# Sequential Analysis of Earth Retaining Structures Using p−y Curves for Subgrade Reaction

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## 요 지

탄성지반에서 지반스프링을 이용하여 굴착단계별로 토류벽의 거동을 검토하였다. 토막선형함수를 이용하여 지반계수에 따른 p−y 특성곡선을 산정할 수 있는 수학적 모델을 사용하였고, 토류벽의 굴착단계는 beam−column 방법에 의해 분석 하였다. 개발된 프로그램의 신뢰도는 예측치와 실제변위의 비교를 통해 검증하였다. 건설단계를 잘 반영하므로, 앵커로 지지된 토류벽의 변위예측이 향상되었다. 분석결과에 따르면 제안된 방법은 민감도해석에 적용되는 계수들의 상대적 중요성의 평가에 효과적으로 이용될 수 있다.

## Abstract

The sequential behavior of earth retaining structure is investigated by using soil springs in elasto−plastic soil. Mathematical model that can be used to construct the p−y curves for subgrade modulus is proposed by using piecewise linear function. The excavation sequence of retaining wall is analyzed by the beam−column method. Reliability on the developed computer program is verfied through the comparison between the prediction and the in−situ measurdments. It is concluded that the proposed method simulates well the construction sequence and thus represents a significant improvement in the prediction of deflections of anchored wall excavation. Based on the results the proposed method can be effectively used for the evaluation of the relative importance of the parameters employed

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in a sensitivity analysis.

Keywords : Earth Retaining Structure, Elasto−Plastic soil, p−y Curve, Beam−Column Method, Construction Sequence.

## 1. Introduction

The major concerns in earth retaining systems are the horizontal displacements, the earth pressure acting on the wall, and the overall stability of the wall. The current analysis and design of a retaining wall are based on the early theory by Rankine and Coulomb. Terzaghi and Peck (1967) and Tschebotarioff (1973) proposed empirical earth pressure diagrams early which were developed from field measurements. These diagrams have been commonly used in the analysis and design of the supported retaining wall systems.

The beam-column method which basically assumes a beam on non-linear supports has been studied for many applications in engineering practice. This approach is based on the beam on elastic foundation models which have been developed for soil-structure interaction problems by Winkler. Here, the deflection at any point on the soil medium is directly proportional to the contact pressure q at that point. Most of the previous studies have used the linear soil support(spring) instead of the non-linear soil supports which are more appropriate and realistic in evaluating the real soil behavior. A more proper way to predict the pressure distribution is to introduce the role of deflections in the calculations with a beam column analysis and a coupled set of load-displacement curves called the nonlinear p-y curves.

A conceptual methodology for retaining wall design by considering soil springs using the p-y curves, the earth pressure-deflection soil model, and the simulation of the construction sequence is proposed.

## 2. Method of Analysis

An anchored wall can be modelled as a beam resting on non-linear soil spring supports as shown in Figure 1. The wall is modelled as a beam with a bending stiffness EI. The non-linear springs are characterized by the non-linear p-y curves and the actual anchor force is considered as the anchor load on the beam.

The governing differential equations for the response of the beam can be formulated from three different stages for proper simulation of the construction sequence as follows :

(1) Unsupported Excavation Stage

$$EIy(x)'''' = q \quad p(y, x) \tag{1}$$

(2) Anchor Stressing Stage (Before anchor is locked-off)

