Fuzzy Pairwise β-continuous Mapping

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ABSTRACT

In this paper, we define a (τ_i, τ_j)-fuzzy β-open set and a fuzzy pairwise fuzzy β-continuous mapping on fuzzy bitopological spaces and study some of their properties.

I. Introduction

The concept of fuzzy set was introduced by Zadeh in his classical paper[13]. Using the concept of a fuzzy set, Chang[4] introduced a fuzzy topological space. Since then various workers have contributed to the development of the theory. Kandil[5] introduced and studied a fuzzy bitopological space as a natural generalization of a fuzzy topological space. In [11], Sampath Kumar introduced and investigated a (τ_i, τ_j)-fuzzy semiopen ((τ_i, τ_j)-fuzzy semiclosed) set and a fuzzy pairwise semicontinuous mapping on fuzzy bitopological spaces. Also, he defined a (τ_i, τ_j)-fuzzy preopen ((τ_i, τ_j)-fuzzy preclosed) set and a fuzzy pairwise precontinuous mapping on fuzzy bitopological spaces and studied some of their basic properties.

In this paper, we first define a (τ_i, τ_j)-fuzzy β-open ((τ_i, τ_j)-fuzzy β-closed) set and a fuzzy pairwise fuzzy β-continuous mapping on fuzzy bitopological spaces and study some of their properties. We show that every (τ_i, τ_j)-fuzzy semiopen set is a (τ_i, τ_j)-fuzzy β-open set and every (τ_i, τ_j)-fuzzy preopen set is a (τ_i, τ_j)-fuzzy β-open set but the converses are not true in general. Any union (respectively intersection) of (τ_i, τ_j)-fuzzy β-open (respectively (τ_i, τ_j)-fuzzy β-closed) sets is a (τ_i, τ_j)-fuzzy β-open (respectively (τ_i, τ_j)-fuzzy β-closed) set. We also show that every fuzzy pairwise semicontinuous mapping is a fuzzy pairwise β-continuous mapping and every fuzzy pairwise precontinuous mapping is a fuzzy pairwise β-continuous mapping. But the converses are not true in general.

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II. Preliminaries

For definitions and results not explained in this paper, we refer to the papers[1, 4, 8, 12, 13] assuming them to be well known.

Let X, Y and Z be nonempty sets and I the unit interval $[0, 1]$. A fuzzy set of X is a mapping from X into I. The empty fuzzy set 0_X is the mapping from X into I which assumes only the value 0, and the set X is denoted by mapping 1_X from X into I which takes the value 1 only. The union $\vee \mu_k$ (respectively intersection $\wedge \mu_k$) of a family $\{\mu_k \mid k \in \Lambda\}$, where Λ is an index set, of fuzzy sets of X is defined to be the mapping sup μ_k(respectively inf μ_k). A member μ of I^X is contained in a member ν of I^X, denoted by $\mu \leq \nu$, if and only if $\mu(x) \leq \nu(x)$ for each x in X. The complement μ^c of a fuzzy set μ of X is $1-\mu$, denoted by $(1-\mu)(x) = 1-\mu(x)$ for each x in X. If μ is a fuzzy set of X and ν is a fuzzy set of Y then $\mu \times \nu$ is a fuzzy set of $X \times Y$, defined by $(\mu \times \nu)(x, y) = \min(\mu(x), \nu(y))$ for each (x, y) in $X \times Y$[1].

Let $f: X \to Y$ be a mapping. If μ is a fuzzy set of X, then $f(\mu)$ is a fuzzy set of Y defined by

$$f(\mu)(y)=\begin{cases}\sup\{\mu(x)\mid x\in f^{-1}(y)\} \text{ if } f^{-1}(y)\neq\phi \text{ for each } y \text{ in } Y,\\ 0 \quad \text{otherwise.}\end{cases}$$

If ν is a fuzzy set of Y, then $f^{-1}(\nu)$ is a fuzzy set of X defined by $f^{-1}(\nu)(x)=\nu(f(x))$ for each x in X.

