# 고유진동주기를 이용한 응답수정계수

# Response Modification Coefficient Using Natural Period

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#### 약 요

구조물의 내진설계는 일반적으로 설계시방서의 스펙트럼을 이용하여 이루어지고 있다. 각 시방서에서 제시 된 스펙트럼은 여러지역에서 발생한 지진파들을 최대 지반가속도로 정규화하여 평탄한 응답을 구하였으며, 구조물의 특성에 따라 증감하여 사용하고 있다. 구조물은 지진하중에 의하여 소성변형을 보이고 있으며, 이러 한 구조물의 소성변형 능력을 고려하여 설계시방서에서는 응답수정계수를 사용하고 있다. 그러나, 이러한 응 답수정계수는 모든 구조물의 고유진동주기에 대하여 일정한 값으로 사용되고 있다.

본 연구에서는 각각의 지진파에 대하여 20개의 인공저진파를 작성하여 평탄한 응답스펙트럼을 구하였다. 구하여진 평균 응답 스펙트럼을 사용하여 구조물의 초기항복강도와 감쇠율의 효과를 측정하였으며, 회기분석 을 통하여 내진설계시 각 구조물에 요구되는 변위연성도를 얻기위한 강도계수를 추정하였다. 또한 현재 사용 되고 있는 설계시방서의 응답수정계수를 구조물 고유진동주기의 함수로 나타내었다.

#### Abstract

In some current procedures, ground motions from different sources have been scaled by their peak ground accelerations and combined to obtain smoothed response spectra for specific regions. As consideration of the inelastic deformation capacity of structure, inelastic deformations are permitted under seismic ground excitation in all codes. In the ATC(Applied Technology Council) and UBC(Uniform Building Code), the inelastic design spectrum is obtained by reducing the elastic design spectrum by a factor that is independent of structural period.

In this study, the average of nonlinear response spectra calculated from a sample of 20 records for each event are constructed to obtain the smoothed response spectra. These response spectra are used to examine the effects of structural strength factors such as the yield strength ratio and damping value. Through the regression analysis of nonlinear response of system for a given damping value and yield strength ratio, the required yield strength for seismic design can be estimated for a certain earthquake event. And a response modification coefficient depending on the natural period for current seismic design specifications are proposed.

Keywords: response modification coefficient, seismic design, damping ratio, yield strength ratio, response spectrum

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#### 1. Introduction

The fundamental problem in seismic design is to estimate the applied lateral forces over the lifetime of a structure under consideration. Structures should be designed to resist those lateral applied forces without severe damage. Traditionally, the basic earthquake design is based on single degree of freedom(SDOF) systems with viscous damping. Current seismic design methods are simplified and are based on smoothed response spectra. To obtain the smoothed response spectra, a sets of acceleration records from particular historical events which have been normalized by their peak values was used. Traditional static methods as well as modified linear spectral methods have been questioned by a number of papers<sup>1,2)</sup>.

Due to the inelastic capacity of structural elements, the basic concept of seismic design has changed from elastic design spectrum to inelastic design spectrum in recent years. Because of practical difficulties, inelastic design response spectra are generally obtained by modifying linear elastic design response spectra using factors that depend on the acceptable level of inelastic response.

The formulas for seismic design contain a strength reduction coefficient for considering the structural inelastic capacity. The use of constant reduction factors over the entire range of periods to construct inelastic design spectrum as currently adopted in the ATC<sup>31</sup> and the UBC<sup>41</sup> was investigated by Mahin and Bertero<sup>51</sup>, and Bertero<sup>22</sup>. Bertero concluded that a reduction factor independent of period can not be justified on the basis of ductility built-up in the structures. It was also suggested that the reduction factors obtained by displacement ductility  $(\mu)$  and  $(2\mu-1)^{1/2}$  for different natural

period in the Newmark-Hall method<sup>6,7)</sup>. This can only be justified if the structure is subjected to relatively very short acceleration pulses and the input energy for the linear elastic structure is the same as that for the inelastic (elasto-plastic) structure. Mahin et al. <sup>5)</sup> concluded that the displacement ductility demands computed for elasto-plastic SDOF system designed using the Newmark-Hall method were on the average less than the specified values.

