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Estimation of P(X<Y) in Marshall and Olkin's Model under Random Censorship

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Abstract

In this paper, we derive the maximum likelihood estimator of $P=P(X<Y)$ and its asymptotic distributions in the Marshall and Olkin's bivariate exponential model with random censored data. Also we compare the bootstrap confidence intervals with asymptotic confidence intervals in view of coverage probabilities.

1. Introduction

Let's consider the stress-strength model in the two component system. In many cases of life testing and reliability analysis, it is realistic to assume that some forms of dependence among the components arise from components depending on common sources of power. As examples, we consider the paired organs like kidneys, eyes, ears or any other paired organs in an individual as two component system. In these cases, each paired organ is correlated each other. Marshall and Olkin(1967) formulated a bivariate extension of the exponential distribution as a model for a system where the failure times of the two components may depend on each other. The bivariate exponential distribution occupies an important place among bivariate life distributions. It has the bivariate loss of memory property and its marginals have the loss of memory property. Also this model is applicable as a failure model for the system where there exists positive probability of simultaneous failure of exponential components.

Arnold(1968) derived unbiased estimators for parameters of Marshall and Olkin's model and their asymptotic properties. Bemis, Bain and Higgins(1972) derived moment type extimators for parameters of Marshall Olkin's model. Awad, Azzam and Hamdon(1981) derived some estimators for P. Kim and Park(1990) obtained the Bayes estimators of parameters and Bayes estimator of P. But estimators and confidence intervals for P under random censored data

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are not derived until now.

Our objective in this paper is to obtain the maximum likelihood estimator(MLE) of P and asymptotic distributions of the MLE. Also we construct some approximate confidence intervals of P under random censored data.

In Section 2, we introduce the notations and preliminaries. In Section 3, we derive the MLE of P under random censored data and asymptotic distributions of the MLE. Also we construct the asymptotic normal confidence interval and the bootstrap confidence intervals. In Section 4, the actual coverage probabilities of these intervals are examined by Monte Carlo simulation and recommendations are made for their use.

2. Notations and Preliminaries

Marshall and Olkin(1967) formulated a bivariate extension of the exponential distribution as a model for a system where the life times of the two components may depend on each other. The random variables (X, Y) are said to follow BVED $(\lambda_1, \lambda_2, \lambda_3)$ if the joint reliability function is given as

$$\begin{aligned} \overline{F}(x, y: \underline{\lambda}) &= P(X > x \ , \ Y > y) \\ &= \exp\{-\lambda_1 x - \lambda_2 y - \lambda_3 (x \vee y)\}, \quad x, y \geq 0, \end{aligned} \tag{2.1}$$

where $\lambda_1, \ \lambda_2, \ \lambda_3 > 0$, $\underline{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ and $x \vee y = \max(x, y)$. It follows that the density function is given as

$$f(x, y : \underline{\lambda}) = \begin{cases} \lambda_1(\lambda_2 + \lambda_3)\overline{F}(x, y : \underline{\lambda}), & y > x > 0, \\ \lambda_2(\lambda_1 + \lambda_3)\overline{F}(x, y : \underline{\lambda}), & x > y > 0, \\ \lambda_3 \overline{F}(x, y : \underline{\lambda}), & x = y > 0. \end{cases} \tag{2.2}$$

Let (X_i, Y_i), $i = 1, 2, \cdots, n$ be $i.i.d.$ pairs of random variables with (2.1), and let T_i, $i = 1, 2, \cdots, n$, be $i.i.d.$ random variables with survival function $\overline{G}(t: \theta) = P_\theta(T_i \geq t)$ with density $g(t : \theta)$. Suppose that the two sequences $\{(X_i, Y_i)\}_{i=1}^n$ and $\{T_i\}_{i=1}^n$ are independent.

