

System Identification of Truss Structures Via Modal Parameters

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모드 특성치에 의한 트러스 구조의 시스템 동일화

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Key Words : Nondestructive(비파괴), Damage Detection(손상발견), System Identification(시스템 동일화), Truss Structure(트러스 구조물), Modal Parameter(모드특성치)

Abstract

본 연구는 진동반응측정을 통해 구조물의 손상을 비파괴적으로 예측하는 분야에 속한다. 건물, 교량, 댐, 해양자켓, 원자력 발전소 등의 구조물에서 시기적절하고 정확한 손상의 발견은 치명적인 구조적 결함의 예방을 위해 필수적이다. 구조물의 진동반응을 측정하여 구조물의 손상발견을 예측하려는 연구는 지난 80년대 이후 활발히 수행되어 오고 있으며, 본 연구는 구조물 고유진동수의 변화로부터 구조물 강성도의 변화를 모니터링하는 연구분야에 속한다.

본 연구의 목적은 현존하는 시스템동일화(system identification)에 의한 손상발견법을 제시하고, 실물 트러스 구조물의 손상위치 및 크기예측의 경우에 대한 제시된 손상발견법의 적합성을 검증하는 것이다. 먼저, 실물실험된 삼차원 트러스교량이 선택되었다. 다음으로, 시스템동일화 개념에 기초한 손상발견법이 요약되었다. 마지막으로, 시스템동일화 손상발견법을 실물트러스에 적용하여 적합성을 실험하였다.

1. INTRODUCTION

Safety inspection of structures using accurate and reliable techniques of damage detection is very important. Damage in critical members undiscovered from various inspection tasks causes not only local or global failures of the

structural systems but also results in catastrophic disasters, such as loss of lives, human suffering, and expenses of infrastructure properties, to our civil society.

During the past decade, many research studies have focused on the possibility of using the vibration characteristics as an indication of

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structural damage.[2,4,6,9,13] More recently, attempts have been made to monitor structural integrity of bridges[3,7] and to investigate the feasibility of damage detection in space structures using changes in modal parameters.[5,14] More recently, attempts have been made to monitor structural integrity of bridges[3,7] and to investigate the feasibility of damage detection in space structures using changes in modal parameters.[5,14] Despite these research efforts, outstanding needs still remain to detect damage in structures: (1) with many members(e.g., complex structures such as 3-D truss structures); (2)for which only few mode shapes are available; and (3) for which baseline (i.e., undamaged state) modal responses are not available (e.g., the majority of existing structures).

The objective of this paper is to examine the feasibility of an existing methodology to locate damage in real, 3-D, truss structures for which modal parameters of a few vibrational modes were measured for only post-damaged state. To achieve this objective, the investigation is performed in three parts. In the first part, we describe a field-tested 3-D truss bridge which was selected as the test structure. Structural dimensions and field test results on the structure are summarized. In the second part, we describe an existing methodology to detect damage in structures with three characteristics: many complex members, a few modes of vibration, and no measured as-built modal responses. We first outline a general scheme of the methodology. Next, we summarize the theory of damage localization and severity estimation. Then we outline the system identification method to build the baseline model (an analytical model representing the as-built state) of the test structure. In the last part, we perform damage detection exercises in the test structure using

the damage detection methodology. We first identify the baseline model of the 3-D truss using the system identification method. Next, we demonstrate the feasibility of the methodology from a damage detection exercise in a FE model which corresponds to the analytical model of the 3-D truss.

2. DESCRIPTION OF STRUCTURE

The selected 3-D truss structure is shown in Fig. 1. [10,15] It consists of ten (10) main structural subsystems which are bottom chords, top chords, middle chords, lower lateral members, vertical members, diagonal members, portals, precast concrete slab, steel stringers, and three water lines.