In this study, 20 artificial earthquake accelerograms for each earthquake event were simulated using the ARMA model proposed by Ellise and Cakmak<sup>8)</sup>. This approach stabilizes the response spectrum for each earthquake events so that the effects of various strength ratios and damping values can be evaluated. This leads to improve the seismic design method, and the modification factor which can be used for spectra in modern practice was proposed.

## 2. Seismic Design Approaches

Based on economical considerations and actual inelastic behavior of structural members under cyclic loading, design procedures have been developed to construct an inelastic response spectra from the elastic response spectra.

In general, the total seismic force that a structure must resist may be written as

$$V = C_s \times W \tag{1}$$

where V is the total seismic applied force often called the "total base shear".  $C_s$  is a seismic coefficient expressed in terms of the system period, soil conditions, seismic zone and type of structure. Details vary in different ap-

proaches. W is the total weight of the structure.

There are two basical methods to determine base shear force for seismic design: static and dynamic analysis. The static analysis method is the simplest form of seismic design in existing design code such as the UBC and the ATC. For MDOF systems, the total base shear is distributed over the height of structure according to given formulas in each code.

Two methods are commonly used to estimate seismic design forces in dynamic analysis. One of these methods is direct time history analysis which consists of constructing a theoretical response model of a structure and calculating the exact dynamic response for given historically measured strong ground motions. The simpler second approach is the approximate dynamic method which is called "design spectrum analysis" and is now commonly adopted in codified design for structures. Seismic design forces are obtained from a response spectrum which is constructed from the smoothed response spectra for an ensemble of earthquake ground motions for a certain seismic area7).

#### 2.1 Static Analysis Method

The static analysis approach is relatively simple. In UBC, the total design base shear, V, in a given direction is derived from:

$$V = \frac{ZIC}{R_w} W \tag{2}$$

where C is a numerical coefficient equal to 1.  $25S/T^{2/3}$ , T is the period, S is a seismic coefficient accounting for the soil profile characteristics at the site, Z is a seismic zone factor with values, I is a factor for the type of occu-

pancy,  $R_w$  is a seismic response modification coefficient to consider inelastic deformation with a range of 4 to 12. The seismic design coefficient, V/W, can be written as

$$\frac{v}{w} = \frac{1.25 \text{ S Z I}}{R_w T^{2/3}}$$
 (3)

In ATC, the total base shear in a given direction derived from:

$$V = \frac{A_v IC}{R_w} W \tag{4}$$

where  $A_v$  is the effective peak ground velocity, C is a numerical coefficient equal to 0.  $8S/T^{2/3}$ .  $R_w$  is a seismic reduction coefficient with almost the same range as UBC, but is slightly different for a few cases. The rest of symbols, S, T and I are the same as in the UBC code. Thus, the seismic design coefficient, V/W, can be given as

$$\frac{V}{W} = \frac{0.8S A_{v} I}{R_{w} T^{2/3}}$$
 (5)

In the ATC approach, two parameters, effective peak acceleration,  $EPA(A_a)$ , and Effective Peak Velocity,  $EPV(A_v)$ , are used to characterize the intensity of design ground excitation for a given seismic region. As summarized by Bertero<sup>2)</sup>, EPA depends both on the type of earthquake considered and the interaction of the dynamic characteristics of the ground motion and of the soil-foundation-superstructure system.

As a result of economic considerations, inelastic deformations are permitted under extreme seismic excitation in all codes. In the ATC and UBC, the inelastic design spectrum is obtained by reducing the elastic design spectrum by a factor that is independent of structural period. The response modification factor,  $R_w$ , in the ATC and the UBC is intended to account for the damping as well as the inelastic deformation capacity of structure. In order words, structural design for elastic design may be made using  $R_w=1$ .

## 2.2 Dynamic Analysis Method

The dynamic analysis method or design spectrum method is based on an appropriate ground motion representation in UBC. A response spectrum is created by solving for the dynamic response in a range of periods for a given acceleration record. For structural design, a plot of the pseudo absolute acceleration given in gravity(g) units versus period is provided.