In the random censoring model, (X, Y) may be censored on the right by the censoring variable T, so that only the random vectors $(X_i^0, Y_i^0, \gamma_i, \rho_i, \delta_i^x, \delta_i^y)$, $i = 1, 2, \cdots, n$ are observed, where $X_i^0 = (X_i \wedge T_i)$, $\delta_i^x = I(X_i \leq T_i)$, $Y_i^0 = (Y_i \wedge T_i)$, $\delta_i^y = I(Y_i \leq T_i)$, $\gamma_i = I(X_i < Y_i)$, $\rho_i = I((X_i = Y_i) > 0)$ and $x \wedge y = \min(x, y)$.

Now, the likelihood function of the sample size n from above is given as

$$L(\underline{X}^0, \underline{Y}^0 : \lambda, \theta)$$
$$= \prod_{i=1}^{n}\Big(\{f(x_i, y_i; \lambda)\overline{G}(t_i; \theta)\}^{\gamma_i\delta_i^x\delta_i^y} \cdot \{f(x_i, y_i; \lambda)\overline{G}(t_i; \theta)\}^{(1-\gamma_i)\delta_i^x\delta_i^y} \cdot$$
$$\{f(x_i, y_i; \lambda)\overline{G}(t_i; \theta)\}^{\rho_i\delta_i^x\delta_i^y} \cdot \{\overline{F}_x(x_i, t_i; \lambda)g(t_i; \theta)\}^{\delta_i^x(1-\delta_i^y)} \cdot$$
$$\{\overline{F}_y(t_i, y_i; \lambda)g(t_i; \theta)\}^{(1-\delta_i^x)\delta_i^y} \cdot \{\overline{F}(t_i, t_i; \lambda)g(t_i; \theta)\}^{(1-\delta_i^x)(1-\delta_i^y)}\Big)$$
$$= \lambda_1^{|D_1|+|D_4|} \cdot \lambda_2^{|D_2|+|D_5|} \cdot \lambda_3^{|D_3|} \cdot (\lambda_1+\lambda_3)^{|D_2|} \cdot (\lambda_2+\lambda_3)^{|D_1|} \cdot$$
$$\prod_{i\in\{D_1\cup D_2\cup D_3\}} \overline{G}(t_i; \theta) \cdot \prod_{i\in\{D_4\cup D_5\cup D_6\}} g(t_i; \theta) \cdot \exp\{-\lambda_1\sum_{i=1}^{n} x_i^0 - \lambda_2\sum_{i=1}^{n} y_i^0 - \lambda_3\sum_{i=1}^{n}(x_i^0\vee y_i^0)\}, \tag{2.3}$$

where $\sum_{i=1}^{n} x_i^0 = \sum_{i\in A} x_i + \sum_{i\in A^c} t_i$, $A = \{x_i^0 \mid \delta_i^x = 1\}$, $\sum_{i=1}^{n} y_i^0 = \sum_{i\in B} y_i + \sum_{i\in B^c} t_i$,

$B = \{y_i^0 \mid \delta_i^y = 1\}$, $\sum_{i=1}^{n}(x_i^0\vee y_i^0) = \sum_{i\in C}(x_i\vee y_i) + \sum_{i\in C^c} t_i$, $C = \{(x_i^0, y_i^0) \mid \delta_i^x\delta_i^y = 1\}$,

$\sum_{i=1}^{n}(x_i^0\wedge y_i^0) = \sum_{i\in D}(x_i\wedge y_i) + \sum_{i\in D^c} t_i$, $D = \{(x_i^0, y_i^0) \mid (1-\delta_i^x)(1-\delta_i^y) = 0\}$,

$$\overline{F}_x(x_i, t_i) = \lim_{dx_i\to 0}\frac{P(x_i < X_i < x_i + dx_i,\ Y_i > t_i)}{dx_i} = \lambda_1\exp\{-\lambda_1 x_i - (\lambda_2+\lambda_3)t_i\},$$

$$\overline{F}_y(t_i, y_i) = \lim_{dy_i\to 0}\frac{P(y_i < Y_i < y_i + dy_i,\ X_i > t_i)}{dy_i} = \lambda_2\exp\{-\lambda_2 y_i - (\lambda_1+\lambda_3)t_i\},$$