A subfamily τ of I^X is called a *fuzzy topology* on X [4] if

(i) 0_X and 1_X belong to τ,

(ii) any union of members of τ is in τ, and

(iii) a finite intersection of members of τ is in τ.

A member of τ is called τ-fuzzy open [$\tau-fo$] set of X and its complement is called τ-fuzzy closed [$\tau-fc$] set. For a fuzzy set μ of X, the τ-closure [τ-Cl] and the τ-interior [τ-Int] are defined, respectively as follows.

$\tau\text{-Cl}\,\mu=\inf\{\nu\mid\nu\geq\mu,\ \nu^c\in\tau\}$ and

$\tau\text{-Int}\,\mu=\sup\{\nu\mid\nu\leq\mu,\ \nu\in\tau\}$.

A system (X, τ_1, τ_2) consisting of a set X with two fuzzy topologies τ_1 and τ_2 on X is called a *fuzzy bitopological space X [fbts X]*[5]. Throughout this paper the indices i, j take values in $\{1, 2\}$ and $i\neq j$, $i=j$ gives the known results in fuzzy topological spaces.

Let μ be a fuzzy set of a *fbts X*. Then μ is called a (τ_i, τ_j)-fuzzy semiopen [(τ_i, τ_j)-*fso*] set of X, if there exists a ν in τ_i such that $\nu\leq\mu\leq\tau_j$-Clν. The complement of a (τ_i, τ_j)-*fso* set is called a (τ_i, τ_j)-fuzzy semiclosed [(τ_i, τ_j)-*fsc*] set[11].

Let $f:(X, \tau_1, \tau_2)\to(Y, \eta_1, \eta_2)$ be a mapping. Then f is called a fuzzy pairwise semicontinuous [*fpsc*] mapping, if $f^{-1}(\nu)$ is a (τ_i, τ_j)-*fso* set of X for each η_i-*fo* set ν of Y.

Let μ be a fuzzy set of a *fbts X*. Then μ is called a (τ_i, τ_j)-fuzzy preopen [(τ_i, τ_j)-*fpo*] (respectively (τ_i, τ_j)-fuzzy preclosed [(τ_i, τ_j)-*fpc*]) set of X, if $\mu\leq\tau_i$-Int (τ_j-Clμ) (respectively τ_i-Cl (τ_j-Intμ)$\leq\mu$).

Let $f:(X, \tau_1, \tau_2)\to(Y, \eta_1, \eta_2)$ be a mapping. Then f is called a fuzzy pairwise precontinuous [*fppc*] mapping, if $f^{-1}(\nu)$ is a (τ_i, τ_j)-*fpo* set of X for each η_i-*fo* set ν of Y[10].

III. (τ_i, τ_j)-fuzzy β-open sets, fuzzy pairwise β-continuous mappings

Now, we define a (τ_i, τ_j)-fuzzy β-open (respectively (τ_i, τ_j)-fuzzy β-closed) set and a fuzzy pairwise β-continuous mapping on fuzzy bitopological spaces and study some of their properties.

Definition 3.1 Let μ a fuzzy set of a *fbts X*. Then μ is called

(i) a (τ_i, τ_j)-fuzzy β-open [(τ_i, τ_j)-$f\beta o$] set of X if $\mu\leq\tau_j$-Cl (τ_i-Int (τ_j-Clμ)),

(ii) a (τ_i, τ_j)-fuzzy β-closed [(τ_i, τ_j)-$f\beta c$] set of X if τ_j-Int (τ_i-Cl (τ_j-Intμ))$\leq\mu$.

From the above definition it is clear that every (τ_i, τ_j)-*fso* (respectively (τ_i, τ_j)-*fsc*) set is a (τ_i, τ_j)-$f\beta o$ (respectively (τ_i, τ_j)-$f\beta c$) set, and every (τ_i, τ_j)-*fpo* (respectively (τ_i, τ_j)-*fpc*) set is a (τ_i, τ_j)-$f\beta o$ (respectively (τ_i, τ_j)-$f\beta c$) set. But the converse are not true in Example 3.2.