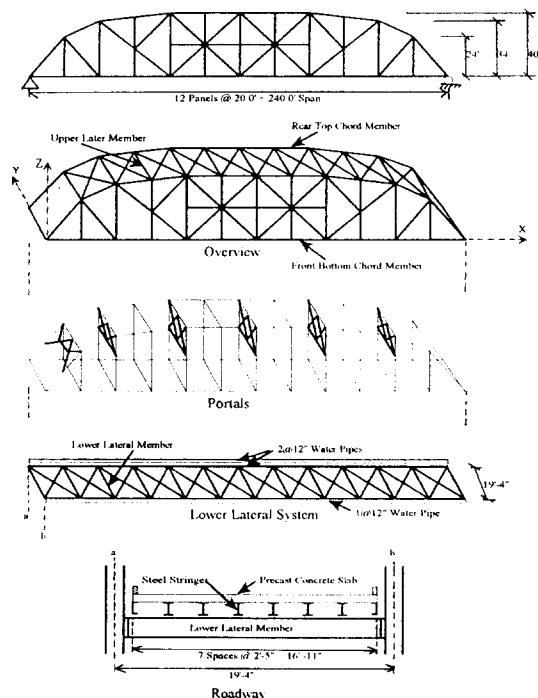


Fig. 1 Schematic of the 3-D Truss Bridge

Vibrational test data of the truss were obtained from two series of tests before and

after damage inflicted.[15] All data were provided by two accelerometers: a fixed accelerometer placed in the z-direction of node 45 (see Fig. 2) and a roving accelerometer that was moved from joint of the bridge. As shown in Fig. 2, accelerations were measured at the total 66 joints

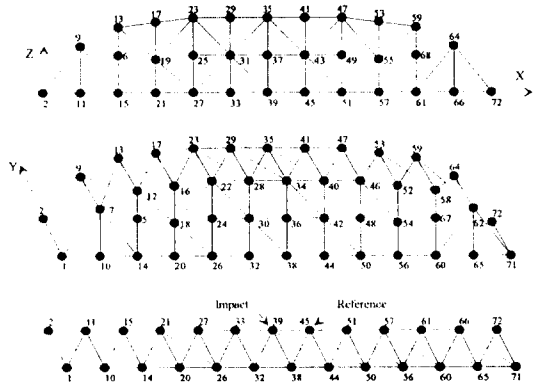


Fig. 2 Accelerometer Reading Locations in the 3-D Truss Bridge

in the bridge. At each joint the roving accelerometer recorded accelerations in the x, y, and z directions. The bridge was excited with an impact from a mass weighing eighty pounds which was dropped about 1.5 feet. The location of excitation was held constant throughout the course of testing. The accelerometers were connected to a multichannel signal analyzer which was coupled to a data acquisition device. Records of acceleration versus time were recorded for each accelerometer. Each accepted record represented the average of at least two responses. In all, approximately 170 time histories were recorded for the bridge. Frequency response functions between the roving accelerometer and the fixed accelerometer were generated. Mode shapes and resonant frequencies were extracted from the frequency response function. The extracted from the frequency response function. The extracted

(post-damage) modal responses of the bridge include resonant frequencies and mode shapes of the first bending mode and the first torsional mode. The resonant frequencies were (1) 2.1875 Hz for the first bending mode and (2) 3.50 Hz for the first torsional mode. The measured mode shapes of those two modes are shown in Fig. 3.

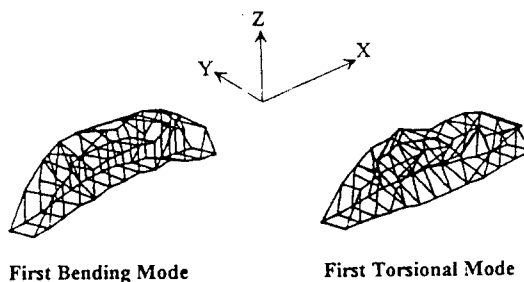


Fig. 3 Measured Mode Shapes of the 3-D Truss

3. DAMAGE DETECTION METHODOLOGY

The scheme described here yields information on the localization and severity of damage directly from changes in mode shapes of structures.[11][12] A series of steps designed to satisfy the objective are schematized in Fig. 4.