$D_1 = \{i \mid \gamma_i\delta_i^x\delta_i^y = 1,\ i = 1,2,\cdots,n\}$, $D_2 = \{i \mid (1-\gamma_i)\delta_i^x\delta_i^y = 1,\ i = 1,2,\cdots,n\}$,

$D_3 = \{i \mid \rho_i\delta_i^x\delta_i^y = 1,\ i = 1,2,\cdots,n\}$, $D_4 = \{i \mid \delta_i^x(1-\delta_i^y) = 1,\ i = 1,2,\cdots,n\}$,

$D_5 = \{i \mid (1-\delta_i^x)\delta_i^y = 1,\ i = 1,2,\cdots,n\}$, $D_6 = \{i \mid (1-\delta_i^x)(1-\delta_i^y) = 1,\ i = 1,2,\cdots,n\}$,

and $|D_i|$ is the number of elements in the set D_i, $i = 1,2,\cdots,6$.

After a little algebra, the log-likelihood function is given by

$$\log L(\underline{X}^0, \underline{Y}^0 : \lambda, \theta)$$
$$= (|D_1| + |D_4|)\log\lambda_1 + (|D_2| + |D_5|)\log\lambda_2 + |D_3|\log\lambda_3$$
$$+ |D_2|\log(\lambda_1+\lambda_3) + |D_1|\log(\lambda_2+\lambda_3)$$

$$+\sum_{i\in\{D_1\cup D_2\cup D_3\}}\log\overline{G}(t_i:\theta)+\sum_{i\in\{D_4\cup D_5\cup D_6\}}\log g(t_i:\theta)$$

$$-\lambda_1\sum_{i=1}^{n}x_i-\lambda_2\sum_{i=1}^{n}y_i-\lambda_3\sum_{i=1}^{n}(x_i\vee y_i).$$

Hence, the log-likelihood equations are given as

$$\frac{\partial\log L(\underline{X}^0,\ \underline{Y}^0:\underline{\lambda},\theta)}{\partial\lambda_1}=\frac{|D_1|+|D_4|}{\lambda_1}+\frac{|D_2|}{\lambda_1+\lambda_3}-\sum_{i=1}^{n}x_i^0=0 \tag{2.4}$$

$$\frac{\partial\log L(\underline{X}^0,\ \underline{Y}^0:\underline{\lambda},\theta)}{\partial\lambda_2}=\frac{|D_2|+|D_5|}{\lambda_2}+\frac{|D_1|}{\lambda_2+\lambda_3}-\sum_{i=1}^{n}y_i^0=0 \tag{2.5}$$

$$\frac{\partial\log L(\underline{X}^0,\ \underline{Y}^0:\underline{\lambda},\theta)}{\partial\lambda_3}=\frac{|D_3|}{\lambda_3}+\frac{|D_1|}{\lambda_2+\lambda_3}+\frac{|D_2|}{\lambda_1+\lambda_3}-\sum_{i=1}^{n}(x_i^0\vee y_i^0)=0 \tag{2.6}$$

$$\frac{\partial\log L(\underline{X}^0,\ \underline{Y}^0:\underline{\lambda},\theta)}{\partial\theta}=\sum_{i\in\{D_1\cup D_2\cup D_3\}}\frac{\frac{\partial}{\partial\theta}\overline{G}(t_i:\theta)}{\overline{G}(t_i:\theta)}+\sum_{i\in\{D_4\cup D_5\cup D_6\}}\frac{\frac{\partial}{\partial\theta}g(t_i:\theta)}{g(t_i:\theta)}. \tag{2.7}$$

