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A New Method for High Resolution DOA Systems

Hak-Lim Ko*, Dai-Tchul Moon** *Regular Members*

고 해상도 DOA 시스템을 위한 새로운 방법 제안

正會員 高 鶴 林*, 文 大 哲**

ABSTRACT

In this paper, we propose a new weighted backward covariance matrix method to enhance the resolution for direction-of-arrival(DOA) estimation. The proposed method (MEVM : modified eigenvector method) is an enhanced covariance matrix method which is an extended form of the conventional covariance matrix. We analyze the effect of using the weighted forward-backward covariance matrix on the performance of the eigenvector method(EVM). By comparing the perturbation angle of the noise-subspace, we show that the spectral estimate obtained using the proposed method is less distorted than the spectral estimate obtained using the conventional EVM.

The simulation results show that the new method is more accurate and has better resolution than the conventional EVM under the same noise conditions.

요 약

본 논문에서는 weighted forward-backward covariance 메트릭스를 이용하여 direction-of-arrival(DOA) estimation의 해상도를 향상시켰다. Weighted forward-backward covariance 메트릭스는 기존의 covariance 메트릭스를 확장시킨 형태로써 동일 잡음조건하에서 고유벡터가 기존 covariance 메트릭스의 고유벡터들 보다 덜 왜곡된다는 것을 시뮬레이션을 이용하여 증명하였다. 따라서 덜 왜곡된 고유벡터는 더욱 정확한 DOA 정보를 갖고 있고 이를 이용하여 구한 spectral estimate이 기존 covariance 메트릭스의 고유벡터를 이용하여 구한 spectral estimate 보다 보다 향상된 해상도를 갖는다. 우리는 이 메트릭스를 eigenvector method(EVM)에 응용, 기존 covariance 메트릭스를 이용하여 구한 EVM과 비교함으로써 weighted forward-backward covariance를 이용하여 구한 DOA estimation이 동일 잡음조건하에서 더욱 정확하고 좋은 해상도를 갖는다는 것을 증명하였습니다.

*LG 반도체
**호서대학교 정보통신공학부
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I. Introduction

High resolution direction-of-arrival(DOA) estimation is widely used for many multisensor systems for applications such as sonar, radar, and seismic exploration. Many high resolution algorithms based on the eigen-decomposition analysis of the observed covariance matrix, such as MVE[2], MUSIC[5], EVM[3], SNLM[1], and SHIRE[4] have been proposed for solving this problem. These algorithms emphasize specific eigenvectors of the observed covariance matrix to obtain the best spectral estimate which provide the DOA information.

The quality of the estimation of the covariance matrix is very important to get the best performance of these algorithms. Since the observed sample covariance matrix is perturbed by correlated noise, the performance estimate of the spectrum using this conventional covariance matrix is degraded when the signal-to-noise ratio(SNR) is low and/or the arrival angles are close to each other. Therefore, Raghunath[11] and Rao[12] used spatial smoothing to enhance the covariance matrix. Moghaddamjoo[10] used spatial filtering and Du and Kirlin[13] used temporal correlations between multiple snapshot to estimate the covariance matrix. Basically, these methods use a similar idea. They extracts more information from the given data set.

We introduce a modified eigenvector method (MEVM). MEVM uses the weighted forward-backward covariance matrix to improve the estimation of the covariance matrix. Likewise, this method extracts more information than conventional method from the given data set. Furthermore, the proposed method reduces the phase error of the estimated DOAs.

In this research, we first analyzed the typical high resolution algorithms noted above. Then, we evaluated the effect of using the forward-backward covariance matrix on the performance of the EVM under various conditions. Our simulation results showed that the performance of the EVM was improved for DOA estimation when we used the weighted forward-backward covariance matrix.

Ⅱ. Comparison of spectral estimates.

The high resolution algorithms for solving the DOA problem emphasize specific eigenvectors of the observed covariance matrix to obtain the best spectral estimate. The estimating spectra for these algorithms have the following general form:

$$P(\theta) = \frac{1}{\sum_{i=1}^{M} w_i \,|\, a^*(\theta) v_i \,|^2} = \frac{1}{\sum_{i=1}^{M} a^*(\theta)\, w_i\, v_i v_i^* \, a(\theta)} = \frac{1}{\sum_{i=1}^{K} w_i \,|\, a^*(\theta) v_i \,|^2 + \sum_{i=K+1}^{M} w_i \,|\, a^*(\theta) v_i \,|^2} \qquad (1)$$

Equation(1) shows that these high resolution algorithms have the same general form and the only difference in each algorithm is a weighting function for the square magnitude of the product of the steering vector, $\mathbf{a}(\theta)$, and the eigenvector, $\mathbf{v}_i$, Table 1 shows the different weighting functions.

