드로네이기법에 의한 고차 유한요소 생성

Higher Order Elements by Delaunay Triangulation

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요 약

드로네이 요소생성기법은 모델링 영역의 모양에 구애받지 않으면서 요소의 크기제어, 재편성, 국지요소생성 등에 있어서 탁월한 기능을 보여주고있다. 그러나 생성되는 요소가 선형삼각형요소임으로해서 비압축성 또는 대변위거동의 근사나 복잡한 형상의 영역의 기하학적 근사에 한계를 갖고 있다. 이를 보완하기 위해 기제시된 드로네이 요소생성 알고리즘을 바탕으로한 6절점 삼각요소 생성알고리즘을 제시하여 본기법의 완성도를 높이고 이를 성형문제에 적용해 보였다.

Abstract

Delaunay triangulation is a very powerful method of mesh generation for its versatility such as handling complex geometries, element density control, and local/global remeshing capability. The limit of generating simplex elements(3-node elements in 2-D) only is resolved by adding generation module of 6-node quadratic elements. Since proposed adjacency does not change from 3-node element mesh to 6-node mesh, generation module can utilize the original simplex element generator. Therefore, versatility of the Delaunay triangulation is preserved. A simple upsetting problem is employed to show the possibility of the algorithm.

Keywords : delaunay triangulation, adjacency, adaptive f.e.m., renumbering density control, 6-node quadratic elements plastic deformation

1. Introduction

As the realm of finite element analysis is broadened, the treatment of mesh(geneation of initial mesh, and remeshing or revision of the mesh locally as well as globally) is getting more attention recently. Grid generation was not considered as one of major concerns in the development of finite element methods from the beginning. The task appeared trivial and

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routine process. But it is not so any more. Solution domain becomes complex, and emerging numerical methods require versatile treatment of the mesh, from initial mesh to revison or remeshing in the course of solution process. Hence an efficient, automatic, and robust grid generation tool becomes an essential part of the finite element analysis.

Finite element approximation gives estimates of a function values at a given set of points. Grid generation gives these points. The points should be able to delineate complex geometries and boundaries. While the points should be close enough to represent changes of function values, they have to be placed apart from each other to the extent that they ignore certain details of the function. Each point is not independent from its neighboring points since in most cases it is required to provide values of derivatives as well as function value itself. As a result, connectivity is introduced to form elements which cover whole domain of the problem. Hence a grid is formulated with the coordinates of nodes and the connectivity of the nodes for an element as basic data.

There are a wide variety of mesh generators each of which has its special attributes over others. Structured mesh generators such as algebraic Serendipity methods,[1,2] algebraic interger net methods,[3] elliptic equation methods,[4,5,6] transfinite interpolation methods,[7,8] and hyperbolic equation methods[9,10] give quadrilateral meshes while unstructured mesh generators like Delaunay methods,[11~17] advancing front methods,[18~20] and octree methods[21] generate triangular meshes. In CFD analysis, hybrid mesh scheme[22] is used to represent flow features efficiently on the boundary layer as well as off the layer region. No single type grid generation method is sufficient to address the challenges of the current and future applications. There are strengths and weaknesses in all approaches. Nonetheless, one can not overlook some of the most apparent incentives such as robustness, and automation of the grid generation method since it will be used for generation of initial mesh, local adaptive mesh, and global revision of the mesh. In this regard, Delaunay triangulation is one of the most efficient and powerful methods among others.

It has been shown that Delaunay triangulation is very powerful method to generate triangular finite elements.[11~17] It is very flexible, adaptive and universal mesh generator in many respects ;

1) It is not restricted by any topology of the domain geometry

2) The whole domain or any of subdomains can be meshed by the same generation algorithm.[17]

3) The density control is simple and easy by the spacings of nodes on the bounary[17,23,24]

In the meantime Delaunay triangulation generates simplex elements only, i. e. linear triangular elements. One of the main advantages of FEM over FDM is capability of handling boundary conditions. Linear elements have many advantages in practice ; it requires not too many memory spaces, and not too much computing time. Yet linear element is not appropriate for a certain class of problems. When it comes to the problems with complex boundary geometry, of large deformation, or of incompressibility constraints, it does not suffice in accomodating the state functions. In this study we add additional algorithm of generating 6-node quadratic triangular elements for the completeness of Delaunay triangular mesh generation method.