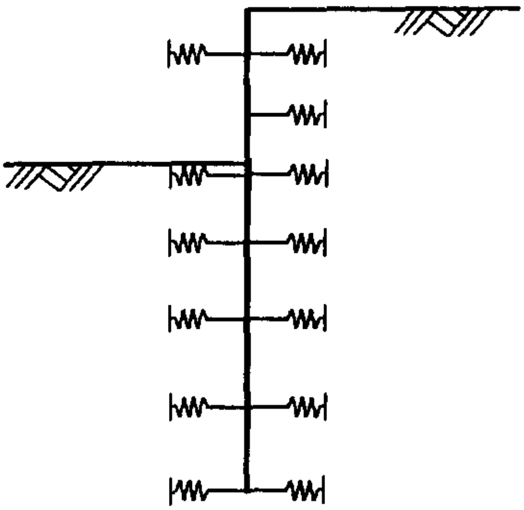


Fig. 1  Modeling of an Anchored Retaining

$$EIy(x)'''' = q - [p(y, x) + R] \tag{2}$$

③ Supported Excavation Stage(After anchor is locked-off)

$$EIy(x)'''' = q - [p(y, x) + R + \frac{AE}{LS} y^*(x)] \tag{3}$$

where $$p(y, x) = K_h \cdot y(x) \tag{4}$$

q = earth pressure at zero deflection of the beam

EI = flexural stiffness of the beam.

y(x) = beam deflection

$K_h$ = Winkler's coefficient of subgrade reaction

R = anchor force

AE = axial stiffness of anchor

L = unbounded anchor length

S = anchor spacing

$y^*(x)$ = deflection after anchor is locked-off

After a beam is divided into a number of discrete segments, the finite difference method is used to solve those equations related to each construction stage by applying the boundary conditions. Next, to solve Eq. (4) the earth pressure deflection curves are idealized as elasto-plastic p-y curves, where p is the load on the wall at a depth x per unit height of wall and per unit width of wall and y is the displacement of the wall at the same depth x. The subgrade reaction modulus is used and finally the p-y curves which represent piecewise linear interpolation are constructed by a series of straight lines joining five data points which are the active coefficient, the at-rest coefficient, the passive coefficient, the reference deflection for the full active and passive earth pressures.

## 3. Modeling of Construction Sequence

The construction sequence for the wall is modelled by dividing a series of unloading and reloading sequences into several stages on the basis of the excavation to the proper excavation level and then stressing the anchor to the final excavation. The unloading and reloading relationship is characterized by a p-y curve (Figure 2) through the following procedure :

1) When a wall is built into the ground, earth pressure at rest is considered to be carried on both active and passive sides.
2) Then, when the first excavation is made down to a certain depth, it causes imbalance in the earth pressures and represents an unloading of the p-y curve : the wall deflects towards the excavation side(A-B-C in Figure 2).
3) Next, the first anchor applies force to the soil through the structural member which causes the wall to move into the soil mass(C-D-E in Figure 2).
4) For the second excavation, the p-y curves which are used in the first stage are shifted by as much as the plastic movement (B-C in Figure 2) which occured in the first construction stage.

As a result of repeating steps 1), 2), 3) and 4) in the aforementioned procedure up to the final construction stage, the deflection obtained in the final construction stage is the total deflection of the wall owing to the entire staged excavation. The deflections from each construction stage are not accumlated, but affect the p-y curves for the subsequent stage. The updated p-y curve can be obtained by shifting the previous p-y curve by an amount of plastic movement represented by an offset distance $w_{offset}(i, j)$. The offset distance $w_{offset}(i, j)$ at the i-th node at the end of the j-th construction stage is used to prepare the p-y curves for the (j+1)-th construction stage. The flow chart for the simulation of the construction sequence is shown in Figure 3.