The design spectra suggested by Newmark and Hall<sup>6,7)</sup> has long been used to describe design earthquakes for seismic structural design. Newmark and Hall used a set of normalized acceleration records to obtained smoothed elastic spectra for SDOF system and they proposed modification formulas depending on the damping value for three period regions. In the Newmark-Hall method, the reduction factors are directly related to a specified displacement ductility,  $\mu$ , for the structures. For short period region(the so-called "acceleration region"), the strength reduction factor was suggested as  $(2\mu-1)^{1/2}$ , where  $\mu$  is the allowable design ductility.

# 3. Response Spectra

# 3.1 Effect of Parameters on Response

Numerous studies of the inelastic response of structures subjected to earthquake ground motion have been conducted. The direct stepby-step integration method was used to estimate the inelastic response of structures in this study. Many researchers have attempted to obtain improved estimates of structural response by using better approximations of the actual behavior of structural members. However, existing load-deformation models do not fully explain or describe the actual inelastical behavior of structures. For the very general conditions, relatively simple response model which is elasto-plastic system were adopted in this study.

To calculate the inelastic response of any nonlinear stiffness model, at least two basic parameters related to the structural system must be assumed. These are the initial yield strength and the damping coefficient. It should be noted that when the yield displacement spectrum is constructed to obtain a specified displacement ductility, the yield displacement is estimated by adjusting and iterating through the time history analysis. Alternatively, if the yield strength ratio, which is defined by the ratio of the initial yield strength, Ry, to the weight of the total system,  $Y=R_v/W$ , is given, the yield displacement can be derived directly. Furthermore, the yield strength ratio may be a useful tool to limit the lateral design force in inelastic design procedures. From the comparision of current design specifications, eight yield strength ratios(0.05, 0.07, 0.1, 0.13, 0.15, 0.2, 0.25 and 0.3) were employed to determine the effects of the yield strength ratio for nonlinear responses.

In elastic analysis, small amounts of damping can substantially reduce the response of system under the cyclic loading. Newmark and Hall recommended various damping values to be employed in the design of structure ranging from a minimum of 0.1% up to a maximum of

20% depending on the stress level and the type and condition of the structure. Based on this information, the six damping values used in the present study were 1, 2, 5, 7, 10 and 15 percent.

Average nonlinear response spectra indicating the displacement ductility was examined from samples of 20 generated accelerograms for each earthquake events. The effects of yield strength ratio, Y, and damping value,  $\beta$ , on the nonlinear response are examined assuming elasto-plastic load-deformation models.

Since inelastic response is directly affected by the initial yield displacement, the yield strength ratio, which provides the initial yield displacement, is an important factor in the nonlinear analysis of the structure. Fig. 1 indicate the variation in response versus yield strength ratio at periods of 0.5 second with the various damping values in case of the El-Centro earthquake event. As suggested by Fig. 1, a linear regression for the logarithm of response ordinates with respect to the logarithm of yield strength ratio with a given damping value can be attempted.

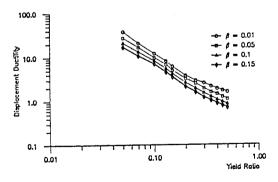


Fig.1 Effect of Yield Strength Ratio

Also, the dependence of nonlinear response spectra on system damping should be established. Fig. 2 show a plot of average spectral ordinates calculated for various yield strength ratios as a function of damping ratio between 1 and 15% for a system with period of 0.5 second in case of the El-Centro earthquake. As shown in Fig. 2, the logarithms of displacement ductility decreased nearly linearly with damping values for a given period. As a result, a linear regression between the logarithm of average response ordinates and damping values for a given yield strength ratio and system period can be made.

