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Energy Dissipation and Mean Crushing Strength of Stiffened Pates in Crushing

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Abstract

The prediction of the crushing strength and corresponding energy dissipation of unstiffened and stiffened plates under axial compression is discussed. Semi-empirical formulae for the crushing strength and dissipation energy of these stiffened plates are derived from the assesment of the structural behavior of unstiffened and stiffened box columns consisted of rectangular plates with longitudinal, transverse and orthogonal stiffeners.

To demonstrate the effectiveness of proposed formulae, they are compared with the existing formulae and experimental results, which are shown in good agreements.

1 Introduction

During last 30 years the crushing behavior of thin-walled structures has been widely studied experimentally and theoretically.

Through the experimental approach using dimensional analysis, some empirical formulae were proposed on the dissipation energy of isotropic rectangular plates by Lee [1], where the coefficient of the energy absorption should be determined as a function of slenderness ratio. Based on the above formulae, he [2] proposed formulae for the longitudinally and transversely stiffened plates by considering volumetric change due to stiffeners.

In the theoretical approach using plastic hinge lines [4,5,6], the mean crushing strength was introduced to analyse the complicated problems. The crushing behavior of thin walled members is characterized that initial ultimate load is far larger than the following loads and collapse process is the same as the first mode, where the dissipation energy due to initial collapse responses of these structres are much less than the subsequent behaviors accompanying large crushing distance.

Later Wierzbicki and Abramowicz [7] introduced the extensional hinge surface element with the symmetric collapse mode and a simplified formula to predict the mean crushing strength of box columns was presented by using rigorous kinematic relations.

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By using the effective crushing distance [8], asymmetric collapse modes were analysed by combining basic folding elements [9]. A new basic folding mechanism of a cruciform element by Amdahl [10] and continuous plate element by Paik and Pedersen [11] has much improved the evaluation of collapse load and dissipation energy of plates.

In this study, based on experimental approach using the dimensional analysis [1,2], the dissipation energy and crushing strength of transversely, longitudinally and orthogonally stiffened plates under axial compressive load are studied and discussed with the existing results.

2 General Expression for Energy Absorption

The absorbed energy of thin-walled structure under axial load $P(\delta)$ can be expressed by integration of $P(\delta)$ over the crushing distance δ .

$$E = \int_0^{\delta} P(\delta) d\delta \tag{1}$$

When the mean crushing strength is introduced, equation (1) can be presented as:

$$E = P_m \delta \tag{2}$$

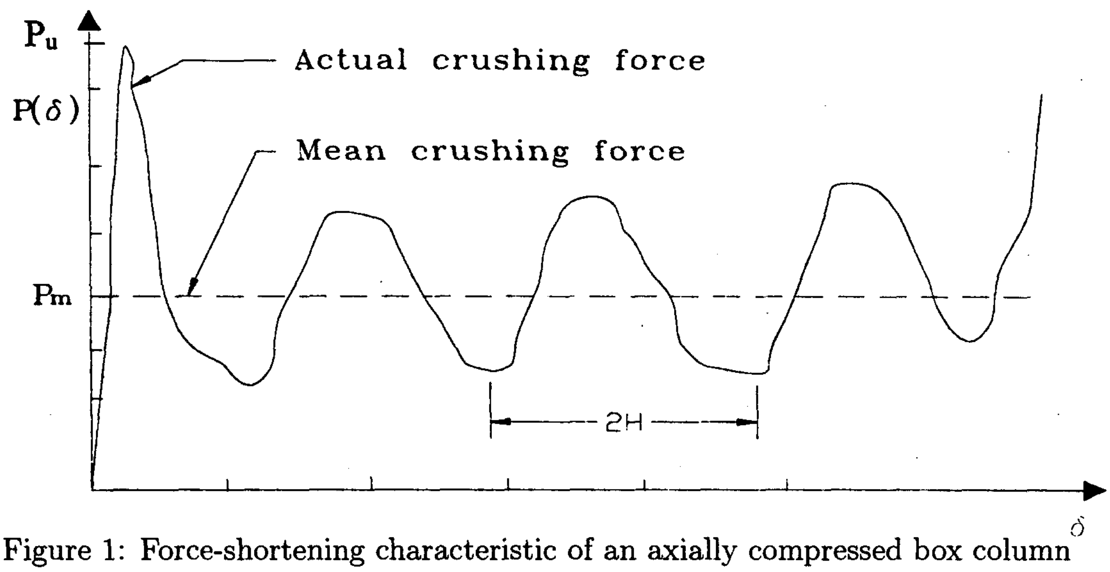


Figure 1: Force-shortening characteristic of an axially compressed box column

The mean crushing load P_m is generally dependent on the ultimate buckling strength of the plate, which is characterized by the geometry and material properties of the plates, i.e., thickness t, width b, length l, yield stress σ_y , elastic modulus E and boundary conditions α_i, which is denoted by the vertical deflection and rotation angle of edges in radian. Then, the mean crushing load P_m can be expressed as follows

$$P_m \propto f(t, b, l, \sigma_y, E, \alpha_i) \tag{3}$$

where, the function f is understood as the ultimate buckling strength, P_u of the plate.