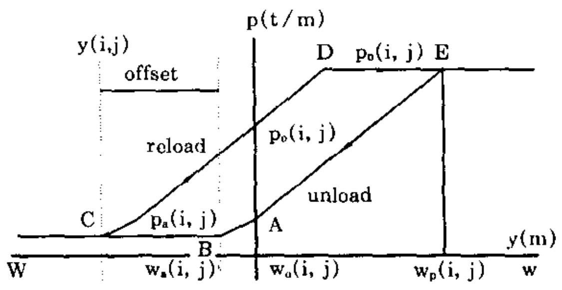



Fig. 2 Non-linear p-y Curve

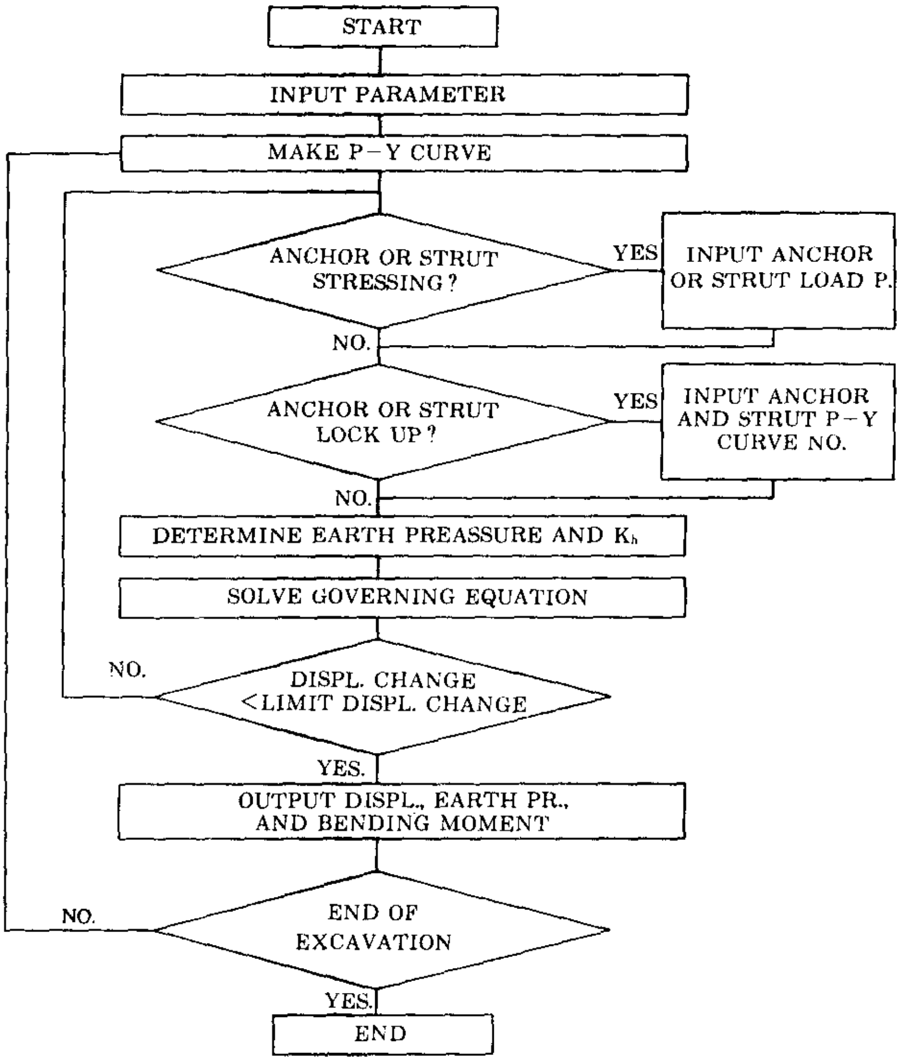



Fig. 3 Flow Chart for Construction Sequence

## 4. Description and Validation of Proposed Model

### 4.1 Description

A computer program for the analysis of anchored walls is developed in this study to predict the behavior of elastic-plastic material in which soil pressures change linearly with lateral displacements until they reach the minimum active pressure or the maximum passive pressure and then remain constant at the limiting conditions. For the analytical model, wall is simplified as a beam element and the soils at both sides of walls are modeled as nonlinear springs by combining the p-y curve. The elasto-plastic analyses are run to take into account the p-y curves for subgrade modulus through an iterative procedure. For each iteration, the beam-column displacements from the previous solution are used to enter the nonlinear p-y curves and solutions are repeated until two successive iterations obtain sets of displacements that agree with an user-specified closure tolerance at all nodal points.