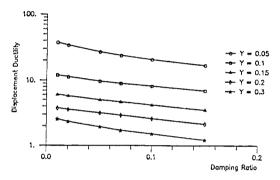


Fig.2 Effect of Damping Ratio

### 3.2 Response Spectra

Regression analysis for average response for 9 earthquake events versus period was performed with various damping values and yield strength ratios. Through the regression analysis, it can be concluded that the damping ratio has an approximately linear relationship and the yield strength ratio has an logarithmic relationship with ductility spectra for all earthquake events. And the formula to estimate displacement ductility may then be written as:

$$\log(\mu) = A_i + B_i \log(T) \tag{6}$$

where,  $A_i = A_{ri} - 1.18log(Y/0.1) - 1.79(\beta - 0.05)$  $B_i = B_{ri} + 0.64 log(Y/0.1) + 1.3(\beta - 0.05)$  and T is the natural period,  $A_{ri}$  and  $B_{ri}$  are the reference values for each event.

To determine the required strength of a structure based on specified limiting values of maximum nonlinear response, Eq.(6) can be rearranged. Assuming specified limiting ductility(or design ductility),  $\mu_D$ , the minimum required yield strength ratio can be calculated in terms of  $A_{ri}$ ,  $B_{ri}$  and damping value  $\beta$  as:

$$\log(\frac{Y}{0.1}) = \frac{\log(\mu_D) - C_2 - C_3 \log(T)}{C_1}$$
 (7)

and

$$Y|\mu_{D} = 10.0 \frac{\log(\mu) - Q - Q \log(T)}{C_{I}} - 1.0}$$
(8)

where 
$$c_1 = 0.64 \log(T) - 1.18$$
  
 $c_2 = A_{ri} - 1.79(\beta - 0.05)$   
 $c_3 = B_{ri} + 1.3(\beta - 0.05)$ 

The required yield strength ratios obtained from UBC, ATC, Newmark-Hall method and Eq. (8) in this study are compared in Fig. 3. Those for UBC and ATC were based on Eq. (3) and Eq. (5), respectively, and the assumptions include that Z=0.4, S=1.5,  $R_w=1.0$  for the elastic system and a moderate case in the California region. Acceleration, velocity and displacement of the ground motion for the

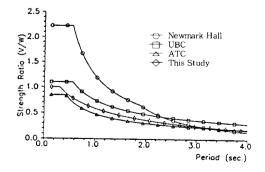


Fig.3 Comparison of Required Strength Ratio

Newmark-Hall method were of 0.822g, 83. 97cm/sec. and 28.62cm, respectively, and those were obtained as the mean plus one standard deviation of the peak ground motions for the six California earthquake events. The values of  $A_{ri}$  and  $B_{ri}$  for California region were estimated as 0.908 and -1.175 in this study

It can be observed in Fig. 3 that the required lateral strength ratio for the Newmark-Hall method is discontinuous, since the slope of the curve changes at the period of 1.82 sec. due to different amplification factors. As well, the required strength is very high in the low period region and relatively low in the high period region. The design strength ratio for the UBC exceeds the strength ratio required in this study. The design strength ratios for ATC are lower than those for both the proposed approach and the UBC.

The goal of constructing an inelastic design spectra is to ensure that structures do not experience more than a specified level of inelastic deformation when subjected to the design earthquake. The average displacement ductility corresponding to each design level of the yield strengh ratio was shown in Fig. 4. In the design spectra used for the UBC and the ATC, it was assumed that the response modification coefficient, R<sub>w</sub>, are 7 for the specified ductility

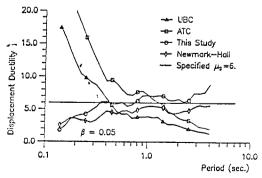


Fig.4 Displacement Ductility Demand ;  $\mu_{\rm D}{=}6$ 

of 6 and others are same as Fig. 3. The values of ground motion for the Newmark-Hall method and the reference values in this study are estimated from actual accelerograms for each earthquake events.

As expected, displacement ductilities calculated using the design spectra of the UBC and the ATC are very high in the low period region and low in high period regions. Although displacement ductility using the Newmark-Hall method show a relatively stable pattern over the range of period, the displacement ductility was always significantly smaller than specified ductility. The similar results were found by Mahin et al.. It seems that the design spectra of the Newmark-Hall method is very conservative and may lead to very strong lateral strength of structures.