By applying the Π theorem for dimensional analysis [2], energy dissipation equation for whole crushing length of plates can be formulated by

$$E = C \cdot \sigma_y \cdot t^2 \cdot \delta = C \cdot \sigma_y \cdot t^2 \cdot b \cdot m \tag{4}$$

where, $C = \phi\{\frac{b}{t}\sqrt{\frac{\sigma_y}{E}}, \frac{\alpha_i}{l}\} = \phi\{\frac{b}{t}, \frac{\alpha_i}{l}\}$ for unit strain

$\delta = b \cdot m = \delta_u \cdot m$ for whole crushing length $l = \delta$

When we introduce a unit crushing length $\delta_u = 2H$, C can be understood as a variable coefficient which is dependent on plate slenderness b/t and m is the number of lobes with unit length δ_u,

$$m = \frac{l}{\delta_u} \qquad l : \text{initial height of the plate} \tag{5}$$

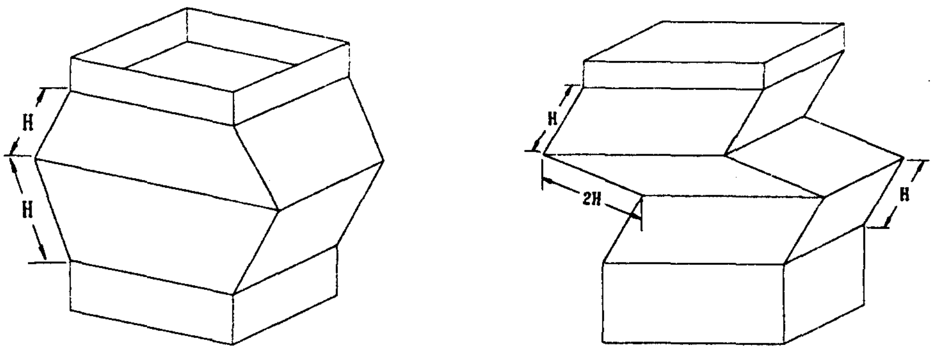
Under the assumption of symmetric collapse mode [7], which is verified experimentally and theoretically [9], unit length is given by

$$\delta_u = 2\sqrt[3]{b^2 \cdot t} \tag{6}$$

The effective unit length δ_{eff} will be adopted as 70 % of initial unit length for unstiffened plate from experimental results [1].

$$\delta_{eff} = 1.4\sqrt[3]{b^2 \cdot t} \tag{7}$$

$$m_{eff} = \frac{l_{eff}}{\delta_{eff}} \tag{8}$$



(a)Symmetric collapse mode (b) Asymmetric collapse mode

Figure 2: Collapse mode

3 Derivation of Energy Absorption and Mean Crushing Strength

3.1 Unstiffened box column

Let slenderness ratio $\frac{b}{t}$ be denoted by β, then C is a function of β ; $C = f(\beta)$. By using the experimental data of unstiffened box column in reference[1], a proper relationship between C and β is obtained as follows

$$C = 0.7622\sqrt{\beta} + 14.911 \tag{9}$$

Therefore the dissipation energy of the unstiffened plate is expressed by

$$E = (0.7622\sqrt{\beta} + 14.911)\sigma_y t^2 b m_{eff} \tag{10}$$

From equation (2) and (10), the mean crushing strength is give by

$$P_m = 0.7143(0.7622\sqrt{\beta} + 14.911)\sigma_y \sqrt[3]{bt^5} \tag{11}$$

3.2 Longitudinally stiffened box column

In the case of a longitudinal stiffener parallel to parent plate in Fig. 3, the interaction between mother plate and stiffener occur only in very narrow zone, thus the contribution to the energy absorption due to the interaction can be considered negligible.

To obtain energy absorption of each member, it is admissible to transform the sectional area A_s of stiffener onto the parent plate.

$$t_{eq} = t + N_l \frac{A_s}{b} \tag{12}$$

where N_l is the number of longitudinal stiffener per one plate

For the case of longitudinal stiffener perpendicular to mother plate in Fig. 3, the interaction between mother plate and stiffener shows the extensional or distorsional behaviors through the plate length.