Table 1. The comparison of weighting functions

	weighting function
MVE	$w_i = \frac{1}{\lambda_i}$
SHIRE	$w_i = \frac{1}{\lambda_i^4 + \lambda_{M-i+1}^4}$
SNLM	$w_i = \frac{\lambda_i^{k-1}}{(\lambda_i^2 + \alpha^2)^k}$
MUSIC	$w_i = 0$ for $i = 1, \cdots, k$, $w_i = 1$ for $i = k+1, \cdots, M$
EVM	$w_i = 0$ for $i = 1, \cdots, k$, $w_i = \frac{1}{\lambda_i}$ for $i = k+1, \cdots, M$

The first-term(for i=1, ⋯, k) of equation (1) relates to the signal-subspace and the second-term(for i = k + 1, ⋯, M) relates to the noise-subspace. The eigenvectors in the signal-subspace(signal-eigenvectors) have the DOA information where the signal are, while the eigenvectors in the noise-subspace(noise-eigenvectors) have the information where they are not. Therefore, the spectral estimate which is obtained using the noise-eigenvectors or using the signal-eigenvectors has better resolution and accuracy than the spectral estmate which is obtained using all the eigenvectors of the covariance matrix. Combining the signal-eigenvectors and the noise-eigenvectors degrades the performance because of the complementary relationship between the two sets of the eigenvectors. Only one set of the eigenvectors is needed for the best estimate of the DOA. However, the signal-eigenvectors are easily distored by the noise[1]. Therefore, MUSIC and EVM use only the noise-eigenvectors to obtain the direction-of-arrivals of the target sources.

The comparison of the above algorithms(Table 1.) is plotted in Fig. 1. We used the same conditions for each algorithm in the simulations. We used a linear array with 10 elements and uniform spacing of 0.5 wavelength between successive sensors. Two sources are located at -2° and 3°. The noise is correlated and the input SNR for the array is 0dB.

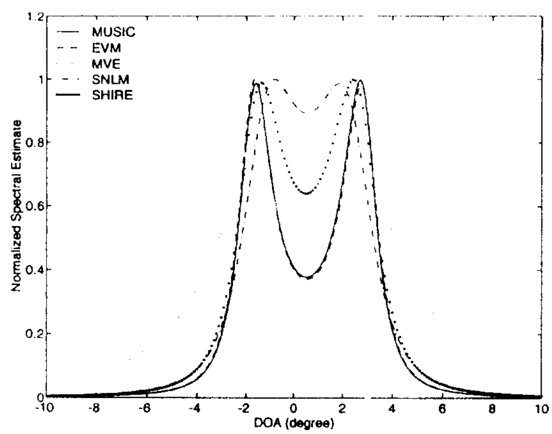


Fig. 1 Performance comparison of algorithms

From Fig. 1, EVM and MUSIC have better resolution and accuracy than the other algorithms. This is because MUSIC and EVM do not combine the signal-eigenvectors and the noise-eigenvectors. Only the noise-eigenvectors are used to estimate the spectra for these algorithms. However, even though the EVM spectral estimate is obtained using the best weighting of the eigenvectors, it is severely degraded when the SNR is low and/or the sources are close to each other. This is true because the eigenvectors of the observed covariance matrix are severely distorted by the noise. Therefore, techniques that can reduce the effect of the noise on the eigenvectors of the observed covariance matrix are important to obtain the best spectral estimate.

Ⅲ. Forward-Backward covariance matrix method

As discussed in section Ⅱ, The eigenvectors of the covariance matrix are divided into two groups. The eigenvectors related to the largest K eigenvalues span the signal-subspace. The remaining M-K eigenvectors related to the smallest M-K eigenvalues span the noise-subspace. In the ideal situation, the eigenvectors have the exact DOA information and they span two disjoint subspace(the signal-subspace and the noise-subspace). However, the eigenvectors are often perturbed by correlated noise. Therefore, the subspaces that are spanned by these eigenvectors are also perturbed.