MLE's $(\hat{\lambda}_1,\hat{\lambda}_2,\hat{\lambda}_3,\hat{\theta})$ of $(\lambda_1,\lambda_2,\lambda_3,\theta)$ are obtained by iterative procedure. From Hanagal and Kale(1992), $\sqrt{n}(\hat{\underline{\lambda}}-\underline{\lambda})$ has the asymptotic trivariate normal distribution with mean vector zero and covariance matrix $\Sigma=((I^{ij}),\ i,j=1,2,3)$ where I^{ij} is the (i,j)th element of inverse matrix of Fisher information $(I=(I_{ij}),i,j=1,2,3)$ with

$$I_{11}=\frac{E(|D_1|)+E(|D_4|)}{\lambda_1^2}+\frac{E(|D_2|)}{(\lambda_1+\lambda_3)^2},\ I_{12}=0,\ I_{13}=\frac{E(|D_2|)}{(\lambda_1+\lambda_3)^2},$$

$$I_{22}=\frac{E(|D_2|)+E(|D_5|)}{\lambda_2^2}+\frac{E(|D_1|)}{(\lambda_2+\lambda_3)^2},\ I_{23}=\frac{E(|D_1|)}{(\lambda_2+\lambda_3)^2},$$

$$I_{33}=\frac{E(|D_1|)}{(\lambda_2+\lambda_3)^2}+\frac{E(|D_2|)}{(\lambda_1+\lambda_3)^2}+\frac{E(|D_1|)}{\lambda_3^2}.$$

3. Asymptotic Properties of Proposed Estimator

If (X,Y) have BVED $(\lambda_1,\lambda_2,\lambda_3)$, then P is represented as follows :

$$P=P(X<Y)=\int\int_{x<y}dF(x,y:\underline{\lambda})=\lambda_1/(\lambda_1+\lambda_2+\lambda_3). \tag{3.1}$$

Applying the invariance properties of MLE, we propose the MLE of P with censored data as follows :

$$\hat{P}=\hat{\lambda}_1/(\hat{\lambda}_1+\hat{\lambda}_2+\hat{\lambda}_3). \tag{3.2}$$

From the result of Hanagal and Kale(1992), we get the following asymptotic property of the proposed estimator $\hat{P}$.

Theorem. Suppose that $\hat{\lambda}_i$, $i=1,2,3$ is MLE's of λ_i in BVED $(\lambda_1,\lambda_2,\lambda_3)$. Then as $n \rightarrow \infty$,

$$\sqrt{n}(\hat{P}-P) \rightarrow^d N(0,\ U\Sigma U^T), \tag{3.3}$$

where $U=\frac{\partial}{\partial \hat{\underline{\lambda}}}\hat{P}\mid_{\hat{\underline{\lambda}}=\underline{\lambda}}=\left(\frac{\lambda-\lambda_1}{\lambda^2},\frac{-\lambda_1}{\lambda^2},\frac{-\lambda_1}{\lambda^2}\right)$, $\Sigma=((I^{ij}),\ i,j=1,2,3)$ and $\rightarrow^d$ denotes the convergence in distribution.

Proof. Since $\hat{\underline{\lambda}}$ has the asymptotic trivariate normal distribution with mean vector $\underline{\lambda}$ and covariance matrix Σ/n, $\hat{P}$ has also the asymptotic normal distribution with mean P and variance $\left(\frac{\partial}{\partial \hat{\underline{\lambda}}}\hat{P}\mid_{\hat{\underline{\lambda}}=\underline{\lambda}}\right)\cdot\frac{\Sigma}{n}\cdot\left(\frac{\partial}{\partial \hat{\underline{\lambda}}}\hat{P}\mid_{\hat{\underline{\lambda}}=\underline{\lambda}}\right)^T$ by applying delta method. Hence the proof is completed.

Using the result of Theorem 1, we can construct $100(1-2\alpha)\%$ confidence interval of P as follows:

$$\left(\hat{P}+z^{(\alpha)}\cdot\sqrt{\hat{U}\cdot\hat{\Sigma}\cdot\ \hat{U}^T/n}\ ,\ \hat{P}+z^{(1-\alpha)}\cdot\sqrt{\hat{U}\cdot\hat{\Sigma}\cdot\ \hat{U}^T/n}\ \right), \tag{3.4}$$

where $\hat{U}$ and $\hat{\Sigma}$ are computed by replacing $\hat{\underline{\lambda}}$ with $\underline{\lambda}$ in U and Σ and $z^{(\alpha)}$ is 100αth percentile of standard normal distribution.