The correction factor G for longitudinal stiffener is introduced to consider additional energy absorption due to interactions shown in Fig. 4 (b) or (c) which is assumed to depend on geometry of plate and stiffener, then it can be numerically represented from the experimental results as follows

$$G = 0.0375N_l + 1.0 \tag{13}$$

Therefore energy absorption and mean crushing strength of longitudinally stiffened box column is expressed by

$$E = G \cdot (0.7622\sqrt{\beta_{eq}} + 14.911)\sigma_y t_{eq}^2 b m_{eff} \tag{14}$$

$$P_m = 0.7143G(0.7622\sqrt{\beta_{eq}} + 14.911)\sigma_y\sqrt[3]{bt_{eq}^5} \tag{15}$$

where $\beta_{eq} = \frac{b}{t_{eq}}$

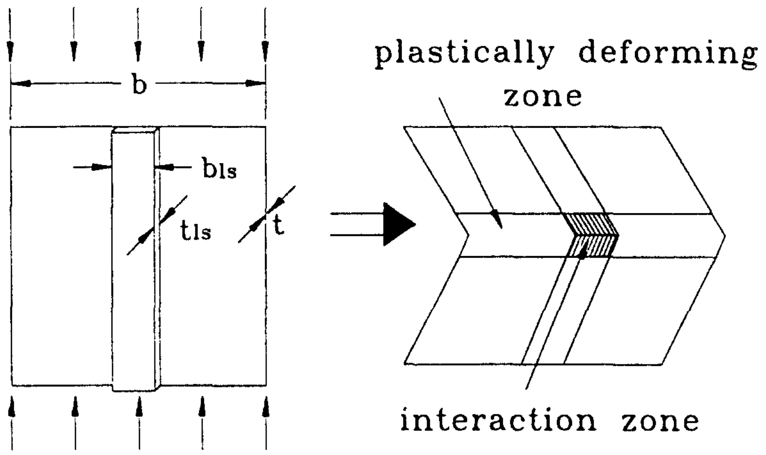


Figure 3: Concept of equivalent thickness for longitudinal stiffener parallel to parent plate

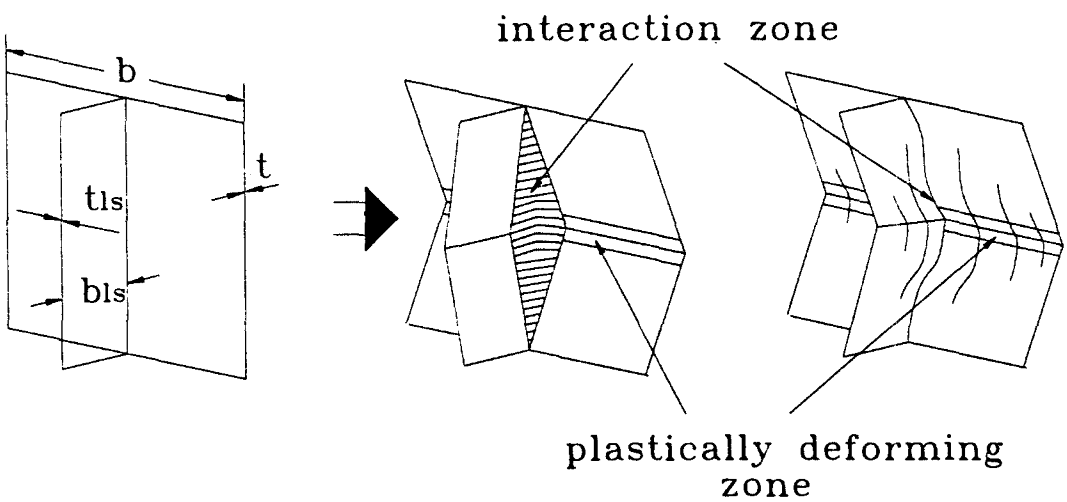


Figure 4: Longitudinal stiffener perpendicular to parent plate

3.3 Transversely stiffened box column

Transverse stiffener has been considered to give little contribution to axial rigidity, but experimental results in the references [2, 3] show that increase of mean crushing strength and energy absorption due to transverse stiffener is remarkable. Thus the concept of equivalent width as shown in Fig. 4 is introduced.

It is expected that the increase of mean crushing strength results from the geometrically isotropic wide plate due to transverse stiffener.