2. Grid Adaptation and Limites of Linear Triangular Elements

Adaptive finite element methods have been extensively studied and applied to solve various linear and nonlinear problems.[25,26] In particular adaptive methods have made successful implementation in solving linear problems which describe more or less steady state phenomena. However, they are not quite appropriate for solving problems defined on varying domains in time and problems whose nature is essentially unsteady. Many of metal forming analysis deal with large deformation and simulation of unsteady processes whose domain is also varying in time. For such a class of problems, application of adaptive refinement methods may not be good enough to obtain accurate simulation results since its adaptive mesh is strongly influenced by the initial mesh while the domain is changing significantly. Naturally adaptive remeshing method draws more attention. It is certain that the adaptive remeshing method needs more computing time, and the adaptive mesh refinement method has much simpler structure. However after realizing difficulties involving many nonlinear problems in which the domain is varying, the advantage of adaptive remeshing methods have been widely recognized over the adaptive mesh refinement methods especially in shape optimization of an elastic structure and large deformation analysis with the Lagrangian formulation.[27,28]

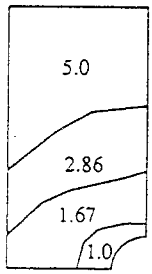
We review one of the adaptive remeshing methods implemented by Yukawa et al.[29] for the analysis of metal forming problem. The strategy of Yukawa is summerized as follow :

1) Develop the initial finite element model.

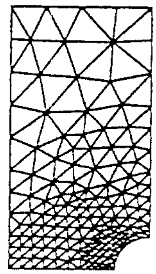
2) Compute the distribution of an estimate of the approximation error, and determine the subdomains by grouping finite elements whose estimated error is in certain ranges.

3) Extract the necessary geometric information of the subdomains defined in the second step, and execute automatic mesh generation for each of the subdomains with different mesh density on.

4) Repeat the second and the third steps until the desired result is obtained.



(a) Subdomain



(b) Subdoman mesh

Fig 1 Adaptive mesh by a subdomain method

Figure 1 shows typical application of the automatic mesh generation method proposed by Yukawa et al.[29] A domain is decompsed into four subdomains, in which different mesh density is defined by the "average" mesh size.

Implementing boundary conditions with linear elements can introduce some serious errors in the finite element approximation of metal forming problems. To minimize geometric discrepancy the number of elements is increased in the region. This error will be reduced greatly by employing quadratic elements.

As far as the rigid plasticity is concerned, there are few works using triangular elements since the 3-node element yields locked solutions due to incompressibility. At least the 4-node quadrilateral element must be applied to deal with such a model, and there are many strikingly successful results published using the 4-or 8/9-node quadrilateral elements with the selective reduced integration methods. However, as Babuska et al.[26] noted that utilization of higher order triangular elements does not imply the locked solution. This fact was not clearly stated in any literature related to finite element methods for (slight) incompressibility, however, Yukawa et al.[29] have shown the successful use of the 6-node elements without using the selective reduced integration scheme in dealing with incompressibility. The convergence of the approximation is assured with the rate of convergence $O(h^2)$ where h is element size. Hence when generation algorithm of 6-node element is added to Delaunay triangulation technique, all of the convenient and useful features of the method can be utilized.

3. Delaunay Triangulation and Density Control

Density control introduced by Yukawa[29] seems too artificial, since it requires several subdomains to accomodate appropriate element gradient on the domain. In that regard, Delaunay triangulation provides natural means of dealing with varying element density.

General procedures,[17, 23, 24] introducing *adjacency* as additional basic data, for a computer coding of Delaunay triangulation with desired density and adaptive remeshing capability are summarized as follows :.

i) Generation of boundary nodes : Boundary nodes are generated by discretizing the lines delineating domain configuration. The lengths of the intervals of the boundary nodes provide control measure of element density. When a specific density is required on the interior, a node with required density function value is inserted at the location.

ii) Construction of the boundary triangulation from the boundary nodes obtained in step i) in such a way that triangles satisfy Delaunay properties.

iii) Locate a position on the interior of the domain where a new node will be assigned through location test, space test, and shape test.[17, 23, 24]

iv) Collect elements of which circumcircles contain the new node, and construct an open block by eliminating those elements.

v) Construct new elements on the block by connecting lines radially from the new node to the boundary segments of the block.

vi) Repeat steps iii)-v) until no more position is found at the step iii).

The three tests in the step iii) provide acceptable regularity and density of the elements. Density function is assumed over the domain and the idea is implemented by assigning density function values at each and every point. Let denote density function $d(p)$ for point p, the lines connected to the node p by Si, and its the other end points by i, respectively. Then density function is defined as follows:

$$d(p) = min\{lengths\ of\ S_i'\ s\} \qquad (1)$$

For a node to be generated its prospective density value can be found either by (1) or by interpolation. Let y be the coordinates of a candidate point q, j the nodes constructing the element which contains point q before remeshing, Lj the area coordinates of the elements containing point q, respectively. Then the interpolation $dd(q)$ of density function value is

$$dd(q) = d_j\, L_j(y)\ where\ d_j = d(j). \qquad (2)$$

For repeated indices Einstein convention is assumed from here and after.