As an alternative method, approximating distributions of $\hat{P}$ can be constructed by the bootstrap method. The bootstrap procedure can be described as follows:

(1) Construct the sample distribution functions for failure time and censoring time based on the MLE's respectively, say, BVED ($\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\lambda}_3$) and $G(t_i;\hat{\theta})$.

(2) Generate B random samples of size m from fixed BVED ($\hat{\lambda}_1$, $\hat{\lambda}_2$, $\hat{\lambda}_3$) and $G(t_i;\hat{\theta})$ respectively. The corresponding samples called the *bootstrap samples* are denoted by $((x_1^{*b},y_1^{*b}),\ (x_2^{*b},y_2^{*b}),\ \cdots,\ (x_m^{*b},y_m^{*b}))$ and $(t_1^{*b},t_2^{*b},\cdots,t_m^{*b})$, respectively, $b=1,\ 2,\ \cdots,\ B$.

(3) Construct the likelihood equations (2.4)-(2.6) based on bootstrap samples, that is, for

$b=1,2,\cdots,B,$

$$\frac{|D_1^{*b}|+|D_4^{*b}|}{\widehat{\lambda_1}}+\frac{|D_2^{*b}|}{\widehat{\lambda_1}+\widehat{\lambda_3}}-\sum_{i=1}^{m}x_i^{0*b}=0 \tag{3.5}$$

$$\frac{|D_2^{*b}|+|D_5^{*b}|}{\widehat{\lambda_2}}+\frac{|D_1^{*b}|}{\widehat{\lambda_2}+\widehat{\lambda_3}}-\sum_{i=1}^{m}y_i^{0*b}=0 \tag{3.6}$$

$$\frac{|D_3^{*b}|}{\widehat{\lambda_3}}+\frac{|D_1^{*b}|}{\widehat{\lambda_2}+\widehat{\lambda_3}}+\frac{|D_2^{*b}|}{\widehat{\lambda_1}+\widehat{\lambda_3}}-\sum_{i=1}^{m}(x_i^{0*b}\vee y_i^{0*b})=0, \tag{3.7}$$

where x_i^{0*b}, y_i^{0*b} and D_j^{*b} are the values of x_i^0, y_i^0 and D_j based on bootstrap samples respectively.

(4) Solve the equations (3.5)-(3.7) and obtain the solutions $\hat{\lambda}_1^{*b}$, $\hat{\lambda}_2^{*b}$, $\hat{\lambda}_3^{*b}$. And construct the bootstrap estimator of P, that is,

$$\widehat{P}^{*b}=\widehat{\lambda_1}^{*b}/(\widehat{\lambda_1}^{*b}+\widehat{\lambda_2}^{*b}+\widehat{\lambda_3}^{*b}). \tag{3.8}$$

From the construction of the bootstrap estimators $\widehat{P}^{*b}$, $b=1,2,\cdots,B$, the cumulative distribution function of $\widehat{P}^*$ can be obtained by $\widehat{F}^*(s)=\sum_{b=1}^{B}I(\widehat{P}^{*b}\le s)/B$, where $I(\cdot)$ is an indicator function.

So we construct some approximated bootstrap confidence intervals using $\widehat{F}^*(s)$, say the percentile method(Efron(1981)), the Bias corrected method(Efron(1982)), the Bias corrected acceleration method, the percentile-t method(Hall(1988)).