The attachment of transverse stiffener in Fig. 4 is considered to bring about increase of width of parent plate and the equivalent width of transversely stiffened plates can be expressed as follows

$$b_{eq} = b + N_t \frac{A_s}{t} \tag{16}$$

where N_t is the number of transverse stiffener per one plate

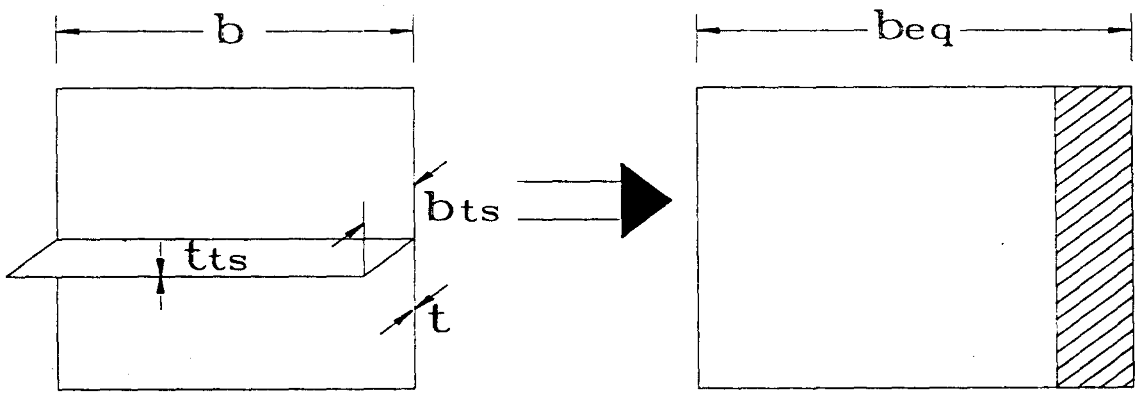


Figure 5: Concept of equivalent width for transverse stiffener perpendicular to parent plate

Concerning the average effective crushing distance, a correction for the transverse stiffeners is needed. Since stiffeners behave as rigid bodies, the thickness of stiffener must be subtracted from effective unit crushing length length

$$\delta^T_{eff} = 1.4\sqrt[3]{b^2_{eq}t}\left(\frac{S_t - t_{ts}}{S_t}\right) \tag{17}$$

where S_t is the span of transverse stiffeners.

The energy absorption and mean crushing strength of unstiffened box column are expressed by

$$E = (0.7622\sqrt{\beta_{eq}} + 14.911)\sigma_y t^2 b_{eq} m^T_{eff} \tag{18}$$

$$P_m = \frac{0.7143 S_t}{S_t - t_{st}}(0.7622\sqrt{\beta_{eq}} + 14.911)\sigma_y \sqrt[3]{b_{eq}t^5} \tag{19}$$

where $\beta_{eq} = \frac{b_{eq}}{t}$, $m^T_{eff} = \frac{l_{eff}}{\delta_{efft}}$

3.4 Orthogonally stiffened box column

As for crushing behavior of orthogonally stiffened box column it is considered that collapse behavior of transverse and longitudinal stiffeners occurs simultaneously. To predict mean crushing strength and energy absorption of orthogonally stiffened box column equations (14),(18) and (15),(19) are reconsidered yielding respectively.

$$E = G(0.7622\sqrt{\beta_{eq}} + 14.911)\sigma_y t_{eq}^2 b_{eq} m_{eff}^T \tag{20}$$

$$P_m = \frac{0.7143GS_t}{S_t - t_{ts}}(0.7622\sqrt{\beta_{eq}} + 14.911)\sigma_y \sqrt[3]{b_{eq} t_{eq}^5} \tag{21}$$

$$\text{where} \qquad \beta_{eq} = \tfrac{b_{eq}}{t_{eq}}$$

4 Discussion

4.1 Experimental models and results

Fig. 6 shows the typical shapes of the selected experimental models conducted by Lee [2] and Paik et al [3] and their dimensions and material properties are listed in Table 1 and 2 respectively. For more details it is recommended to refer to references [2] and [3].