Example 3.2 Let μ and ν be fuzzy sets of $X=\{a, b\}$ defined as follows;

$\mu(a)=0.5,\ \ \mu(b)=0.6,$

$\nu(a)=0.5,\ \ \nu(b)=0.3.$

Consider fuzzy topologies $\tau_1=\{0_X, \mu, 1_X\}$ and $\tau_2=\{0_X, \nu, 1_X\}$. Then every fuzzy set of X is a (τ_i, τ_j)-$f\beta o$ (respectively (τ_i, τ_j)-$f\beta c$) set, but ν, μ^c are not (τ_i, τ_j)-*fso* sets and μ, ν^c are not (τ_i, τ_j)-*fsc* sets. Also, ν^c is not a (τ_i, τ_j)-*fpo* set and ν is not a (τ_i, τ_j)-*fpc* set. ■

Theorem 3.3 (i) Any union of (τ_i, τ_j)-$f\beta o$ sets is a (τ_i, τ_j)-$f\beta o$ set.

(ii) Any intersection of (τ_i, τ_j)-$f\beta c$ sets is a (τ_i, τ_j)-$f\beta c$ set.

Proof. We prove (i) for (τ_i, τ_j)-$f\beta o$ sets, the other proof is similar. Let $\{\mu_k\}$ be a collection of (τ_i, τ_j)-$f\beta o$ sets of a *fbts* X. Then $\mu_k \le \tau_j$-Cl $(\tau_i$-Int $(\tau_j$-Cl$\mu_k))$ for each k, and by Lemma 3. 1 of [1], we have

$$\vee\mu_k \le \vee(\tau_j\text{-Cl}\,(\tau_i\text{-Int}\,(\tau_j\text{-Cl}\,\mu_k))) \le \tau_j\text{-Cl}\,(\tau_i\text{-Int}\,(\tau_j\text{-Cl}\,(\vee\mu_k))).$$

This show that $\vee\mu_k$ is a (τ_i, τ_j)-$f\beta o$ set. ■

It is clear that the intersection (respectively union) of any two (τ_i, τ_j)-$f\beta o$ (respectively (τ_i, τ_j)-$f\beta c$) sets need not be a (τ_i, τ_j)-$f\beta o$ (respectively (τ_i, τ_j)-$f\beta c$) set. Even the intersection (respectively union) of a (τ_i, τ_j)-$f\beta o$ (respectively (τ_i, τ_j)-$f\beta c$) set with a τ_i-fo (respectively τ_i-fc) set may fail to be a (τ_i, τ_j)-$f\beta o$ (respectively (τ_i, τ_j)-$f\beta c$) set.

Example 3.4 Let μ and ν be fuzzy sets of $X=\{a, b\}$ defined as follows;

$\mu(a)=0.1,\ \ \mu(b)=0.8,$
$\nu(a)=0.8,\ \ \nu(b)=0.3.$

Let $\tau_1=\{0_X, \mu, 1_X\}$ and $\tau_2=\{0_X, \nu, 1_X\}$ be fuzzy topologies on X. Then μ and ν are (τ_i, τ_j)-$f\beta o$ sets, but $\mu\wedge\nu$ is not a (τ_i, τ_j)-$f\beta o$ set and $\mu^c\vee\nu^c$ is not a (τ_i, τ_j)-$f\beta c$ set of *fbts* X. ■

Theorem 3.5 If μ is both a (τ_i, τ_j)-$f\beta o$ set and a (τ_j, τ_i)-fsc set of a *fbts* X, then μ is a (τ_i, τ_j)-fso set.

Proof. Since μ is a (τ_i, τ_j)-$f\beta o$ set, $\mu \le \tau_j$-Cl $(\tau_i$-Int $(\tau_j$-Cl$\mu))$. But μ is a (τ_j, τ_i)-fsc set, hence τ_i-Int $(\tau_j$-Cl $\mu) \le \mu$. Thus τ_i-Int $(\tau_j$-Cl$\mu) \le \mu \le \tau_j$-Cl $(\tau_i$-Int $(\tau_j$-Cl $\mu))$. Hence μ is a (τ_i, τ_j)-fso set. ■

Corollary 3.6 If μ is both a (τ_i, τ_j)-$f\beta c$ set and a (τ_j, τ_i)-fso set of a *fbts* X, then μ is a (τ_i, τ_j)-fsc set.

