# The Comparative Power Evaluation of Parametric Versus Nonparametric Methods<sup>1)</sup>

# Young Hun Choi<sup>2)</sup>

## **Abstract**

The simulation study shows that the rank transform test has relatively superior power advantages over the parametric analysis of variance test in many cases for a  $2^3$  factorial design, particularly with heavy-tailed distributions of the error terms. However the rank transform test should be cautiously used when all main effects and interactions related to a testing effect are possibly present at the same time.

#### 1. Introduction

One advantage of the rank transform method is to allow existing parametric test based on data replaced with their corresponding ranks. In short the usual parametric procedure is performed on the ranks. The potential of the degree to which the rank transform procedure is being used in the analysis of experimental designs seems great.

Although many simulation papers have shown that the rank transform approach is fairly proper for many given circumstances[Conover and Iman (1976), Pirie and Rauch (1984), Pavur and Nath (1986), Choi (1995)], it seems to be controversial under some situations[Blair, Sawilowsky and Higgins (1987)]. Further until recently, the rank transform procedure has not provided useful solutions for problems involving more complicated linear models, particularly factorial designs with high-order interactions. Due to complexity of theoretical development for the rank transform method[Hora and Conover (1984)], the simulation study as well as the theory for tests based on ranks over a three-way layout with interactions has not concretely been considered.

Thus statistical examination will be thoroughly made to investigate the power difference between the usual analysis of variance F test and a rank transform test(FR) by using computer generated simulation technique. Namely the primary concern of this paper is a Monte Carlo simulation study comparing the power of the tests in a three-way layout, especially in a  $2^3$  factorial experiment.

<sup>1)</sup> This paper was partially supported by Hanshin University Research Fund, 1995.

<sup>2)</sup> Associate Professor, Department of Statistics, Hanshin University, 447-791, Korea.

## 2. Description of the Procedures

Computer generated Monte Carlo methods are employed to compare the power properties of the rank transformed ANOVA test to those of the normal parametric test for main effects and interaction effects in the context of a balanced  $2^3$  fixed effects factorial design. For simplicity the model chosen for data generation is a three factor linear model as follows:

$$X_{ijkn} = \mu + \alpha_i + \beta_j + \gamma_k + \alpha\beta_{ij} + \alpha\gamma_{ik} + \beta\gamma_{jk} + \alpha\beta\gamma_{ijk} + e_{ijkn},$$
for  $i, j, k = 1, 2, n = 1, 2, \dots, N$ .

where  $\mu$ ,  $a_i$ ,  $\beta_j$ ,  $\gamma_k$ ,  $a\beta_{ij}$ ,  $a\gamma_{ik}$ ,  $\beta\gamma_{jk}$ ,  $a\beta\gamma_{ijk}$  are the overall mean of zero, main effect of the ith level of factor A, main effect of the jth level of factor B, main effect of the kth level of factor C, interaction effect by ith level of factor A and jth level of factor B, interaction effect by ith level of factor C and interaction effect by ith level of factor A, jth level of factor B and kth level of factor C respectively. The error terms,  $e_{ijkn}$ , are assumed to be independently and identically distributed with continuous distribution functions. Four underlying distributions such as standard normal, exponential, double exponential and uniform distributions are chosen for the error terms. This process is carried out for N=2 and N=10 observations per cell.

In this article to test for the null hypothesis of no B main effects  $H_0: \beta_j = 0$  for all j and the null hypothesis of no AB interaction effects  $H_0: \alpha\beta_{ij} = 0$  for all i and j, the symbols F and FR will denote the usual ANOVA test statistic and F statistic on the ranks of the data respectively. The A, B and C main effects are formed by setting  $\alpha_1 = c$ ,  $\alpha_2 = -c$ ,  $\beta_1 = c$ ,  $\beta_2 = -c$  and  $\gamma_1 = c$ ,  $\gamma_2 = -c$ , respectively, where c representing the effect size takes the values from 0.25 to 1.00 by 0.25. Meanwhile AB interactions are created by setting  $\alpha\beta_{11} = \alpha\beta_{22} = c$  and  $\alpha\beta_{12} = \alpha\beta_{21} = -c$ . AC and BC interactions are formed in a similar manner. Further ABC interactions can be created by setting  $\alpha\beta\gamma_{111} = \alpha\beta\gamma_{122} = \alpha\beta\gamma_{212} = \alpha\beta\gamma_{221} = c$  and  $\alpha\beta\gamma_{112} = \alpha\beta\gamma_{121} = \alpha\beta\gamma_{211} = \alpha\beta\gamma_{222} = -c$  with c taking on the values just mentioned above. However we regard ABC interactions, which are generally known as complicated to interpret, as negligible or meaningless effects having value of 0 in order to clarify the simulation results.