Table 1: Dimensions and material properties of selected experimental models[1,2]

Model No.	l (mm)	b (mm)	t (mm)	b_{eq} (mm)	t_{eq} (mm)	β_{eq}	b_{ls} (mm)	t_{ls} (mm)	b_{ts} (mm)	t_{ts} (mm)	S_l (mm)	S_t (mm)	N_l	N_t	σ_y (N/mm^2)
1	600.0	200.0	3.2	-	-	62.50	-	-	-	-	-	-	-	-	286.0
2	450.0	300.0	1.6	-	-	187.50	-	-	-	-	-	-	-	-	286.0
3	600.0	300.0	3.2	367.5	-	114.84	-	-	15.0	1.6	-	60.0	-	9	286.0
4	600.0	300.0	3.2	367.5	3.28	112.04	15.0	1.6	15.0	1.6	150.0	60.0	1	9	286.0
5	600.0	300.0	3.2	367.5	3.44	106.83	15.0	1.6	15.0	1.6	75.0	60.0	3	9	286.0

Table 2: Dimensions and material properties of selected experimental models[3]

Model No.	l (mm)	b (mm)	t (mm)	b_{eq} (mm)	t_{eq} (mm)	β_{eq}	b_{ls} (mm)	t_{ls} (mm)	b_{ts} (mm)	t_{ts} (mm)	S_l (mm)	S_t (mm)	N_l	N_t	σ_y (N/mm^2)
LS-7	450.0	100.0	2.8	-	3.22	31.06	15.0	2.8	-	-	50.0	-	1	-	310.8
TS-7	450.0	100.0	4.2	120	-	28.57	-	-	20.0	4.2	-	225.0	-	1	366.8

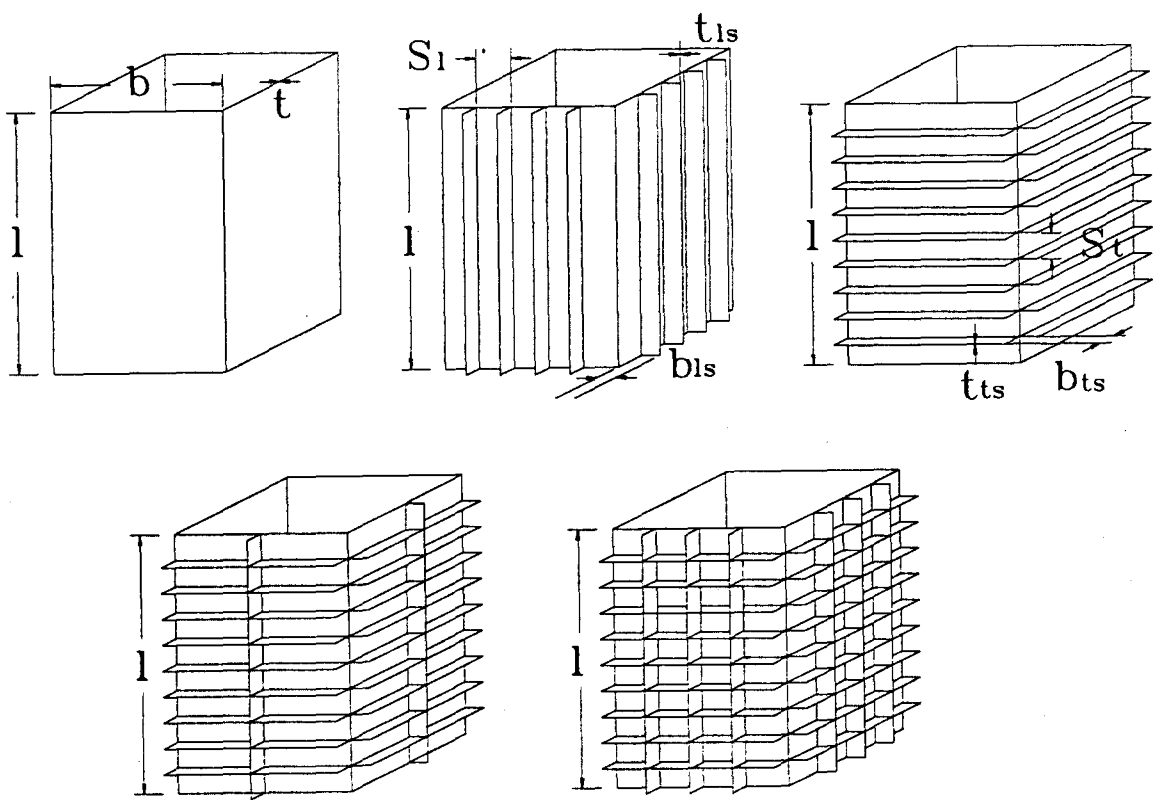


Figure 6: Typical shapes of experimental models[1,2]

4.2 Comparison of unstiffened box column with experimental results

Normalized crushing strength predictions have been derived by Wierzbicki et al [7] for unstiffened box columns showing symmetric collapse mode [7] and asymmetric collapse mode by Abramowicz et al [9] as function of slenderness ratio.