Proof. Since μ is a (τ_i, τ_j)-$f\beta c$ set, τ_j-Int $(\tau_i$-Cl $(\tau_j$-Int$\mu)) \le \mu$. But μ is a (τ_j, τ_i)-fso set, hence $\mu \le \tau_i$-Cl $(\tau_j$-Int$\mu)$. Thus τ_j-Int $(\tau_i$-Cl $(\tau_j$-Int$\mu)) \le \mu \le \tau_i$-Cl $(\tau_j$-Int$\mu)$. Hence μ is a (τ_i, τ_j)-fsc set. ■

Theorem 3.7 Let (X, τ_1, τ_2) and (Y, η_1, η_2) be *fbts*'s such that X is product related to Y[1]. Then the product $\mu\times\nu$ of a (τ_i, τ_j)-$f\beta o$ set μ of X and a (η_i, η_j)-$f\beta o$ set ν of Y is a (σ_i, σ_j)-$f\beta o$ set in the fuzzy product bitopological space $(X\times Y, \sigma_1, \sigma_2)$, where σ_k is the fuzzy product topology[12] generated by τ_k and η_k $(k=1, 2)$.

Proof. Since μ is a (τ_i, τ_j)-$f\beta o$ set of X and ν is a (η_i, η_j)-$f\beta o$ set of Y, $\mu \le \tau_j$-Cl $(\tau_i$-Int $(\tau_j$-Cl$\mu))$ and $\nu \le \eta_j$-Cl $(\eta_i$-Int $(\eta_j$-Cl$\nu))$. Then, by Theorem 3.10 of [1], we have

$$\begin{aligned}\mu\times\nu &\le \tau_j\text{-Cl}\,(\tau_i\text{-Int}\,(\tau_j\text{-Cl}\,\mu))\times\eta_j\text{-Cl}\,(\eta_i\text{-Int}\,(\eta_j\text{-Cl}\,\nu))\\ &=\sigma_j\text{-Cl}\,(\sigma_i\text{-Int}\,(\sigma_j\text{-Cl}\,(\mu\times\nu))).\end{aligned}$$

Thus $\mu\times\nu$ is a (σ_i, σ_j)-$f\beta o$ set. ■

Definition 3.8 Let $f:(X, \tau_1, \tau_2)\rightarrow(Y, \eta_1, \eta_2)$ be a mapping. Then f is called a fuzzy pairwise β-continuous [*fp*β*c*] mapping, if $f^{-1}(\nu)$ is a (τ_i, τ_j)-$f\beta o$ set of a X for each η_i-fo set of Y.

From the above definition it is clear that every *fpsc* mapping is a *fp*β*c* mapping, and every *fppc* mapping is a *fp*β*c* mapping. But the converses are not true in Example 3.9.

Example 3.9 Let μ and ν be fuzzy sets of X in Example 3.2. Consider fuzzy topologies $\tau_1=\{0_X, \mu, 1_X\}$, $\tau_2=\{0_X, \nu, 1_X\}$, $\eta_1=\{0_X, \mu, \nu, \mu^c, \nu^c, 1_X\}$ and $\eta_2=\{0_X, \nu, 1_X\}$ and the identity mapping $i_X:(X, \tau_1, \tau_2)\rightarrow(X, \eta_1, \eta_2)$. Then i_X is neither a *fpsc* mapping nor a *fppc* mapping, but i_X is a *fp*β*c* mapping. ■

The following theorem provides several characterization of *fp*β*c* mappings.

Theorem 3.10 Let $f:(X, \tau_1, \tau_2)\rightarrow(Y, \eta_1, \eta_2)$ be a mapping. Then the following statements are pairwise equivalent:

(i) f is a *fp*β*c* mapping.

(ii) The inverse image of each η_i-fc set of Y is a (τ_i, τ_j)-$f\beta c$ set of X.