In the initial triangulation(boundary triangulation), all the existing points are on the boundary and their density index values are given by (1). In the space test for a candidate node q, if

$$dd(q) < \alpha\, d(q) \quad (3)$$

for a certain factor α, then point q will be denied as a new node at the location. In (3), $d(q)$ is evaluated assuming remeshing is done on the block, and $dd(q)$ is evaluated before remeshing. In practice $\alpha=\sqrt{2}$ appears to be appropriate for preserving desired element size.

The three tests used for the criteria to generate a new node are not enough to provide acceptable shape regularity. At the end of mesh generation, a smoothing process is carried out to regularize the mesh. For global regularization we use well known method of barycenter at each nodal point. The basic formula to relocate node p at new coordinates x_p is as follow.

$$x_p = \frac{a_i}{nA}\, {}_pX^i \quad (4)$$

where

n=the number of elements around node p

a_i=area of triangle ${}_p\triangle^i$ which is sharing node p

A=sum of ai around node p

${}_pX^i$=coordinates of areacenter of triangle ${}_p\triangle^i$

The quality of improvement is assessed by the shape quality index Qi defined as follow.

$$Q_i = f\frac{a_i}{L^2} \quad (5)$$

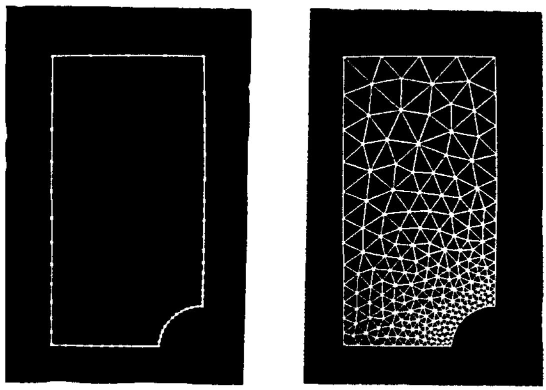
where

L=maximum side length of the triangle ${}_p\triangle^i$

f=normalizing factor.

The normalizing factor f assumes the value of $4/\sqrt{3}$ on the basis that the quality index Qi of the equillateral triangle be unity.

The implementation of Delaunay triangulation is shown in Figure 2. Comparing to the subdomain method shown in Figure 1, Delaunay triangulation method has much simpler setting. The only setting is discretization of boundary lines by varying intervals according to density requiremens of the given problems as shown in Figure 2(a). The resulting mesh is shown as Figure 2(b), and we can observe that the density variation and the size gradient of the elements are very smooth comparing to the mesh by the subdomain method of Figure 1.



(a) Boundary discretization (b) Delaunay mesh

Fig 2 Density control in the Delaunay Tringulation

4. Generation of Six Node Elements

The use of adjacency was newly introduced to identify remeshing block in the process of triagulation.[17]. Data for adjacency are kept in array NEXT :

NEXT(I, J) = element number adjacent to

I-th side of element J

NEXT(I, J) = 0 if side I of element J is on the boundary

In addition to saving time of element search, the array helps construct the block to be revised through the whole iterative process of mesh generation. The array NEXT will be used again effectively in generating 6-node quadratic triangular mesh from the 3-node mesh net for identifying element sides to put mid-nodes and completing connectivities of 6-node elements. The process is similar to that of adaptive refinement scheme used in the previous work.[23, 24] First, we construct 3-node triangular mesh net from Delaunay triangulation, after then we modify the 3-node base net into 6-node element net. There are two possible ways of transformation ;

1) Composition of four 3-node triangular elements into one 6-node triangular element.

2) Generation of mid-side nodes in each 3-node triangular element.

In the first method there is certain limits such that the number of triangles in 3-node triangular mesh net be the multiples of 4 so that the entire elements can be transformed to 6-node elsments without any of 3-node elements left. In the second method there is no particular constraint involved. Moreover data for the adjacency array NEXT are identical in 3- and 6-node triangular mesh systems. The second method is more attractive for use. The followings are agreed upon in generation of 6-node triangular elements.

· No gradient is allowed between nodes within an element, i. e. a mid-side node is located at the center of each side.

· Mid-side nodes on the domain boundary are projected exactly on the boundary lines where as other mid side nodes stay on the original straight lines.