### 4.2 Validation

The validity of the proposed model was tested by comparing some of the predicted results with the measured ones by Briaud and Kim(1993). A full scale permanent ground anchor wall was constructed on the National Geotechnical Experimentation site which is in the city of College Station, Texas. The wall had permanent soil anchors, soldier beams, wood-lagging and wales. The site characterization, the wall data and the anchor data are summarized in Table 1. The excavated retaining wall is presented in Figure 4. and the measured deflections and bending moments at three stages are compared with the results by the proposed method as shown in Figure 5 and Figure 6. Reasonably good agreement is obtained between the present approach and the observed ones with the construction sequence.

Table 1. Input Data(Texas Case)

| Soil Data | | Wall Data | | Anchor Data | |
|---|---|---|---|---|---|
| Unit Weight | 115 pcf | Wall Height | 30ft | Unbonded Length | 15ft |
| Friction Angle | 32° | EI(Lateral Stiffness) | $1.25\times10^{10}$ (1b−in²) | Stiffness of Tendon | $1.157\times10^{10}$ (1b−in²) |
| N Value | 15/0.3m | Diameter of Pile | 0.8in | Lock−off Load | 68.6kips |
| − | − | − | − | Angle of Inclination | 30° |

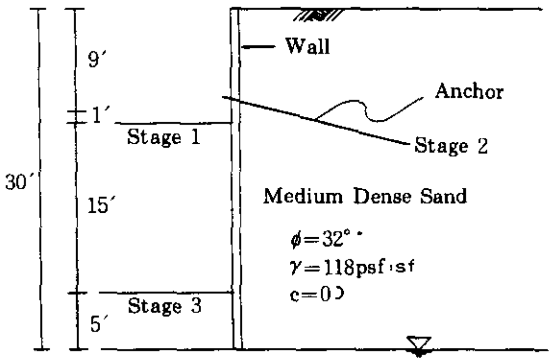
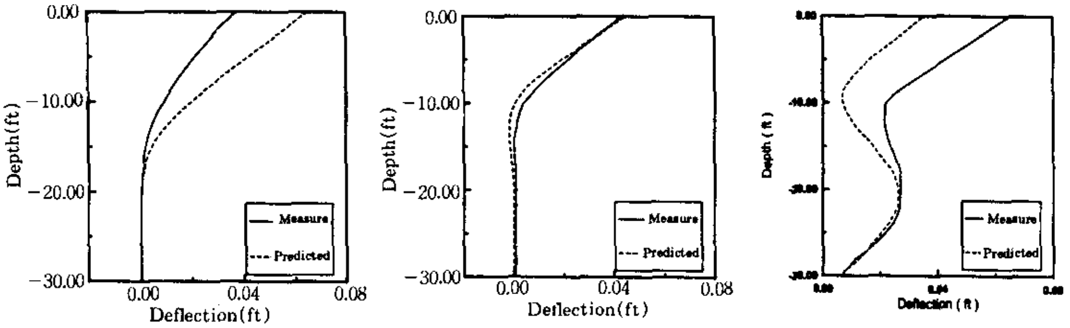
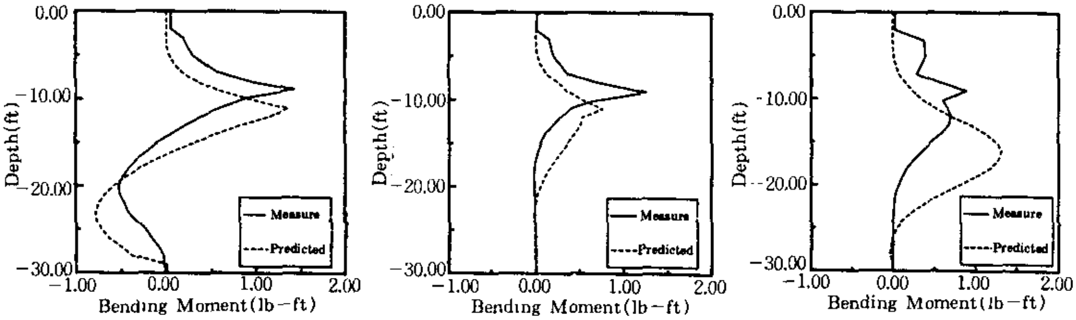