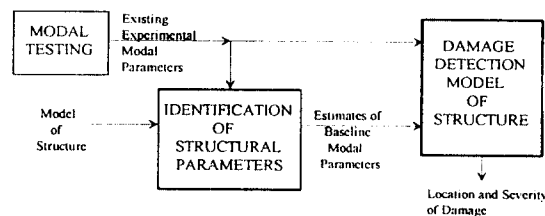


Fig. 4 Schematic of Approach used to Detect Damage in Structures Without Baseline Modal Data

3.1 Theory of Damage Localization and Severity Estimation

For a linear skeletal structure with NE elements and N nodes and then for NM modes

of vibration, a damage index β_j of j^{th} member is defined as [11][12]

$$\beta_j = \frac{\sum_{i=1}^{NM} \left\{ \left(\Phi_i^{*T} C_{jo} \Phi_i^* + \sum_{k=1}^{NE} \Phi_i^{*T} C_{ko} \Phi_i^* \right) K_i \right\}}{\sum_{i=1}^{NM} \left\{ \left(\Phi_i^T C_{jo} \Phi_i + \sum_{k=1}^{NE} \Phi_i^T C_{ko} \Phi_i \right) K_i^* \right\}}, \quad (1)$$

$\beta_j \geq 0$

where damage is indicated at the j^{th} member if $\beta_j > 1$.

$$K_i = \Phi_i^T C \Phi_i, \quad K_{ij} = \Phi_i^T C_j \Phi_i \quad (2)$$

$$K_{ij}^* = \Phi_i^{*T} C_j^* \Phi_i^*, \quad K_i^* = \Phi_i^{*T} C^* \Phi_i^* \quad (3)$$

where Φ_i is the i^{th} modal vector, C is the system stiffness matrix, C_j is the contribution of j^{th} member to the system stiffness matrix; K_i is the i^{th} modal stiffness; and K_{ij} is the contribution of the j^{th} modal stiffness. Also, a subsequently damaged structure is characterized by asterisks.

The predicted location j is classified into one of two groups (i.e., undamaged and damaged locations). First, the values of the indicator β_j ($j=1,2,3, \dots, NE$) are normalized according to the rule

$$Z_j = \frac{\beta_j - \bar{\beta}}{\sigma_\beta} \quad (4)$$

in which the terms $\bar{\beta}$ and σ_β represent, respectively, the mean and the standard deviation of the collection of β_j values. Next, the damage pattern is classified via a statistical pattern recognition technique [8].

$$\begin{aligned} &\text{Choose } H_1 : \text{ when } Z_j \geq K \\ &\text{Choose } H_0 : \text{ when } Z_j < K \end{aligned} \quad (5)$$

where the null hypothesis (i.e., H_0) is: the structure is not damaged at the j^{th} location. The alternate hypothesis (i.e., H_1) is: the structure is damaged at the j^{th} location.

Next, damage severity in the element j can be estimated by

$$\alpha_j = \frac{[\Phi_i^{*T} C_{jo} \Phi_i^*] K_i^*}{[\Phi_i^{*T} C_{jo} \Phi_i^*] K_i} - 1, \quad \alpha_j \geq -1 \quad (6)$$

where α_j is the fractional change in the stiffness of the j^{th} member.

3.2 System Identification Algorithm

Here, we outline an algorithm to identify a baseline model of a structure. Consider a linear skeletal structure with NE members and N nodes. Suppose k_j^* is the unknown stiffness of the j^{th} member of the structure for which M eigenvalues are known. Also, suppose k_j is a known stiffness of the j^{th} member of a finite element (FE) model for which the corresponding set of M eigenvalues are known. Then, relative to the FE model, the fractional stiffness change of the j^{th} member of the structure, α_j , and the stiffnesses are related according to the following equation

$$k_j^* = k_j(1 + \alpha_j) \quad (7)$$

The fractional stiffness change of NE members is defined as

$$\alpha = F^{-1} Z \quad (8)$$

where α is a $NE \times 1$ matrix containing the fractional changes in stiffnesses between the FE model and the structure, Z is a $M \times 1$ matrix containing the fractional changes in eigenvalues between the two systems, and F is a $M \times NE$ sensitivity matrix relating the fractional changes in stiffnesses to the fractional changes in eigenvalues [11][12].