(iii) τ_j-Int $(\tau_i$-Cl $(\tau_j$-Int $(f^{-1}(\nu)))) \leq f^{-1}(\eta_i$-Cl$\nu)$ for each fuzzy set ν of X.

(iv) $f(\tau_j$-Int $(\tau_i$-Cl $(\tau_j$-Int$\mu))) \leq \eta_i$-Cl $(f(\mu))$ for each fuzzy set μ of X.

Proof. (i) implies (ii): Let δ be a η_i-*fc* set of Y. Then δ^c is a η_i-*fo* set of Y. Thus $f^{-1}(\delta^c)$ is a (τ_i, τ_j)-*fβo* set of X. But $f^{-1}(\delta^c) = (f^{-1}(\delta))^c$. Therefore $f^{-1}(\delta)$ is a (τ_i, τ_j)-*fβc* set of X.

(ii) implies (iii): Let ν be any fuzzy set of Y. Then $f^{-1}(\eta_i$-Cl$\nu)$ is a (τ_i, τ_j)-*fβc* set of X. Hence $f^{-1}(\eta_i$-Cl $\nu) \geq \tau_j$-Int $(\tau_i$-Cl $(\tau_j$-Int $(f^{-1}(\eta_i$-Cl$\nu))))$.

$\geq \tau_j$-Int $(\tau_i$-Cl $(\tau_j$-Int $(f^{-1}(\nu))))$.

(iii) implies (iv): Let μ be any fuzzy set of X. Then, by (iii), we have

τ_j-Int $(\tau_i$-Cl $(\tau_j$-Int $f^{-1}(f(\mu)))) \leq f^{-1}(\eta_i$-Cl $(f(\mu)))$.

Hence $f(\tau_j$-Int $(\tau_i$-Cl $(\tau_j$-Int$\mu))) \leq \eta_i$-Cl $(f(\mu))$.

(iv) implies (i): Let λ be a η_i-*fo* set of Y. Then λ^c is a η_i-*fc* set. Thus

$f(\tau_j$-Int $(\tau_i$-Cl $(\tau_j$-Int $(f^{-1}(\lambda^c))))) \leq \eta_i$-Cl $(f(f^{-1}(\lambda^c)))$
$\leq \eta_i$-Cl $(\lambda^c) = \lambda^c$.

So τ_j-Int $(\tau_i$-Cl $(\tau_j$-Int $f^{-1}(\lambda^c))) \leq f^{-1}(\lambda^c)$, that is, $f^{-1}(\lambda^c)$ is a (τ_i, τ_j)-*fβc* set of X. But $f^{-1}(\lambda^c) = f^{-1}(\lambda))^c$. Therefore $f^{-1}(\lambda)$ is a (τ_i, τ_j)-*fβo* set of X and consequently, f is a *fpβc* mapping. ■

Theorem 3.11 Let (X_1, τ_1, τ_2), $(X_2, \omega_1, \omega_2)$, (Y_1, η_1, η_2) and $(Y_2, \sigma_1, \sigma_2)$ be *fbts*'s such that X_1 is product related to X_2[1]. Then the product $f_1 \times f_2 : (X_1 \times X_2, \theta_1, \theta_2) \to (Y_1 \times Y_2, \rho_1, \rho_2)$, where θ_k (respectively ρ_k) is the fuzzy product topology generated by τ_k and ω_k (respectively η_k and ρ_k) $(k=1, 2)$, of *fpβc* mappings $f_1 : (X_1, \tau_1, \tau_2) \to (Y_1, \eta_1, \eta_2)$ and $f_2 : (X_2, \omega_1, \omega_2) \to (Y_2, \sigma_1, \sigma_2)$, is a *fpβc* mapping.