Fig. 4 Construction Sequence of Anchored Wall Excavation (Texas Case)




(a) Stage 1 (b) Stage 2 (c) Stage 3

Fig. 5 Comparison of Present Approach with Field Data (Deflection)




(a) Stage 1 (b) Stage 2 (c) Stage 3

Fig. 6 Comparison of Present Approach with Field Data (Bending Moment)

## 5. Sensitivity Analysis

The proposed method is based on the non-linear soil-structure interaction model. Since the results of the proposed method are strongly influenced by the horizontal subgrade reaction coefficient, the sensitivity analysis is studied in detail in the following section.

### 5.1 Methods Available for Estimation of Subgrade Reaction Coefficient

Methods to extimate horizontal subgrade reaction coefficient have been studied. They can generally be categorized into two different types : Subgrade reaction methods and p-y curve methods

① Estimation by subgrade reaction constant

In these methods, the subgrade reaction constant is modified to determine the subgrade reaction coefficient as shown in Eq. (5).

$$K_h = n_h \times \frac{D}{z} \tag{5}$$

where, D = parameter related to the deflection shape of the wall

z = the depth where $K_h$ is calculated

Depending on the extimation methods of subgrade reaction constant, these methods can be classified as follows :

Method 1 : Initial subgrade reaction constant proposed by Terzaghi(1954)

Method 2 : Subgrade reaction constant calculated by in-situ N value.

② Estimation by using p-y curves

In p-y curve methods the subgrade reaction coefficient is obtained by the slope of the p-y curve. Here, Rankine's theory is used for calculating the maximum passive and minimum active earth pressure. The subgrade reaction cofficient does not vary with the wall deflection shape but varies with the magnitude of wall deflection. Depending on the amount of movement (plastic limit) required to reach the limiting conditions, these methods can be classified as follows :

Method 3 : Using plastic limit calculated by FEM analysis of earth retaining structure (Fang, 1991)

Method 4 : Using subgrade reaction coefficient calculated by in-situ N value.(proposed method)

To determine the plastic limit by using subgrade reaction coefficient, the property of plastic limit is used. Based on the experimental study(Fang, 1991) it is shown that the movement required to reach the maximum passive earth pressure is ten times as large as the movement required to reach the minimum active earth pressure. The maximum passive and minimum active earth pressure are determined by Rankine's theory. The plastic limit at the middle point of each soil layer is calculated from the subgrade reaction coefficient determined by the in-situ N value and the maximum passive and the minimum active earth pressure(Figure 7). The plastic limit does not vary with depth at the same soil layer whereas the active and passive earth pressure vary with depth. Therefore the subgrade reaction coefficient at all nodal points can be evaluated by considering the varying depth.

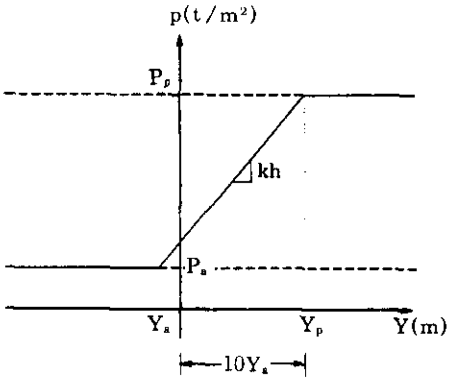



Fig. 7 Calculation of Plastic Limits Required to Reach the Minimum and Maximum Earth Pressure

Based on the four different methods, the sensitivity analysis was performed for the following cases.