The first step of methodology adopted for this study is to generate pseudo random deviates using C program, then yield the given main and interaction effects by adding or subtracting constants corresponding to the effect size. Next the classical ANOVA F statistic is computed. After comparing the critical values with respect to the significance level of 0.05, we calculate the proportion of rejections under the given effets for the power. In addition the procedure is repeated after replacing original observations with their respective ranks. For each case

10,000 repetitions are accomplished in the experiment.

For standard normal random variates we use Box and Muller (1958)'s transformation. For exponential random variates we generate uniform random variates and use the inverse transform method to convert to exponential random variates[Marsaglia (1961)]. exponential random variates we generate exponential random variates and use the composition method to yield double exponential random variates[Law and Kelton (1991)]. random variates we use the linear congruential method directly.

### Simulation Results

The simulation study is conducted over a variety of experimental situations. We wish to investigate the effects of allowing the magnitude of the nuisance parameters to vary. We also consider several fashions in which main effects and interaction terms are constructed.

The results reported in [Table 1] through [Table 4] are representative of a variety of configurations considered. The power of various occasions including certain specific cases is presented. In these tables the second column labeled "c" indicates the value of constant employed in generating the given effects. The "Statistic" column exhibits which of the two statistical tests is being considered; F represents the usual ANOVA statistic and FR represents the rank transform statistic.

[Table 1] and [Table 2] examine the power of tests for main effects(i.e. B effect for our case) when sampling is from n=2 and n=10 observations per cell respectively. For the power analysis, the following nine different situations are considered;

① 
$$\beta = c$$
,  $\alpha = \gamma = \alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0$ , ②  $\alpha = \beta = c$ ,  $\gamma = \alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0$ , ③  $\alpha = \beta = \gamma = c$ ,  $\alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0$ , ③  $\alpha = \beta = c$ ,  $\alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0$ , ③  $\alpha = 1.5 c$ ,  $\beta = c$ ,  $\gamma = 0.5 c$ ,  $\alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0$ , ④  $\beta = \alpha\beta = c$ ,  $\alpha = \gamma = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0$ , ⑤  $\alpha = \beta = \gamma = \alpha\gamma = c$ ,  $\alpha\beta = \beta\gamma = \alpha\beta\gamma = 0$ , ⑤  $\alpha = \beta = \gamma = \alpha\beta = c$ ,  $\alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0$ , ⑥  $\alpha = \beta = \gamma = \alpha\beta = c$ ,  $\alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0$ , ⑥  $\alpha = \beta = \gamma = \alpha\beta\gamma = 0$ .

The results of [Table 1] show that in all cases the power of the parametric F statistic remains the same regardless of the manner in which effects are constructed. The results also indicate that the power of the rank transformed FR statistic is superior in certain respects to the parametric F statistic. First of all in the absence of interactions(cases ①,②, ③<sup>1</sup>, ③<sup>2</sup>) the rank transform test appears to have greater or moderate power than the parametric test without regard to the manner in which main effects are constructed. presence of interactions if the number of main effects and interactions is small(case 4), the power of rank transform test is greater or moderate than that of the parametric test. Further if three main effects and interactions without a testing effect are simultaneously present(case (5), the rank transform test still maintains the power advantage. However if three main effects and interactions including a testing effect are simultaneously present (cases \$\sigma^2\$, \$\sigma^1\$,

(6)<sup>2</sup>), the rank transform test seems to lose the power.

In general increases in the number of effects are associated with decreases in the power of the rank transform test. Besides increases in the value of effect size and sample size show manifest increases in the power of both the normal theory(parametric) test and the rank transform test. Note that in the nonnormal populations such as exponential and double exponential ones, the rank transform test indicates pronounced power advantage over the normal theory test, especially for the small effect size. Note also that as compared with other distributions, the power of light-tailed distributions like the uniform distribution increases rapidly.

The results of [Table 2] agree with those seen in [Table 1]. Furthermore it is interesting to note that in all instances the rank transform test produces reasonable power nearly equal to the normal theory test. The performance of the rank transform test is remarkably improved with increse in sample size.