The above rationale produces the following 6-step algorithm:

1. Collect measured frequencies and mode shapes of a target structure;

2. Select an initial FE model of the structure, utilizing all possible knowledge about the design and construction of the structure;
3. Compute the sensitivity matrix of the FE model;
4. Compute the fractional changes in eigenvalues between the FE model and the target structure;
5. Fine-tune the FE model by first solving Eq. 8 to estimate stiffness changes and next solving Eq. 7 to update the stiffness parameters of the FE model; and
6. Repeat Steps 2 to 5 until $Z \cong \alpha \cong 0$ (i.e., as they approach zero) when the parameters of the FE model are identified.

The converged FE model is the baseline model. It has the frequencies of the damaged(i.e., target) structure but none of its members are damaged. Furthermore, the mode shapes of the baseline model differs from those of the damaged structure.

Table 1. Sensitivity Used to Fine-tune the FE Model

Mode	Sensitivity	
	Group II	Group III
1(First Bending)	0.5609	0.4391
2(First Torsion)	0.0913	0.9087

4. DAMAGE PREDICTION PRACTICE

4.1 Identification of Baseline Model

To identify a realistic analytical model of the 3-D truss, we fine-tuned a FE model (which is selected as an initial guess) using a system identification concept that combines experimental and analytical responses of the structure. As shown in Fig. 5, the FE model consisted of

eleven (11) subsystems. Each subsystem was assigned to one of three element groups(Group I, II, and III)based on the availability of structural parameters (e.g., stiffness of flexibility) and on the sensitivity of structural responses to vibrational modes.

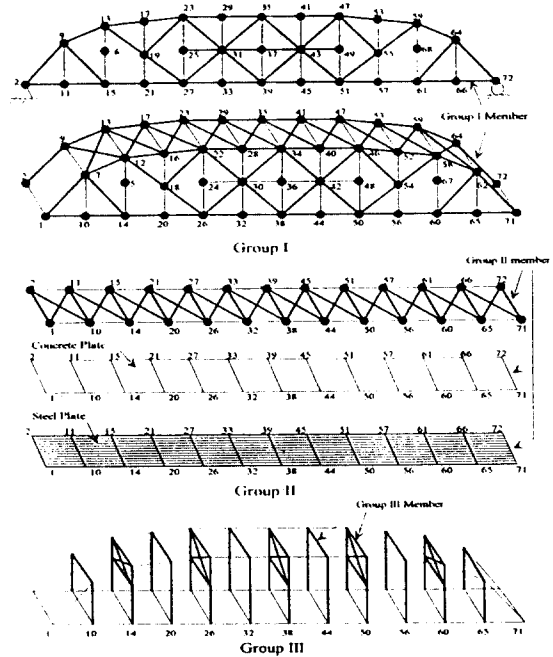


Fig. 5 Schematic of Baseline Model of the 3-D Truss

Group I represents elements “for which sufficient information on structural parameters is available from field-measurements and as-built designs” and it includes the top, bottom, and middle chords, the diagonal truss members, and the three water lines. Group II represents elements “which have insufficient information on structural parameters” and “which react relatively sensitive to motions of the first bending mode” and it includes the lower lateral members, the pre-cast concrete slab, and the steel stringers. Group III represents elements which have insufficient information on structural

parameters" and "which react relatively sensitive to motions of the first torsional mode" and it includes the upper lateral members, the vertical truss members, and the portal members.