Case 1

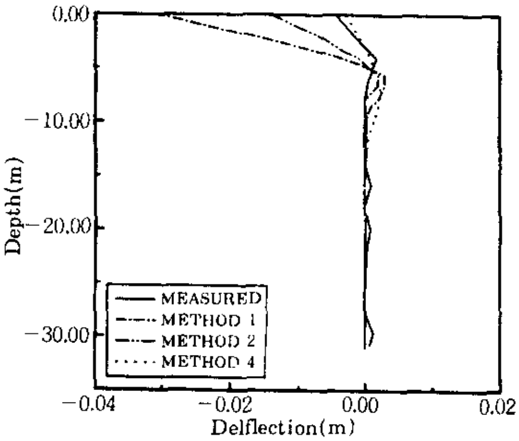
This case is Samsung insurance co. building construction site in Seoul, Korea. The wall had temporary ground anchors, soldier beam(H-pile : 300×300×15×15), and wood-laggings. The soil data and anchor data are shown in Table 2. The basic parameters employed in four methods are shown in Table 3. Figure 8 shows the predicted and measured lateral displacement in the retaining wall.

Table 2. Input Data(Case 1)

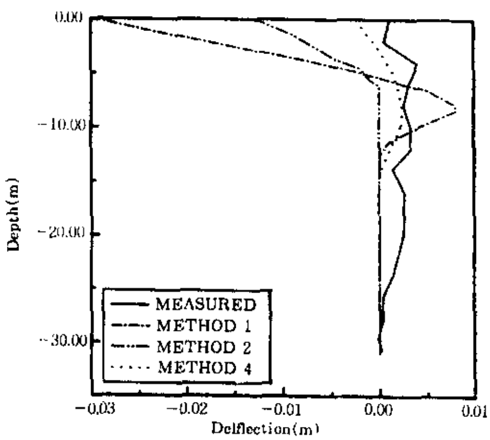
| Soil Data | | | | | | Anchor Data | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Fill | Clay | R.S. | W.R. | H.R. | | No.1 | No.2 | No.3 |
| Depth(m) | 4.5 | 6.5 | 12.5 | 14.0 | 35 | Depth(m) | 1.4 | 3.4 | 6.1 |
| $\gamma_{sat}$(t/m$^3$) | 1.8 | 1.8 | 1.9 | 2.0 | 2.1 | Pre-stress(ton) | 30 | 30 | 30 |
| Friction Angle | 28° | 26° | 32° | 36° | 38° | Stiffness of Tendon(t-m$^2$) | 9870 | 9870 | 9870 |
| c(t/m$^2$) | 0 | 5.0 | 0.0 | 3.0 | 5.0 | Free-Length(m) | 17.0 | 16.0 | 15.0 |
| – | – | – | – | – | | Angle of Inclination | 40° | 35° | 32.5° |

Table 3. Parameters Used for the Sensitivity Analysis(Case 1)

| | Method 1 | Method 2 | Method 3 | | Method 4 | |
|---|---|---|---|---|---|---|
| | Reaction Const. | Reaction Const. | $y_q$ | $y_p$ | $y_q$ | $y_p$ |
| Fill | 250 | 500 | 0.024 | 0.24 | 0.0013 | 0.013 |
| Clay | 1750 | 2000 | 0.120 | 0.24 | 0.0025 | 0.025 |
| Residual Soil | 360 | 2750 | 0.012 | 0.12 | 0.0021 | 0.021 |
| Weak Rock | 1200 | 4480 | – | – | – | – |
| Hard Rock | 2500 | 5280 | – | – | – | – |



(a) Stage 3



(b) Stage 4

Fig. 8 Comparison of Deflection Shape

Case 2

This case is Gung-Ang Ⅱ-Bo building construction site in Seoul, Korea. The wall had temporary ground anchor, soldier beam(H-pile : 300×300×15×15), and wood-laggings. The soil data and anchor data are shown in Table 4. the parameters employed are shown in Table 5. Figure 9 shows the predicted and measured lateral displacement in the retaining wall.