[Table 1]. Power of tests for B main effect when sampling is from n=2 and for .

```
① all cases of F

① \beta = c, \alpha = \gamma = \alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

② \alpha = \beta = c, \gamma = \alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

③ \alpha = \beta = \gamma = c, \alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

③ \alpha = 1.5 c, \beta = c, \gamma = 0.5 c, \alpha\beta = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

④ \beta = \alpha\beta = c, \alpha = \gamma = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

⑤ \alpha = \beta = \gamma = \alpha\gamma = c, \alpha\beta = \beta\gamma = \alpha\beta\gamma = 0

⑤ \alpha = \beta = \gamma = \alpha\beta = c, \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

⑥ \alpha = \beta = \gamma = \alpha\beta = \alpha\gamma = c, \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

⑥ \alpha = \beta = \gamma = \alpha\beta = \alpha\gamma = c, \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

⑥ \alpha = 1.5 c, \beta = c, \gamma = 0.5 c, \alpha\beta = 0.5 c, \alpha\gamma = 1.5 c, \beta\gamma = \alpha\beta\gamma = 0
```

		Statistic											
Population	c	<b>0</b> F	①FR	②FR	31FR	3°FR	4 FR	51FR	⑤ <sup>2</sup> FR	€¹FR	6°FR		
Normal	0.25	0.145	0.145	0.143	0.142	0.142	0.141	0.143	0.142	0.134	0.137		
	0.50	0.426	0.420	0.405	0.399	0.392	0.406	0.402	0.369	0.305	0.356		
	0.75	0.756	0.742	0.717	0.707	0.706	0.716	0.703	0.630	0.481	0.618		
	1.00	0.942	0.935	0.914	0.903	0.911	0.914	0.901	0.824	0.643	0.822		
Exponential	0.25	0.177	0.256	0.234	0.224	0.215	0.233	0.225	0.194	0.179	0.193		
	0.50	0.498	0.597	0.550	0.526	0.523	0.553	0.528	0.451	0.369	0.455		
	0.75	0.776	0.822	0.785	0.758	0.771	0.785	0.762	0.684	0.550	0.684		
	1.00	0.917	0.927	0.907	0.889	0.908	0.906	0.888	0.824	0.680	0.836		
Double	0.25	0.106	0.126	0.121	0.121	0.117	0.119	0.116	0.118	0.111	0.111		
Exponential	0.50	0.279	0.323	0.304	0.294	0.288	0.304	0.299	0.272	0.233	0.265		
-	0.75	0.514	0.563	0.527	0.511	0.506	0.534	0.516	0.464	0.372	0.456		
	1.00	0.727	0.762	0.727	0.709	0.709	0.730	0.708	0.637	0.496	0.632		

Uniform	0.25	0.869	0.821	0.803	0.789	0.798	0.803	0.781	0.702	0.528	0.701
	0.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.938	0.999
	0.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.938	1.000
	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.938	1.000

[Table 2]. Power of tests for B main effect when sampling is from n=10 and for the same cases as [Table 1].

						Stat	istic				
Population	c	<b>@</b> F	①FR	②FR	31FR			⑤¹FR	⑤ <sup>2</sup> FR	<b>⑥¹FR</b>	© <sup>2</sup> FR
Normal	0.25	0.602	0.581	0.579	0.580	0.579	0.581	0.578	0.576	0.547	0.565
	0.50	0.992	0.990	0.989	0.988	0.988	0.990	0.987	0.985	0.962	0.979
	0.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.989	1.000
	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Exponential	0.25	0.619	0.876	0.839	0.815	0.801	0.845	0.810	0.780	0.720	0.744
	0.50	0.988	1.000	1.000	0.998	0.998	0.999	0.998	0.997	0.983	0.993
	0.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000
	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Double	0.25	0.364	0.480	0.469	0.462	0.454	0.468	0.455	0.449	0.422	0.433
Exponential		0.880	0.953	0.937	0.928	0.923	0.940	0.925	0.912	0.863	0.891
23.12	0.75	0.994	0.999	0.998	0.997	0.998	0.998	3 0.997	0.996	0.983	0.993
	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.998	1.000
Uniform	0.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
001111	0.50	1.000	1.000				1.000	1.000	1.000	1.000	1.000
	0.75	1.000	1.000				1.000	1.000	1.000	1.000	1.000
	1.00	1.000	1.000							1.000	1.000