In Fig. 2, because of the correlated noise, the eigenvector v becomes v′. Note that, the perturbed eigen-

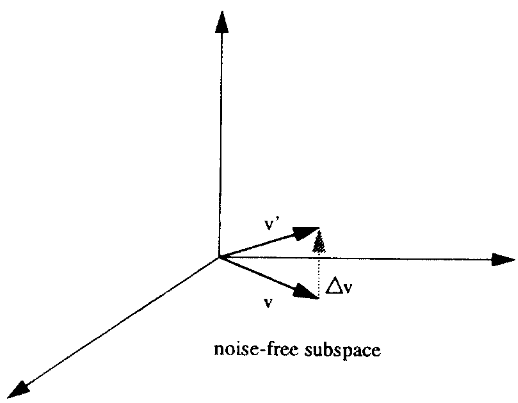


Fig 2. The relation between a noise-free eigenvector and noised eigenvector

vector v′ is no longer in the noise-free subspace. Since the basis of the subspace is the eigenvectors of the covariance matrix, the perturbed eigenvectors span the other subspace(called perturbed subspace). In other words, the perturbed noise-eigenvectors are in the perturbed noise-subspace and the spectrum which is obtained using these eigenvectors does not have good resolution and accuracy.

In this paper, we develop a new subspace which is less perturbed by the correlated noise. Even though this is an empirical results, the spectral estimate which is obtained using the eigenvectors in this subspace has better resolution and accuracy than the spectral estimate which is obtained using the eigenvectors in the conventional subspace. This is because the eigenvectors in this subspace are less perturbed than those in the conventional subspace under the same noise condition. We get this subspace from a weighted forward-backward covariance matrix.

The weighted forward-backward covariance matrix has the following form:

$$C = \omega R + (1-\omega) P \bar{R} P \tag{2}$$

where, ω is a weighting coefficient, R is the conventional covariance matrix, $\bar{R}$ is a complex conjugate of R, and P is an inverse diagonal matrix. Note that, when $\omega = 1$, C becames the conventional covariance matrix R. Therefore, the weighted forward-backward covariance matrix can be thought of as a generalization of the conventional covariance matrix.

We used a perturbation angle to represent the quantity of the distortion of the eigenvectors. The perturbation angle is the angle between the noise-free subspace which is spanned by the noise-free eigenvectors and the noisy subspace which is spanned by the noisy eigenvectors. It is clear that small perturbation angle represents a small distortion of the eigenvectors. Therefore, if we can find the eigenvectors which have a smaller perturbation angle for the same noise conditions, then the spectral estimate obtained using these eigenvectors will have better resolution and accuracy. We simulated the perturbation angle of the eigenvectors of the weighted forward-backward covariance matrix with various values for ω to find the best ω. The results are plotted in Fig. 3.

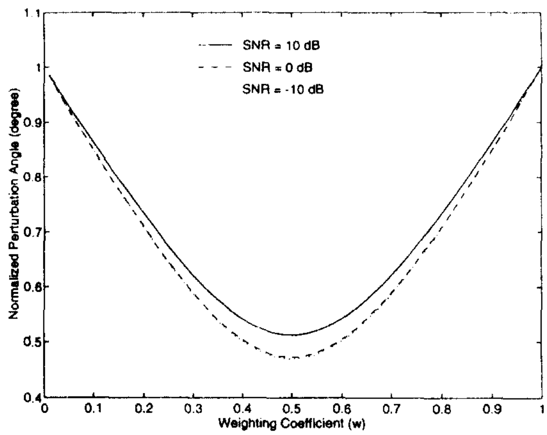
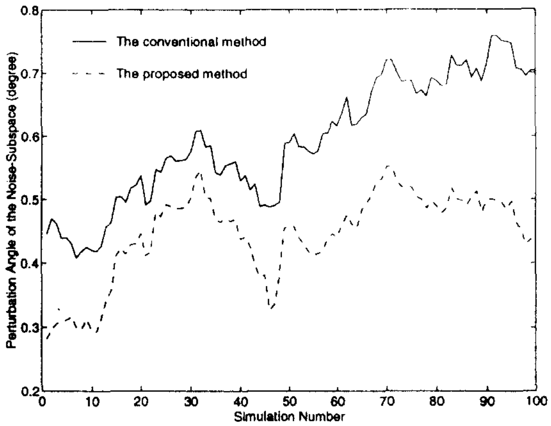


Fig 3. The perturbation angle when ω varies. The smallest perturbation angle is obtained at $\omega = \frac{1}{2}$