Table 4. Input Data(Case 2)

| Soil Data | | | | | | | | Anchor Data | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Fill | Clay | R.S. | W.R. | Weak R. | H.R. | | | No.1 | No.2 | No.3 |
| Depth(m) | 1.6 | 3.3 | 17.0 | 19.5 | 23.0 | 35 | | Depth(m) | 1.5 | 3.7 | 5.9 |
| $\gamma_{sat}(t/m^3)$ | 1.8 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | | Pre-stress(ton) | 30 | 30 | 30 |
| Friction Angle | 28° | 28° | 32° | 35° | 37° | 40° | | Stiffness of Tendon($t-m^2$) | 9870 | 9870 | 9870 |
| $c(t/m^2)$ | 0.0 | 12.2 | 0.0 | 0.0 | 5.0 | 5.0 | | Free−Length(m) | 17.0 | 16.0 | 15.0 |
| − | − | − | − | − | − | − | | Angle of Inclinat-ion | 40° | 35° | 32.5° |

Table 5. Parameters used for the Sensitivity Analysis(Case 2)

| | Method 1 | Method 2 | Method 3 | | Method 4 | |
|---|---|---|---|---|---|---|
| | Reaction Const. | Reaction Const. | $y_a$ | $y_p$ | $y_a$ | $y_p$ |
| Fill | 250 | 500 | 0.034 | 0.34 | 0.0015 | 0.015 |
| Clay | 3000 | 500 | 0.340 | 0.68 | 0.0013 | 0.013 |
| Residual Soil | 300 | 1500 | 0.017 | 0.17 | 0.0060 | 0.060 |
| Residual Soil | 310 | 1500 | 0.017 | 0.17 | 0.0980 | 0.080 |
| Weathered R. | 450 | 3500 | − | − | − | − |
| Weak Rock | 1800 | 4800 | − | − | − | − |
| Hard Rock | 2500 | 7000 | − | − | − | − |

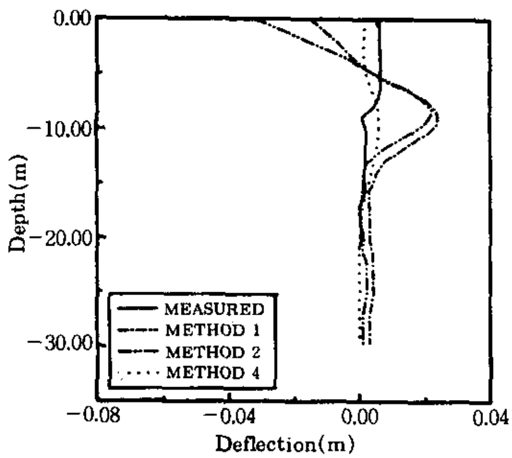


Figure. 9 Case 2

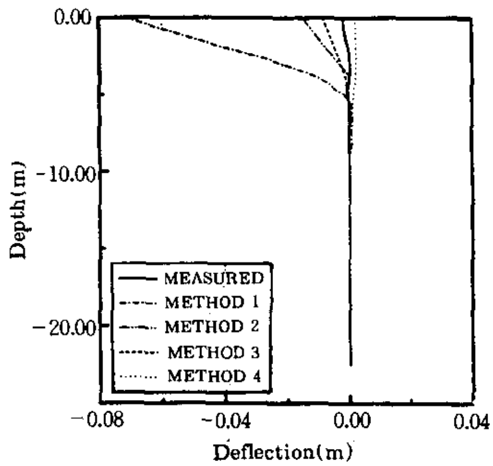


Figure. 10 Case 3

Case 3

The behavior of the two-row ground anchor wall built at Yonsei University was analyzed by the proposed method. The site characterization, the wall data and the anchor data are summarized in Table 2. The horizontal deflections were measured from inclinometer wells instrumented about 1m behind a driven soldier beam. Figure 10 shows the predicted and measured lateral displacement in the retaining wall.