$$\frac{P_m}{M_0} = 52.22\sqrt[3]{\frac{b}{t}} \qquad \text{(Symmetric)} \tag{22}$$

$$\frac{P_m}{M_0} = 43.61\sqrt[3]{\frac{b}{t}} + 3.79\sqrt[3]{(\frac{b}{t})^2} + 2.6 \qquad \text{(Asymmetric)} \tag{23}$$

,where $M_0 = \frac{1}{4}\sigma_y t^2$

A theoretical formula for unstiffened box column has also been developed by Paik and Pedersen [11]

$$\frac{P_m}{M_0} = 31.31\sqrt{\frac{b}{t}} + 5.875 \tag{24}$$

The approximate semi-empirical new formula of equation (11) can be rewritten in normalized form as

$$\frac{P_m}{M_0} = 2.8571(0.7622\sqrt{\frac{b}{t}} + 14.911)\sqrt[3]{\frac{b}{t}} \tag{25}$$

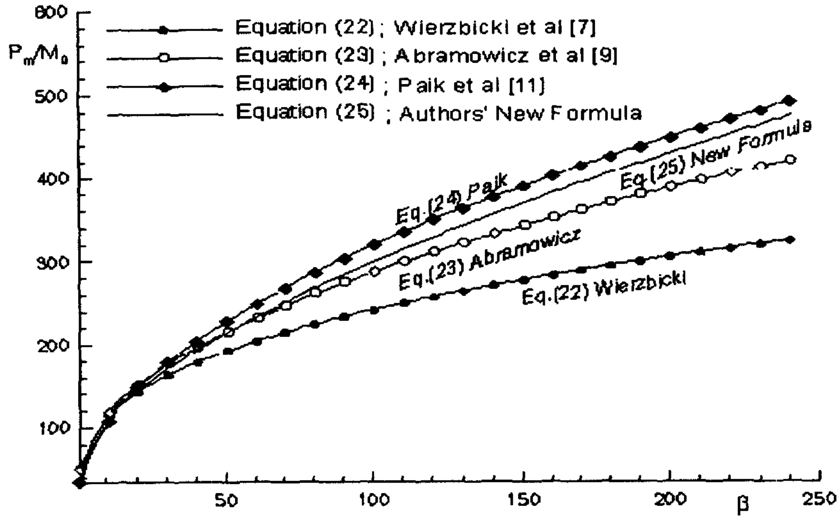


Figure 7: The curves of normalized mean crushing strength versus slenderness $(1 \leq \beta \leq 250)$

Equations in Fig. 7, (22), (23) and (24) are compared with the proposed new equation (25). For the range of 1 <= *beta* <= 100, equation (25) agrees well with equation (23) while approaching to equation (24) for the range of *beta* > 100.

4.3 Comparison of longitudinally stiffened box column with experimental results

Since model No. LS-7 has only one longitudinal stiffener in Fig. 8 influence of correction factor G on mean crushing strength is very small but in the case of box column with many longitudinal stiffeners, mean crushing strength increases dependending upon the number of stiffeners.

4.4 Comparison of transversely stiffened box column with experimental results

In Fig. 9, dotted lines represent the theoretical prediction when the correction factor for compressibility and equivalent width are not considered. Corrected mean crushing strength (dashed lines) are in good agreement with experimental results in both energy and strength.