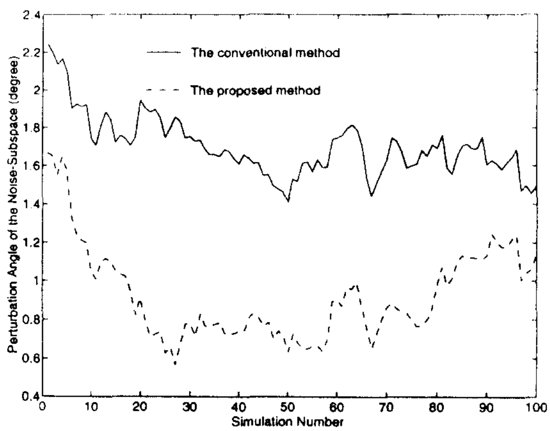
In the simulations, the different correlated noises were used with the SNRs equal to −10dB, 0 dB, and 10dB. Fig. 3 shows the perturbation angle between the noise-free subspace and the noisy subspace when ω varies. From Fig. 3, the smallest perturbation angle is obtained when $\omega = \frac{1}{2}$ regardless of the SNRs. Note that, the perturbation angle at $\omega = 1$ is the same as the angle of the conventional covariance matrix and the perturbation angle is large at this point. Therefore, we choose $\omega = \frac{1}{2}$.

Ⅳ. Result of simulations

We compared the perturbation angle for the forward-backward covariance matrix(C) and for the conventional covariance method(R). In the simulations, as discussed in section Ⅱ, we used a linear array with 10 elements and uniform spacing of 0.5 wavelength between successive sensors. One source was located at 3°. The noise was correlated and the SNRs were 5dB, 0dB, −5dB and −10dB respectively. We collected one hundred snapshots of data and used in each simulation run. We ran the simulation one hundred times for each SNR. Fig. 4 shows the comparison of the perturbation angles of the eigenvectors of C and the eigenvectors of R. It shows that the perturbation angle of the eigenvectors for the forward-backward covariance matrix are always less than those for the conventional covariance matrix under the same noise conditions.



(a) When SNR = 5 dB



(b) When SNR = 0 dB

Fig 4. The eigenvector perturbation angle when SNR = 5dB and 0dB

Table 2 shows the mean-values of the perturbation angle for the noise-subspace(simulation results). The value, mean1 and mean2, are the mean values of the eigenvector perturbation angles for the conventional covariance matrix and for the proposed enhanced covariance matrix, respectively. Table 2 shows that the noise-subspace using the forward-backward covariance matrix has a smaller perturbation angle than that for the noise-subspace using the conventional covariance matrix.

Table 2. The comparison of the eigenvector perturbation angle.

SNR(dB)	Mean 1 (degree)	Mean 2 (degree)
5	0.5925	0.4448
0	1.7041	0.9278
−5	4.3134	2.6881
−10	16.1729	8.8334

We performed several computer simulations to examine the performance of the our new method which we call the modified EVM(MEVM). We considered two separate conditions:

1. When the input SNR to the array is low, and
2. When two sources are close to each other.

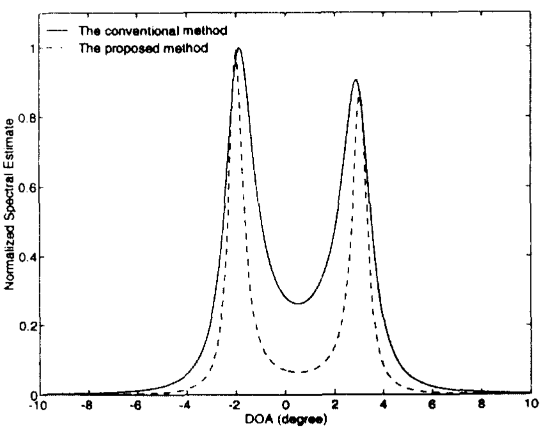


Fig 5. The comparison of the spectral estimates when SNR = 0dB, two signals are at −2° and 3°. Both methods detect the DOAs.

