g \circ f)^{-1}(u) \leq i_a(g \circ f)^{-1}(i_c(c(u)))$. Since f is a fuzzy preopen onto mapping, $g^{-1}(u) \leq i_b(g^{-1}(i_c(c(u))))$.

For the converse, let u be a preopen L-fuzzy set in Z. Then by Proposition 2.4 in [2], we obtain the follwing implications:

$$\begin{aligned}(g \circ f)^{-1}(u) &\leq f^{-1}(i_b(g^{-1}(i_c(c(u))))) \\ &\leq i_a(f^{-1}(g^{-1}(i_c(c(u)))\end{aligned}$$

Therefore $(g \circ f)$ is L-fuzzy almost precontinuous.

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