Proof. For convenience, we denote $\lambda = \vee_{m,n}(\mu_m \times \nu_n)$, where μ_m's are η_i-*fo* sets of Y_1 and ν_n's are σ_i-*fo* sets of Y_2. Then λ is a ρ_i-*fo* set of $Y_1 \times Y_2$. By Lemma 2.1 and 2.3 of [1], we have

$$(f_1 \times f_2)^{-1}(\lambda) = \vee_{m,n}((f_1 \times f_2)^{-1}(\mu_m \times \nu_n)) = \vee_{m,n}(f_1^{-1}(\mu_m) \times f_2^{-1}(\nu_n)).$$

Since f_1 and f_2 are *fpβc* mapping, $f_1^{-1}(\mu_m)$'s are (τ_i, τ_j)-*fβo* sets of X_1 and $f_2^{-1}(\nu_n)$'s are (ω_i, ω_j)-*fβo* sets of X_2. By Theorem 3.3 and 3.7, it follows that $(f_1 \times f_2)^{-1}(\lambda)$ is a (θ_i, θ_j)-*fβo* set. Therefore $f_1 \times f_2$ is a *fpβc* mapping. ■

Theorem 3.12 Let (X, τ_1, τ_2), $(X_1, \eta_1^{(1)}, \eta_2^{(1)})$ and $(X_2, \eta_1^{(2)}, \eta_2^{(2)})$ be *fbts*'s and $\pi_k : (X_1 \times X_2, \theta_1, \theta_2) \to (X_k, \eta_1^{(k)}, \eta_2^{(k)})$ $(k=1, 2)$ be the projections. If $f : X \to X_1 \times X_2$ is a *fpβc* mapping, then so is $\pi_k \circ f$.

Proof. For a fuzzy open set μ of X_k, we have $(\pi_k \circ f)^{-1}(\mu) = f^{-1}(\pi_k^{-1}(\mu))$. Since π_k is *fpc* and f is *fpβc*, $(\pi_k \circ f)^{-1}(\mu)$ is a (τ_i, τ_j)-*fβo* set of X. Hence $\pi_k \circ f$ is a *fpβc* mapping. ■

Theorem 3.13 Let X_1 and X_2 be *fbts*'s such that X_1 is product related to X_2 and let $f : (X_1, \tau_1, \tau_2) \to (X_2, \eta_1, \eta_2)$ be a mapping. If the graph mapping $g : (X_1, \tau_1, \tau_2) \to (X_1 \times X_2, \theta_1, \theta_2)$ of f defined by $g(x) = (x, f(x))$ is a *fpβc* mapping, then f is a *fpβc* mapping.

Proof. Let ν be a η_i-*fo* set of X_2. Then by Lemma 2.4 of [1] we have $f^{-1}(\nu) = 1 \wedge f^{-1}(\nu) = g^{-1}(1 \times \nu)$. Since g is a *fpβc* mapping and $1 \times \nu$ is a θ_i-*fo* set of $X_1 \times X_2$, $f^{-1}(\nu)$ is a (τ_i, τ_j)-*fβo* set of X_1. Hence f is a *fpβc* mapping. ■

But the converse of above theorem is not true in Example 3.14.

Example 3.14 Let μ and ν be fuzzy sets of X in Example 3.4. Consider fuzzy topologies $\tau_1 = \{0_X, \mu, 1_X\}$, $\tau_2 = \{0_X, \nu, 1_X\}$, $\eta_1 = \{0_X, \nu, 1_X\}$ and $\eta_2 = \{0_X, \mu^c, 1_X\}$ and the identity mapping $i_X : (X, \tau_1, \tau_2) \to (X, \eta_1, \eta_2)$. Then i_X is a *fpβc* mapping, but its graph mapping g is not a *fpβc* mapping. Now, $\mu \times \nu$ is a (θ_i, θ_j)-*fo* set of the fuzzy product space $(X\times X, \theta_1, \theta_2)$, where θ_k is the fuzzy product topology generated by τ_k and η_k ($k=1, 2$). But $g^{-1}(\mu\times\nu)=\mu\wedge i_X^{-1}(\nu)=\mu\wedge\nu$ is not a (τ_i, τ_j)-$f\beta o$ set of X. ■

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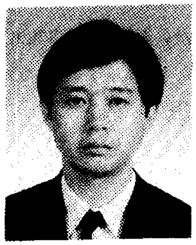

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