4. A Simulation Study

To investigate the performances of MLE and approximate confidence intervals for $P(X<Y)$, we assume that censoring time T_i has the exponential distribution with mean θ^{-1}. Then $g(t_i;\theta)=\theta\exp(-\theta t_i)$, $\overline{G}(t_i;\theta)=P\{T_i>(x_i\vee y_i)\}=\exp\{-\theta(x_i\vee y_i)\}$ and $\hat{\theta}$ is given as $(|D_4|+|D_5|+|D_6|)/\sum_{i=1}^{n}(x_i^0\vee y_i^0)$. We define the censoring rate (ω) by $\omega=P(X>t$ or $Y>t)$. The Marshall and Olkin's bivariate exponential random numbers were generated by the method proposed by Friday and Patil(1977). The values of λ_1,λ_2 and λ_3 are selected so that the values of P are 0.1, 0.3, 0.5, 0.7 and 0.9. The sample sizes n are 10, 20 and 40 and censoring rates (ω) are 0%, 25%, 50%. The used confidence levels $(1-2\alpha)$ are 0.90 and 0.95. Since simulation results of nominal level 0.90 and 0.95 are similar, we don't report in the Tables for 0.95. The Monte Carlo samplings were repeated 500 times. For each independent random samples the approximated confidence intervals were constructed by each method with bootstrap replications $B=1000$ times.

From table 1, we can note that MLE's and Biases tend to decrease as sample size increases for fixed censoring rate. For the most cases of sample size, MLE's and Biases tend to increase as censoring rate increases.

From table 2, we can observe that the coverage probabilities of all intervals tend to become true confidence level $(1-2\alpha)$ as n increases and ω decreases. As a whole, the approximation to the nominal confidence level 90% of bootstrap method is often close to or better than that of normal-theory method under small sample size. From the viewpoint of censoring rate, the coverage probabilities of all approximated intervals are very sensitive for large value of P.

Table 1. Biases and MSE's for MLE of P(X<Y)

ω	n	MLE	P = 0.1	P = 0.3	P = 0.5	P = 0.7	P= 0.9
0%	10	Bias	0.0054	-0.0107	0.0028	-0.0030	-0.0007
		MSE	0.0089	0.0176	0.0172	0.0104	0.0035
	20	Bias	0.0035	0.0023	-0.0053	-0.0057	0.0002
		MSE	0.0044	0.0084	0.0097	0.0055	0.0014
	40	Bias	0.0026	-0.0021	-0.0090	0.0025	-0.0017
		MSE	0.0026	0.0048	0.0048	0.0027	0.0006
25%	10	Bias	0.0078	-0.0082	0.0226	0.0080	0.0068
		MSE	0.0113	0.0219	0.0221	0.0133	0.0042
	20	Bias	-0.0019	0.0014	0.0219	0.0173	0.0122
		MSE	0.0050	0.0111	0.0111	0.0070	0.0014
	40	Bias	-0.0008	0.0027	0.0273	0.0288	0.0165
		MSE	0.0027	0.0054	0.0054	0.0035	0.0008
50%	10	Bias	0.0135	0.0005	0.0090	0.0289	0.0132
		MSE	0.0174	0.0233	0.0297	0.0145	0.0055
	20	Bias	-0.0073	0.0078	0.0328	0.0405	0.0201
		MSE	0.0061	0.0145	0.0156	0.0081	0.0024
	40	Bias	-0.0016	0.0144	0.0302	0.0494	0.0307
		MSE	0.0036	0.0090	0.0074	0.0041	0.0012

Table 2. Actual Coverage Probabilities of P(X<Y)