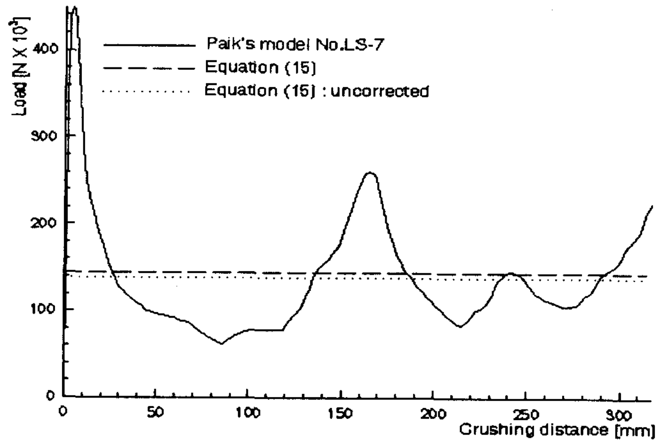
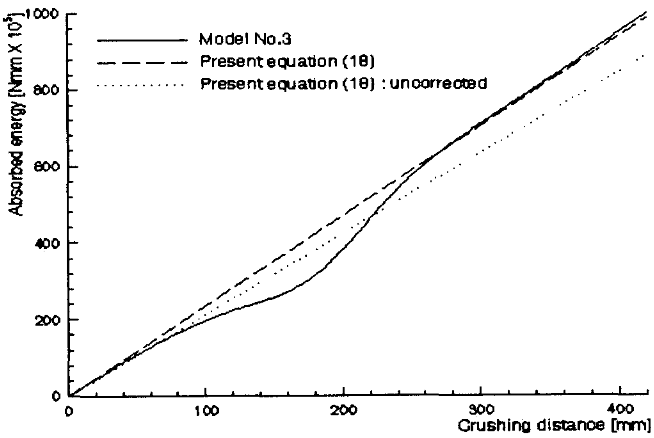
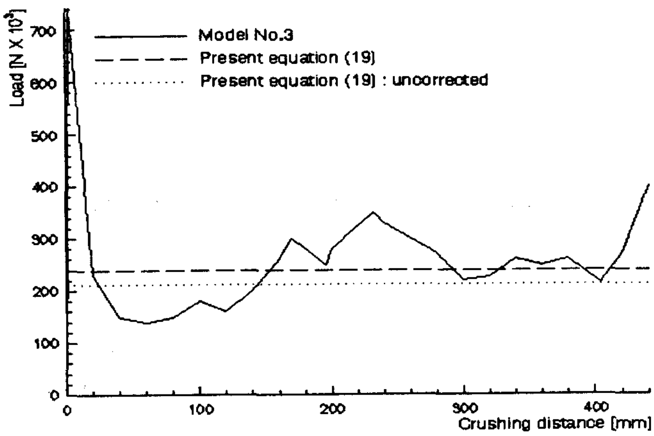


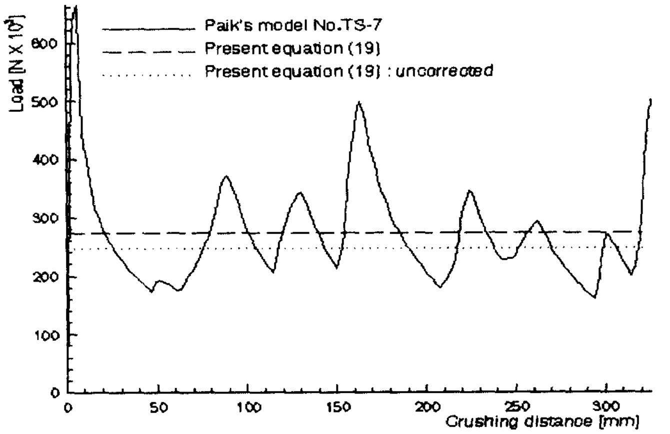
Figure 8: Comparison of present formula with experimental results for longitudinally stiffened box column



(a) Absorbed energy (β_{eq}=114.84)



(b) Crushing strength (β_{eq}=114.84)



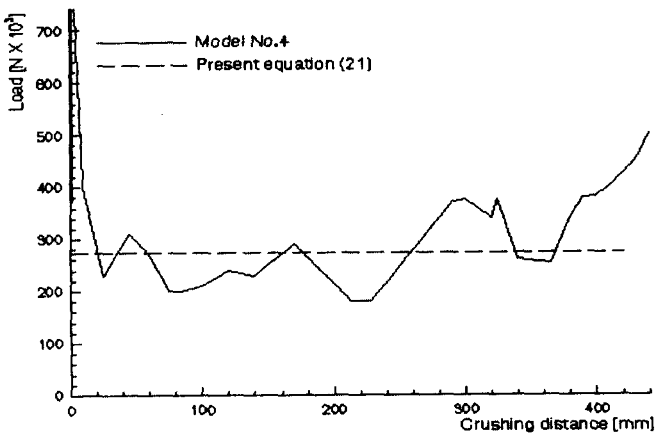
(c) Crushing strength (β_{eq}=28.57)

Figure 9: Comparison of present formula with experimental results for transversely stiffened box column

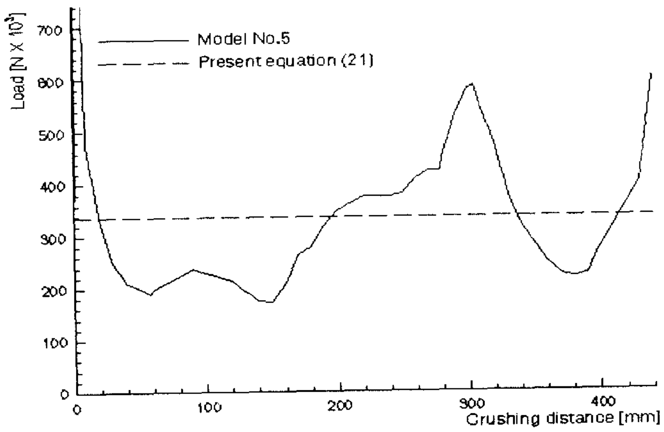
4.5 Comparison of orthogonally stiffened box column with experimental results

For the analysis of orthogonally stiffened plates, the combination of the concepts for longitudinally stiffened plate and transversely stiffened plate introduced in 4.3 and 4.4 each is used to evaluate the structural dissipation energy and mean crushing strength.