Meanwhile [Table 3] and [Table 4] report the power of tests for interaction effects(i.e. AB effect for our case) when sampling is from n=2 and n=10 observations per cell respectively. Likewise the following nine different situations are considered for the power analysis;

```
\beta \gamma = \alpha \beta \gamma = 0, 6^2 \alpha = c, \beta = 0.5c, \gamma = 1.5c, \alpha \beta = c, \alpha \gamma = 0.5c, \beta \gamma = \alpha \beta \gamma = 0.
```

The results of [Table 3] and [Table 4] show that when there are only interaction effects

and there exist interaction effects with one main effect(cases ①,②,③), the power of the rank transform test is excellent or very similar with that of the normal theory test. Specifically it is shown that the rank transform procedure is superior to the parametric procedure under the heavy-tailed distributions like the exponential and double exponential distributions of the error terms.

On the other hand when there exist interactions with two or three main effects related to a testing effect simultaneously(cases  $\textcircled{4}^1 \sim \textcircled{6}^2$ ), the power of the rank transform test tends to be decreased somewhat(especially for the cases of  $\textcircled{4}^2$  and  $\textcircled{6}^1$ ). [A side comment seems appropriate here. If there exist interactions with main effects unrelated to a testing effect simultaneously, the rank transform test obviously maintains much of its substantial power even though the results are not presented here.] This result closely agrees with that of Thompson (1991)'s paper which may be applicable to a three-way layout. However note that when the distribution of the error terms is heavy-tailed with small effect size or large sample size, the power of the rank transform test is still prominent over the parametric test.

In overall these results of testing for interaction effects quite agree with those of testing for main effects mentioned above. That is, the power of testing for main effects and interaction effects behaves in a similar fashion for most instances.

[Table 3]. Power of tests for AB interaction effects when sampling is from n=2 and for m=1 all cases of m=1

```
① \alpha\beta = c, \alpha = \beta = \gamma = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

② \alpha\beta = \alpha\gamma = c, \alpha = \beta = \gamma = \beta\gamma = \alpha\beta\gamma = 0

③ \beta = \alpha\beta = c, \alpha = \gamma = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

④¹ \beta = \gamma = \alpha\beta = c, \alpha = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

④² \alpha = \beta = \alpha\beta = c, \gamma = \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

⑤¹ \alpha = \beta = \gamma = \alpha\beta = c, \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

⑤² \alpha = c, \beta = 0.5c, \gamma = 1.5c, \alpha\beta = c, \alpha\gamma = \beta\gamma = \alpha\beta\gamma = 0

⑥¹ \alpha = \beta = \gamma = \alpha\beta = \alpha\gamma = c, \beta\gamma = \alpha\beta\gamma = 0

⑥² \alpha = c, \beta = 0.5c, \gamma = 1.5c, \alpha\beta = c, \alpha\gamma = 0.5c, \beta\gamma = \alpha\beta\gamma = 0
```

		Statistic										
Population	с	<b>0</b> F	①FR	②FR	③FR	<b>⊕</b> ¹FR	<b>⊕</b> <sup>2</sup> FR	⑤¹FR	⑤ <sup>2</sup> FR	€¹FR	6°FR	
Normal	0.25	0.148	0.149	0.150	0.151	0.149	0.146	0.145	0.144	0.134	0.140	
	0.50	0.427	0.419	0.409	0.409	0.399	0.351	0.375	0.391	0.304	0.365	
	0.75	0.752	0.739	0.714	0.714	0.704	0.504	0.629	0.675	0.484	0.636	
	1.00	0.938	0.930	0.910	0.914	0.898	0.550	0.817	0.862	0.646	0.830	
Exponential	0.25	0.177	0.254	0.232	0.234	0.223	0.185	0.192	0.205	0.180	0.201	
	0.50	0.495	0.595	0.545	0.547	0.519	0.361	0.451	0.499	0.374	0.469	
	0.75	0.773	0.823	0.784	0.782	0.761	0.471	0.679	0.730	0.546	0.702	
	1.00	0.917	0.931	0.912	0.911	0.895	0.523	0.832	0.872	0.675	0.847	

Double Exponential	0.25 0.50 0.75 1.00	0.106 0.274 0.503 0.724	0.313 0.552	0.300 0.520	0.125 0.296 0.527 0.721	0.290 0.509	0.252 0.385	0.268 0.457	0.281 0.484	0.228 0.364	0.118 0.266 0.455 0.632
Uniform	0.25 0.50 0.75 1.00	0.866 1.000 1.000 1.000	1.000 1.000	1.000			0.554 0.554	1.000 1.000		0.522 0.937 0.937 0.937	0.723 0.992 1.000 1.000

[Table 4]. Power of tests for AB interaction effects when sampling is from n=10and for the same cases as [Table 3].