Displacement ductility using the proposed strength ratio show quite stable results for all cases over the period range from 0.45 sec. to 2.5 sec. The displacement ductility were lower than specified ductilities at the period lower than near 0.4 second.

# Strength Reduction Factor for Nonlinear Design Spectra

Inelastic behavior is admitted in earthquake design and a response modification coefficient is used to construct the nonlinear design spectra from elastic design spectra. In the UBC and ATC, a deamplification coefficient, R<sub>w</sub>, is introduced to consider both damping effects and the inelastic ductility demand. In the current practice, these are assumed to be constant over the entire range of periods,

In this study the strength reduction factor can be found from Eq.(8), as a function of natural period. If  $Y_E$  and  $Y_D$  are strength ratios

corresponding to ductility equal to one and the value specified, respectively, the strength reduction factor " $F_D$ " can be defined as the ratio of  $Y_E$  to  $Y_D$ . This can be written as:

$$F_{\rm D} = \frac{Y_{\rm E}}{Y_{\rm D}} = 10.0^{\frac{-\log(\mu D)}{0.84\log(T) - 1.18}}$$
 (9)

Even though allowable displacement ductility for structural systems are not clearly stated in the UBC and ATC, the ATC mentioned that a moment resisting concrete structure with a system reduction factor of 4.5 and 7 can be assumed to have a displacement ductility of 4 and 6, respectively.

Fig. 5 shows strength reduction factor versus system period for specified displacement ductilities of 4 and 6. Even though the formular of the proposed reduction factor is an exponential form as shown in Eq.(9), the curves turn out to be nearly linear with respect to period over the range investigated. If constant modofication coefficient in the ATC and UBC are used over the entire range of periods, relatively conservative results in response demand will be obtained for low periods and unconservative results for high periods. Accordingly, to adjust the ATC and UBC methods to the results of this study, the coefficient R<sub>w</sub> should be made as a function of period.

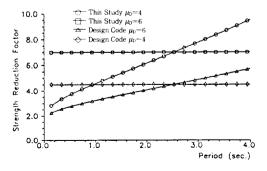


Fig.5 Strength Reduction Factor Proposed

To show the validity of the proposed period dependent reduction factor, the design spectra of the UBC and the ATC were modified by using the proposed strength reduction factor instead of response modification coefficient. Average ductility demand using samples of 20 for El-Centro with the yield force ratio based on the modified design spectra are evaluated, and the results are compared with ductility demand from original design formulas in the ATC and UBC. The results are shown in Fig. 6 and 7.

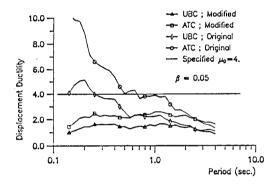


Fig.6 Displacement Ductility Demand;  $\mu_0=4$ 

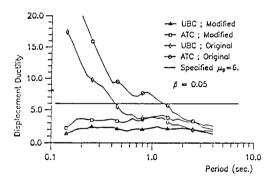


Fig.7 Displacement Ductility Demand;  $\mu_D=6$ 

It was shown that displacement ductility demand from both the modified ATC and UBC for the El-Centro event are smaller than the specified ductility over the entire periods. Displacement ductility demands for both the modified UBC and the modified ATC are significantly improved to provide more st-

able values over the entire range of periods and are almost parallel to the period axis,

#### 5. Conclusion

The objective of this study was to evaluate the nonlinear response of single degree of freedom systems to seismic loading with particular attention to the effects of yield strength ratios and damping values. In the current practice, inelastic design spectra are developed by reducing elastic spectra by constant factors over the entire range of system period. This approach yields relatively high displacement ductility in low period regions and low ductility in high period regions. Through regression analysis, a strength reduction factor for use with current design approaches was developed as a function of the natural period of structure.

Average displacement ductility using required yield strength ratio in this study showed quite stable results and good agreement to specified displacement ductility initially given over the period range. The average displacement ductility from the modified design spectra of UBC and ATC by using period dependent reduction factor proposed in this study were improved to provide more stable values over the range of periods.

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