$P(X<Y)$	n	ω	Normal	Percentile	BC	BCa	Percentile-t
0.5	10	0%	0.8280	0.8840	0.9140	0.9160	0.8780
		25%	0.6940	0.7340	0.7680	0.7880	0.7860
		50%	0.6280	0.7240	0.7440	0.7320	0.7440
	20	0%	0.8560	0.8840	0.9120	0.9100	0.9040
		25%	0.7420	0.7540	0.8520	0.8680	0.8560
		50%	0.6540	0.7280	0.7580	0.7500	0.7980
	40	0%	0.8820	0.8980	0.9020	0.8980	0.9000
		25%	0.7820	0.7920	0.8620	0.8680	0.8760
		50%	0.7060	0.7650	0.7600	0.7820	0.7900
0.7	10	0%	0.8360	0.8760	0.8740	0.8840	0.9160
		25%	0.5960	0.6600	0.7420	0.7580	0.8040
		50%	0.4780	0.5620	0.7000	0.6980	0.6880
	20	0%	0.8840	0.9080	0.9100	0.9160	0.8840
		25%	0.7040	0.6420	0.8440	0.8460	0.8580
		50%	0.5640	0.5820	0.7200	0.7080	0.7220
	40	0%	0.9040	0.9080	0.9020	0.9080	0.8920
		25%	0.7020	0.7780	0.8680	0.8660	0.8720
		50%	0.6660	0.6500	0.7520	0.7420	0.7400
0.9	10	0%	0.8300	0.8480	0.8240	0.8720	0.8600
		25%	0.5680	0.6700	0.7880	0.7960	0.7980
		50%	0.3280	0.3640	0.4620	0.4440	0.4620
	20	0%	0.8580	0.8720	0.8840	0.8860	0.8880
		25%	0.7020	0.6920	0.8020	0.8000	0.7980
		50%	0.5040	0.5320	0.5880	0.6480	0.6200
	40	0%	0.9080	0.9120	0.9020	0.9160	0.9140
		25%	0.6980	0.7020	0.8520	0.8600	0.8480
		50%	0.6380	0.6620	0.6340	0.6880	0.6980

Normal denote asymptotic normal method
Percentile denote percentile method
BC denote bias corrected method
BCa denote biased corrected acceleration method
Percentile-t denote percentile-t method

References

[1] Arnold, B.C. (1968). Parameter Estimation for a Multivariate Exponential Distribution, *Journal of the American Statistical Association*, 62, 848-852.

[2] Awad, A.M., Azzam, M.M. and Hamdon, M.A. (1981). Some Inference Results on P(X<Y) in the Bivariate Exponential Model, *Communication in Statistics Theory and Methods*, A10(24), 2515-2525.

[3] Bemis, B.M., Bain, L.T. and Higgins, J.J. (1972). Estimation and Hypothesis Testing for the Parameters of a Bivariate Exponential Distribution, *Journal of the American Statistical Association*, 67, 927-929.

[4] Efron, B. (1981). Nonparametric Standard Error and Confidence intervals(With discussion), *Canadian Journal of Statistics*, 9, 139-172.

[5] Efron, B. (1982). The Jackknife, the Bootstrap and Other Resampling Plans, *CBMS-NSF Regional Conference Series in Applied Mathematics*, SIAM, 38.

[6] Efron, B. (1987). Better Bootstrap Confidence Intervals(With Discussion), *Journal of the American Statistical Association*, 82, 171-200.

[7] Friday, D.S. and Patil, G.P. (1977). A Bivariate Exponential Model with Applications to Reliability and Computer Generation of Random Variables, *The Theory and Applications of Reliability*, ed. Tsokos, C.P. abd Shimi, I.N., Academic, 527-549.

[8] Hall, P. (1988). Theoretical Comparison of Bootstrap Confidence Intervals(with Discussion), *The Annals of Statistics*, 16, 927-953.

[9] Hanagal, D.D. and Kale, B.K. (1992). Some Inference Results in Bivariate Exponential Distributions Based on Censored Samples, *Communication in Statistics-Theory and Methods*, 21(5), 1273-1295.

[10] Kim, J.J. and Park, E.S. (1990). A Study on Estimators of Parameters and Pr(X<Y) in Marshall and Olkin's Bivariate Exponential Model, *Journal of the Korean Society for Quality Control*, 18, 101-116.

[11] Marshall, A.W. and Olkin, I. (1967). A Multivariate Exponential Distribution, *Journal of the American Statistical Association*, 62, 30-44.

[12] Proschan, F. and Sullo, P. (1976). Estimating the Parameters of a Multivariate Exponential Distribution, *Journal of the American Statistical Association*, 71, 465-472.