Initial values of material and geometric properties of the FE model were estimated as follows (For the cross-sectional area and the second moment of area, see Reference [10] for details): for elements of Subsystems 1 to 8, Poisson's ratio $\nu=0.3$; the elastic modulus $E=2.03 \times 10^6 \text{ kg/cm}^2$; and the linear mass density $\rho=7,850 \text{ kg/m}^3$. For elements of Subsystem 11, $E=0$; $\rho=1,070 \text{ kg/m}^3$; and the radius of the pipe section $r=15.24 \text{ cm}$. For plate elements of Subsystem 9, $E=0.25 \times 10^6 \text{ kg/cm}^2$; $\nu=0.15$; $\rho=2,420 \text{ kg/m}^3$; and the plate thickness $t=0.51 \text{ cm}$. For plate elements of Subsystem 10, $E=2.03 \times 10^6 \text{ kg/cm}^2$; $\nu=0.3$; $\rho=7,850 \text{ kg/m}^3$; and the thickness of steel plate $t=0.51 \text{ cm}$.

Two element groups, Group II and Group III, were selected to fine-tune the initial FE model. The two element groups were selected on the basis of the following reasons: (1) elements in those groups have insufficient information on the geometric and material properties; (2) elements in Group II react relatively sensitive to the first flexural motion; and (3) elements in Group III react relatively sensitive to the first torsional motion. The flexural rigidity and the torsional rigidity were selected for the fine-tuning exercise. Note that Group I was not selected since elements in the group have relatively sufficient data on structural parameters which were obtained from the as-built information.

The baseline model was identified as follows: First the F-matrix of the initial FE model was computed as shown in Table 3. For a given mode, each sensitivity represents the fraction of modal energy stored in the particular member type. Next, the FE model was fine-tuned by solving Eq. 7 to estimate stiffness changes and

then solving to update the stiffness parameters of the FE model. The whole procedure was then repeated until the stiffness changes are reduced to zero. The results, using two frequencies and five iterations, are listed in Table 2. After the iterations, the frequencies were identified within one-percent-error-range of the target values. The values of the material properties (the elastic moduli) under the five iterations are summarized in Table 3. Note that the values of the effective elastic moduli represent stiffness parameters of Group II and Group III assuming that the geometric properties remain constant. Based on this results, the FE model after the fifth iteration is selected as the baseline model. Typical numerically generated mode shapes of the first two modes are shown in Fig. 6. Natural frequencies of those two modes are 2.1946 Hz for the first flexural mode and 3.4761 Hz for the first torsional mode.

From Fig. 3 and Fig. 6, detailed comparison between the experimental mode shapes and the

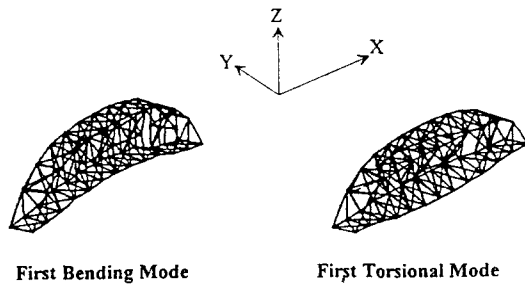


Fig. 6 Mode Shapes of Baseline Model of the 3-D Truss

Table 2. Values of Frequencies for Five Iterations

Mode	Initial-Guess	Iteration Number					Target
		1	2	3	4	5	
1	2.3730	2.2268	2.2042	2.1982	2.1958	2.1946	2.1850
2	2.7854	2.9955	3.2445	3.3797	3.4462	3.4761	3.5000

Table 3. Values of Elastic Moduli (psi) for Element Groups After Five Iterations

Element Group	Iteration Number					
	Initial-Guess	1	2	3	4	5
Group II	29.0E6	4.87E6	2.87E6	2.35E6	2.15E6	2.05E6
Group III	29.0E6	49.9E6	72.01E6	86.30E6	93.93E6	97.6026

baseline mode shapes could be made by plotting the z-component of the two modes. The results are shown in Figs. 7-8 . It is observed that the baseline mode shapes show good identification to the experimental mode shapes except several nodes where deflections are occurred. So we conclude that those two sets of mode shapes are identical. Note that assessing the effect of the noises shown in the figures is the topic of another on-going research.