In Fig.10, a comparison of the present proposed formula for mean crushing strength is made with experimental results which shows in good agreements.



(a) Crushing strength (β_{eq}=112.04)



(b) Crushing strength (β_{eq}=106.83)

Figure 10: Comparison of present formula with experimental results for orthogonally stiffened box column

5 Conclusions

In the present study, mean crushing strength and absorbed energy dependent on the slenderness ratio are obtained for unstiffened plates of square box column from the experimental data by Lee [1]. Under the assumption that interactions for longitudinally stiffened plates are the linear function of the number of longitudinal stiffener, predictions for mean crushing strength and absorbed energy are formulated. For the assesment of increase of mean crushing strength due to transverse stiffener, concept of equivalent width is introduced and for rigid body behavior of transverse stiffener, correction factor for effective crushing distance is considered.

Present formulae for unstiffened and longitudinally, transversely and orthogonally stiffened box column are respectively compared with the theoretical formulae [7,9,11] and experimental results [1,2,3] and it is observed that new predictions are in good agreements with them.

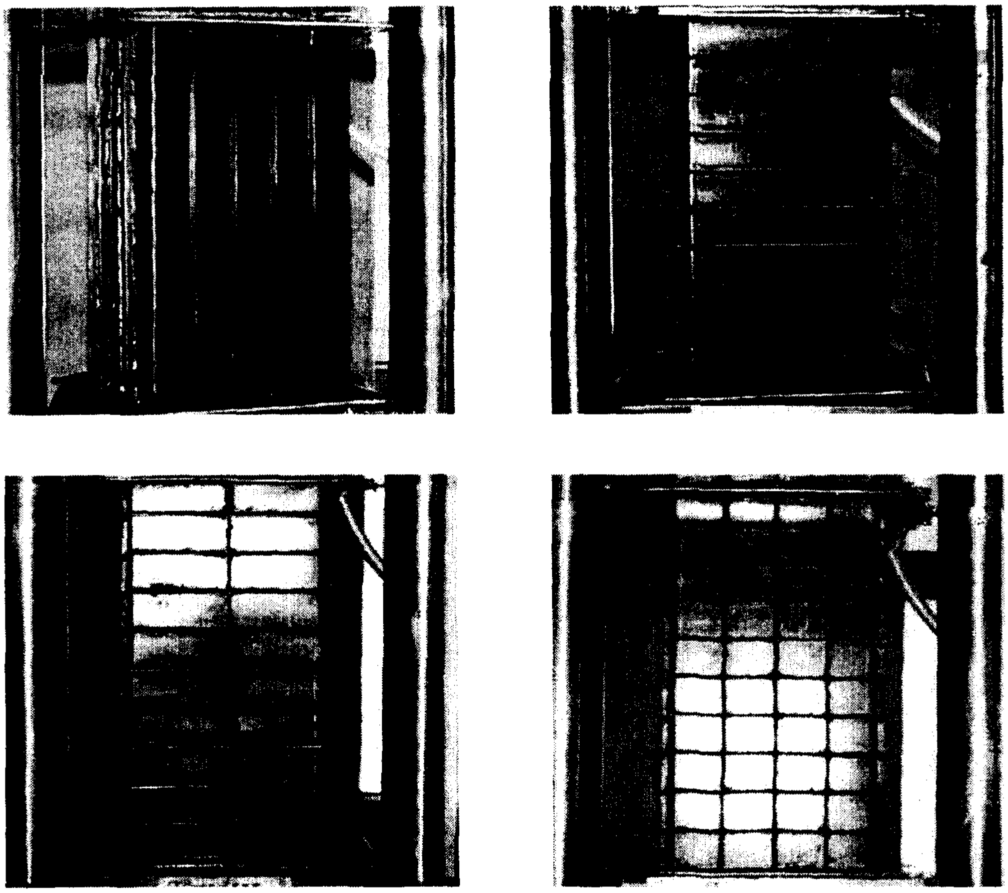


Figure 11: Test models of rectangular plates with transverse, longitudinal and orthogonal stiffeners

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