						Stat	tistic				
Population	c	<b>0</b> F	①FR	②FR	3FR	<b>⊕</b> ¹FR	4ºFR	⑤¹FR	⑤²FR	€¹FR	© <sup>2</sup> FR
Normal	0.25	0.602	0.581	0.580	0.583	0.582	0.577	0.575	0.576	0.547	0.565
	0.50	0.994	0.991	0.991	0.991	0.990	0.988	0.989	0.990	0.965	0.984
	0.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	1.000
	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Exponential	0.25	0.619	0.880	0.842	0.843	0.817	0.778	0.782	0.790	0.721	0.766
_	0.50	0.988	1.000	0.999	0.999	0.998	0.992	0.995	0.996	0.982	0.994
	0.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Double	0.25	0.364	0.479	0.464	0.467	0.458	0.455	0.447	0.446	0.418	0.435
Exponential	0.50	0.878	0.956	0.942	0.942	0.932	0.912	0.914	0.917	0.866	0.902
_	0.75	0.995	0.999	0.999	0.999	0.998	0.992	0.996	0.997	0.984	0.996
	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Uniform	0.25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	0.75	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.00	1.000	1.000		1.000	1.000	1.000	1.000	1.000	1.000	1.000

## 4. Conclusions

The results of this simulation study exhibit that the rank transform test in a  $2^3$  factorial design has considerable power advantages over the parametric test in many cases. Generally if the effect size or the number of effects is small, the sample size is large and the distribution of error terms is heavy-tailed, the rank transform procedure appears to maintain much of its substantial power and is more desirable. Further when the distribution of the error terms is heavy tailed with small effect size or large sample size, the power of the rank transform test is superior to that of the parametric test.

However the rank transform test should be especially approached with caution when three main effects and interactions related to a testing effect exist at the same time.

## References

- [1] Blair, R. C., Sawilowsky, S. S. and Higgins, J. J. (1987). Limitations of the Rank Transform Statistic in Tests for Interactions, *Communications in Statistics, Part B, Simulation and Computation*, Vol. 16, 1133-1145.
- [2] Box, G. E. P. and Muller, M. E. (1958). A Note on the Generation of Random Normal Deviates, *The Annals of Mathematical Statistics*, Vol. 29, 610-611.
- [3] Choi, Y. H. (1995). Simulation Analysis of Type I Error and Power for F Test and Rank Transformed F Test in 2<sup>2</sup> Factorial ANOVA, *The Korean Journal of Applied Statistics*, Vol. 8, No. 2, 87-97.
- [4] Conover, W. J. and Iman, R. L. (1976). On Some Alternative Procedure Using Ranks for the Analysis of Experimental Designs, *Communications in Statistics*, A5, 1349-1368.
- [5] Hora, S. C. and Conover, W. J. (1984). The F Statistic in the Two-Way Layout with Rank Score Transformed Data, Journal of the American Statistical Association, Vol. 79, 668-673.
- [6] Law, A. M. and Kelton, W. D. (1991). Simulation Modeling and Analysis, McGraw-Hill, Inc..
- [7] Marsaglia, G. (1961). Generating Exponential Random Variables, *Annals of Mathematical Statistics*, Vol. 32, 899–902.
- [8] Pavur, R. and Nath, R. (1986). Parametric Versus Rank Transform Procedures in the Two-Way Factorial Experiment: A Comparative Study, *Journal of Statistical Computation Simulation*, Vol. 23, 231-240.
- [9] Pirie, W. R. and Rauch, H. L. (1984). Simulated Efficiencies of Tests and Estimators from General Linear Models Analysis based on Ranks: The Two-Way Layout with Interaction, Journal of Statistical Computation Simulation, Vol. 20, 197-204.
- [10] Thompson, G. L. (1991). A Note on the Rank Transform for Interactions, Biometrika, Vol. 78, 697-701.