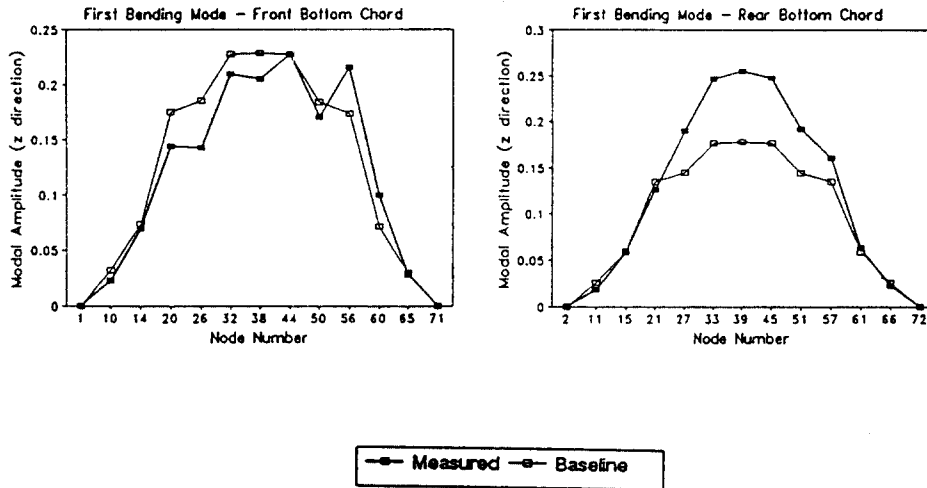


Fig. 7 Vertical Motion of the First Flexural Mode for Bottom Chords of the 3-D Truss

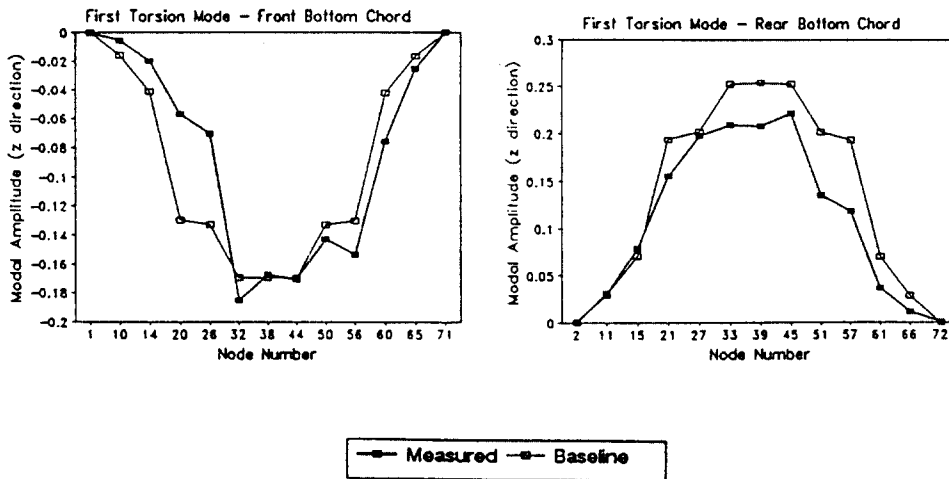


Fig. 8 Vertical Motion of the First Torsional Mode for Bottom Chords of the 3-D Truss

4.2 Damage Localization and Severity Estimation in Baseline Model

The pre-damage and post-damage modal parameters for the baseline model were generated numerically using the software.[1] Here, fifteen damage cases were investigated. The simulations of the damage locations and the corresponding severities of damage are listed in Table 4. In all cases, damage was simulated in the structure by reducing the elastic moduli of the appropriate members of the FE model shown in Fig. 2. Note that these damage cases form the sample of our population for future statistical analyses. The pre-damage and post-damage frequencies of the first two modes are listed in Table 5. The fifteen cases were programmed as follows: Cases 1-3 damaged on the top chords; Cases 4-6 damaged in the bottom chords; Cases 7-9 damaged in the vertical truss members; Cases 10-12 damaged in the diagonal truss members; and Cases 13-15 damaged in the