Table 6. Input Data(Case 3)

| Soil Data | | | | | Wall Data | | Anchor Data | | |
|---|---|---|---|---|---|---|---|---|---|
| | Fill | R.S. | W.R. | H.R. | | | | No.1 | No.2 |
| Depth(m) | 3.5 | 1.2 | 2.7 | 14.6 | Wall Hight | 22.2m | Depth(m) | 2.1 | 4.9 |
| Unit Weight ($t/m^3$) | 1.8 | 1.8 | 1.9 | 2.1 | EI($t-m^2$) | 2793 | Unbonded Length(m) | 12.5 | 11.5 |
| Friction Angle | 29° | 30° | 35° | 40° | Diameter of Pile(m) | 0.3 | Lock-off Load(ton) | 38 | 38 |
| Cohesion ($t/m^2$) | 0 | 0 | 2.0 | 6.0 | – | – | Stiffness of Tendon($t-m^2$) | 9870 | 9870 |
| – | – | – | – | – | – | – | Angle of Inclination | 30° | 30° |

Table 7. Parameters used for the Sensitivity Analysis(Case 3)

| | Method 1 | Method 2 | Method 3 | | Method 4 | |
|---|---|---|---|---|---|---|
| | Reaction Const. | Reaction Const. | $y_a$ | $y_p$ | $y_a$ | $y_p$ |
| Residual Soil | 400 | 2000 | 0.0094 | 0.094 | 0.0006 | 0.005 |
| Residual Soil | 800 | 3000 | 0.0047 | 0.047 | 0.0012 | 0.012 |
| Weak Rock | 1800 | 5000 | – | – | – | – |
| Hard Rock | 3000 | 7000 | – | – | – | – |

## 5.2 Comparison of Wall Deflections Based on Analysis Methods

Three case histories are analyzed by four methods. The results of sensitivity studies are as follows :

① Method using subgrade reaction constant(Method 1 and Method 2)

Based on the results from method 1 and method 2, though there are differences in the subgrade reaction constants used for the input data, the variation of wall deflection is relatively small since the subgrade reaction coefficients determined by wall deflection shape have unique values, soil is still in the elastic state and the predicted wall deflection is overestimated when compared with the measured deflection.

Thus the method is applicable to the preliminary design of retaining wall with suitable safety factor.

② Method using a p-y curve(Method 3 and Method 4)

Compared to method 4, the subgrade reaction coefficent calculated by method 3 is relatively small. The reason for this is that if the deflection of the wall is larger than the plastic limit, the subgrade reaction coefficient becomes zero and as a result, the soil springs break down.

The wall deflection predicted by method 4 produced good predictions for all case histories. This is because most of the previous studies(methods 1, 2, and 3) have not led to firm analytical solutions since the effect on the subgrade reaction coefficient by the many complex construction sequences and the plastic limit have not been clearly examined: the subgrade reaction coefficient is the most important factor in anchored wall analysis. As a result, the prediction by these methods has some inaccuracy compared with the measured ones. However, method of using p-y curve is effective in simulating the construction sequence of anchored wall excavation, but is sensitive to the plastic limit and underestimates the wall deflection, Thus this method can be effectively used to perform back calculation analysis for evaluating the relative importance of the parameters in the anchored wall structures.

## 6. Conclusion

In this study the beam-column method for anchored walls is proposed and the results of the analysis are compared with case histories. Based on the results, the following conclusions are drawn :

1) The non-linear p-y curves based on the theory of active and passive earth pressure may be applied to earth retaining walls with low wall stiffness, let alone the high stiffness of the pile.
2) The prediction of the horizontal deformation by shifting p-y curves by as much as the permanent movement is necessary to simulate the construction sequence of anchored wall excavation.
3) The wall deflection predicted by method 4 which uses subgrade reaction coefficient calculated by the N value, leads to good predictions for all case histories.

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