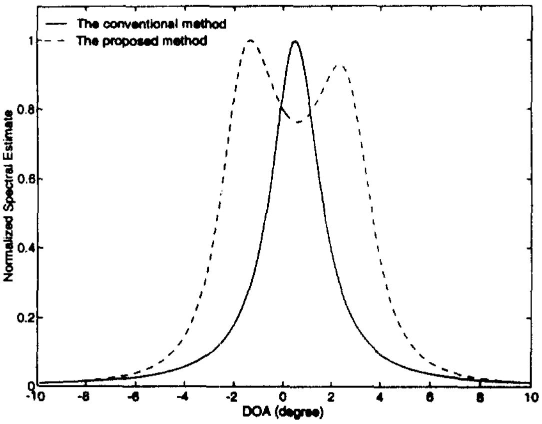


Fig 6. The comparison of the spectral estimates when SNR $= -5$dB, two signals are at $-2°$ and $3°$. Only the MEVM detects the correct DOAs.

For the first condition, Fig. 5 shows the spectral estimates when the input SNR is moderate(0 dB). Both methods detect the DOAs for moderate input SNR. Fig. 6 shows the spectral estimate when input SNR becomes low(-5 dB). Only the MEVM detects the correct DOAs when the input SNR is low.

For the second condition, Fig. 5 shows that both methods detect DOAs when two sources are $-2°$and $3°$. Fig. 7 shows the spectral estimates when two sources are at $-2°$and $2°$. The spectral estimates for the conventional method merge into a single estimate while the MEVM still detects the correct DOAs. Fig. 8 shows the spectral estimates when two sources are closer($-2°$and $1.5°$). The MEVM still detects the DOAs even though two sources are very close to each other. Therefore, the MEVM has the better resolution and accuracy than the conventional method under the same noise conditions.

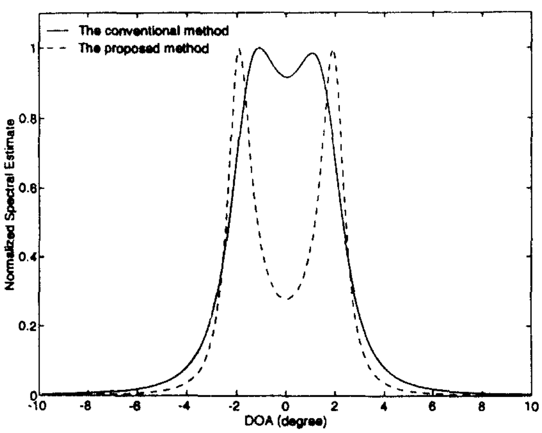


Fig 7. The comparison of the spectral estimates when two signals are at $-2°$ and $2°$, SNR $=0$dB. The conventional EVM becames unstable.

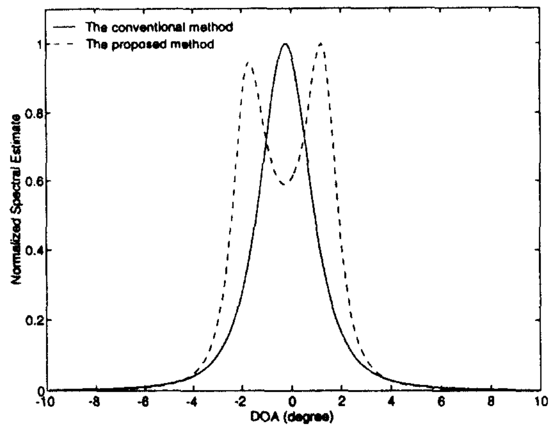


Fig 8. The comparison of the spectral estimates when two signals are at $-2°$ and $1.5°$, SNR $=0$dB. The MEVM still detects the correct DOAs.

V. Conclusions.

In this paper, we proposed a new method which not only uses the best weighting for the eigenvectors of the covariance matrix but also enhances the estimate of the covariance matrix to obtain the best spectral estimate.

We evaluated the performance of the MEVM and compared the results with the conventional EVM method. Our results show that the MEVM has better resolution and accuracy than the conventional covariance matrix method when the input SNR to the array is low and/or the sources are close to each other.

The superiority of the MEVM has been shown by our simulation results.

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高 鶴 林(Hak-Lim Ko) 정회원

1960년 10월 30일생

1983년 2월 : 숭실대학교 전자공학과(학사)

1986년 8월 : Fairleigh Dickinson University Electrical Engineering(석사)

1995년 8월 : North Carolina State University Electrical and Computer Engineering(박사)

1995년 9월~1996년 2월 : LG반도체 책임 연구원

1996년 3월~현재 : 호서대학교 정보통신공학부 전임강사

文 大 哲(Dai-Tchul Moon) 정회원

한국통신학회논문집 제 18권 11호 참조

현재 : 호서대학교 정보통신공학부 교수