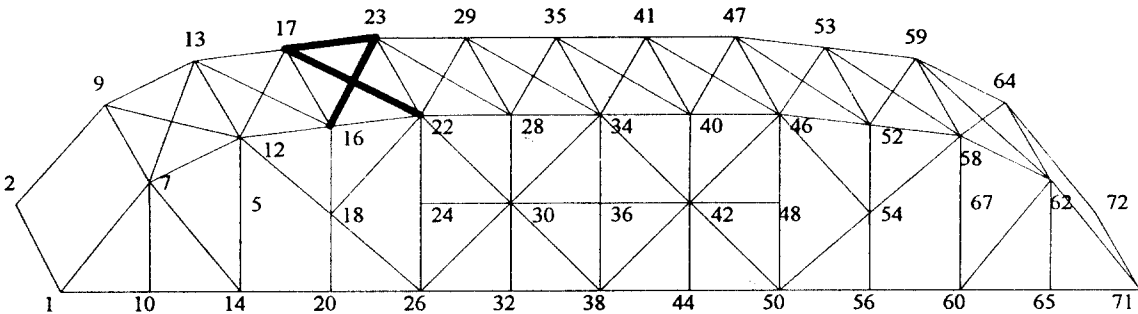
lower lateral members.

Table 5. Natural Frequencies of Baseline Model

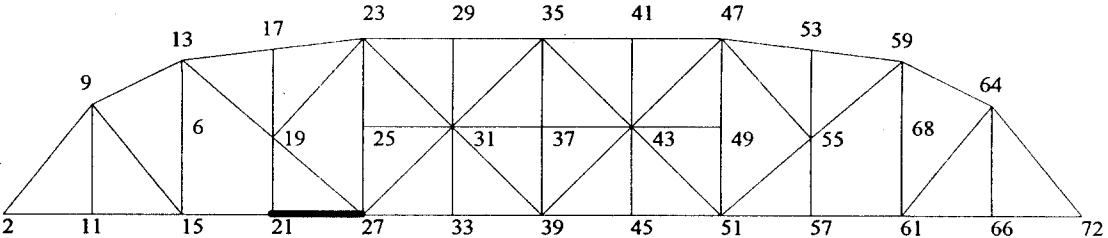
Damage Case	Frequency(Hz)	
	Model (First Bending)	Mode2 (First Torsion)
Reference	2.1946	3.4761
1	2.1891	3.4738
2	2.1865	3.4695
3	2.1862	3.4681
4	2.1904	3.4761
5	2.1914	3.4761
6	2.1899	3.4749
7	2.1946	3.4755
8	2.1946	3.4685
9	2.1946	3.4740
10	2.1946	3.4760
11	2.1946	3.4758
12	2.1946	3.4755
13	2.1946	3.4760
14	2.1946	3.4781
15	2.1946	3.4760

Table 4. Damage Localization and Severity Estimation Estimation in Baseline Model

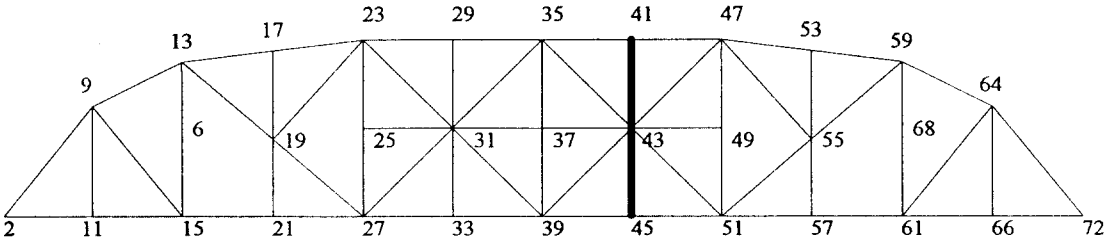
Case NO.	Simulated Damage		Predicted Damage					
	Between Nodes	Damage Magnitude	Prediction One		Prediction Two		Prediction Three	
			Location	Magnitude	Location	Magnitude	Location	Magnitude
1	9-13	-0.25	9-13	-0.33	9-12	-0.18	7-13	-0.18
2	17-23	-0.25	17-23	-0.35	17-22	-0.14	16-23	-0.24
3	29-35	-0.25	29-35	-0.35	29-34	-0.17	28-35	-0.20
4	11-15	-0.25	11-15	-0.30				
5	21-27	-0.25	21-27	-0.34				
6	33-39	-0.25	33-39	-0.40				
7	17-19	-0.25	17-19	-0.45				
8	35-37	-0.25	35-37	-0.33	34-37	-0.15		
9	41-43	-0.25	41-43	-0.34	43-45	-0.14		
10	5-13	-0.25	5-13	-0.23				
11	23-24	-0.25	23-24	-0.23				
12	35-36	-0.25	35-36	-0.22				
13	14-15	-0.25	14-15	-0.17				
14	26-27	-0.25	26-27	-0.19				
15	38-39	-0.25	38-39	-0.19				



(a) Damage Case 2 (damage simulated in Nodes 17-19)



(b) Damage Case 5 (damage simulated in Nodes 21-27)



(c) Damage Case 9 (damage simulated in Nodes 41-43)

Fig. 9 Locations of Simulated and Predicted Damage in the 3-D Truss : Dark lines represent the predicted locations of damage.

Damage in the FE model was located and sized in five steps. Firstly, we selected the truss model as the damage detection model of the FE model. Secondly, we computed sensitivities of the damage detection model for both the pre-damage and post-damage cases. Thirdly, the indicator Eq. 1 was computed and the criteria Eq. 5 was established as follows: (1) select H_0 (i.e., no damage at location j) if $Z_j < 2$. This criterion corresponds to a one-tailed test at a significance level of 0.023 (i.e., 97.7 percent confidence level). Fourthly, we used the latter criterion to scan potential locations of damage. The predicted and inflicted locations of damage for the fifteen damage cases are summarized in Table 4. Finally, damage severity at each predicted damage location was estimated from Eq. 6. The simulated and predicted severities of damage for each predicted location are also summarized in Table 4. Figs. 9(a), 9(b), and 9(c) visualize three typical cases (Cases 2, 5, and 9) among all damage prediction results.

It is observed that the methodology is feasible to locate and size damage in the complex model of 211 members and of only two vibrational modes available. Total twenty-three locations were predicted. All fifteen locations of simulated damage were predicted correctly. Eight false-alarmed locations (i.e., 8 locations in which damage was not simulated) were predicted. The magnitudes of damage at the correctly predicted locations were estimated within approximately ± 20 percent-error-range of the exact magnitudes of damage. Note that only two modes of vibrations were used to locate and size damage in the complex model of 211 members.

5. SUMMARY AND CONCLUSION

An existing methodology of damage localization was experimentally tested to examine its

feasibility to locate and estimate severities of damage in highly complex structures for which a few vibrational modes were measured only for post-damaged state of operation. To achieve this objective, the investigation was performed in three parts. In the first part, we described a field-tested 3-D truss bridge which was selected as the test structure. Structural dimensions and field experimental modal tests performed on the structure were summarized. In the second part, we described the existing methodology to detect damage in structures with three characteristics: many complex members, a few modes of vibration, and no measured as-built modal responses. We first outlined a general scheme of the methodology. Next, we summarized the theory of damage localization and severity estimation. We also outlined the system identification method to build the baseline model (an analytical model representing the as-built state) of the test structure. In the last part, we performed damage detection exercises in the test structure using the damage detection methodology. We first identified the baseline model of the 3-D truss using the system identification method. Next, we demonstrated the feasibility of the methodology from a damage detection exercise in a FE model which corresponds to the analytical model of the 3-D truss.

Results of the baseline modeling and damage detection exercises in the analytical FE model demonstrated the feasibility of the methodology. Using the methodology, we analyzed to define potential locations and severities of damage in the real 3-D truss-type bridge for which post-damage mode shapes of the initial two modes of vibrations were measured experimentally. On-going research by the author includes the assessment of the methodology's practicality and the model-uncertainty impact on the damage detection practice of the 3-D truss

structures with limited structural and modal information.

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