The algorithm is straightforward with some ideas introduced in the previous works.[23] Geometric data of elements are coordinates of nodes and their connectivity within elements. Let the coordinate vector of node k be denoted by $x(k)$. The coordinates of mid node k between nodes i and j are given by

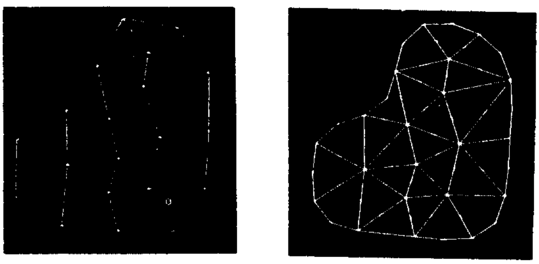
$$x(k) = \frac{x(i)+x(j)}{2} \qquad (6)$$

Generation of mid-side node k must be done one time only for side i-j. Scanning every side of all the elements, the sides on the interior of the domain appear twice while those on the boundary appear only once. Double generation can be avoided by starting generation of mid side nodes from lower element number with the use of adjacency array NEXT. On a side of the element, a mid-side node shall be generated only if its adjacent element number is greater than its own element number or if the side is on the boundary. The sides on the boundary are easily identified since their NEXT values are set by the value of zero. The generation algorithm is stated as follows. Here NX is the number of nodes, NEL the number of elements in 3-node base net by Delaunay triangulation for a given domain. The connectivity is established at the same time as mid-node is generated by filling in the array NEXT of existing connectivity data with mid-node number.

```
i) Set NE=1
ii) do I=1, 3
        if (NEXT(I,NE)>NE) then
          NX=NX+1
          compute x(NX) by equation (6)
          fill in connectivity of elements NE
          and NEXT(I,NE)
        elseif (NEXT(I,NE)=0) then
          NX=NX+1
```

compute x(NX) by projection on the boundary
fill in connectivity of element NE
endif
enddo
iii) NE=NE+1
iv) if (NE>NEL) then
stop
else
goto ii)
endif

From 3-node triangular base mesh net of Figure 3(a), 6-node triangular mesh of Figure 3 (b) is generated following the algorithm. The real lines on the boundary should be curved ones in Figure 3(b). We observe that mid-side nodes on the boundary are projected on the exact boundary line and mid-side nodes on the interior of the domain stay on the straight lines.



(a) 3-node base net (b) 6-node mesh

Fig 3 3-node base net and 6 node element mesh

5. Renumbering

Due to the nature of automatic node generation scheme in Delaunay triangultion, node ordering is arbitrary.

The node numbers are assigned in generating order of the nodes of which generating locations are not controlable. Inability to control node numbering results in possibly big a half bandwidth and tremendous increase of total band profile of the stiffness matrices. Hence it is inevitable to employ node renumbering scheme to reduce the bandwidth for successful use of the algorithm. When it comes to the element numbering process, one can not expect better situation than that of node numbering. The efficacy of conventional banded matrix solvers is irrelavent with element ordering. But when the frontal solver is used, ordering of elements is critical.[30] In this study renumbering schemes for nodes and elements are proposed to cope with the problems of uncontroled arbitrariness of orderings.

5.1 Node Renumbering

Conceptual principle of node numbering is such that maximum difference of node numbers in each element over all the elements be minimal. Let the mesh of initial numbering system be denoted by T_h, the element of number j by Ej, the number of nodes by NX, and the number of elements by NEL, respectively. We also define several concepts which will be used in subsequent discussions.

· Nodes s and t are members of the nodes in generating T_h.

· A node s is considered a neighbor of node t if nodes s and t are used in constructing an element of T_h.

· Degree of node is the number of elements in the collection of neighbors of node t and denoted by $dgr(t)$.

Neighbors of nodes t constitute members of neighbor level 1, denoted by $N_1(t)$. The number of members in $N_1(t)$ equals to $dgr(t)$. $N_1(t)$ is nodal version of adjacent element array NEXT used in generating Delaunay triangulation.

· Neighbors of nodes in $N_1(t)$ constitute members of the neighbor level 2 of node t denoted by $N_2(t)$. The nodes of $N_1(t)$ are excluded from $N_2(t)$, so are for the case of arbitrary neighbor level $N_k(t)$. That is to say that no node appears twice in the list of $N_k(t)$ for any level k.

· Nodes in the union of all the neighbor levels are called descendants of node t. Those are virtually all the nodes constituting the whole meshes except node t itself and the union is denoted by $D(t)$.

· Depth p or $p(t)$ of the descendants of node t is the largest level value of neighbor level k constituting the descendants, i. e.

$$D(t) = \bigcup_{k=1}^{p} N_k(t) \qquad (7)$$

Some of the concepts above are depicted in Figure 4. Following is schematic algorithm to achieve optimum node numbering for reducing total matrix profile as well as half bandwidth. This will be done in two steps, firstly finding the starting node of sequential numbering and secondly assigning new numbers to old nodes.

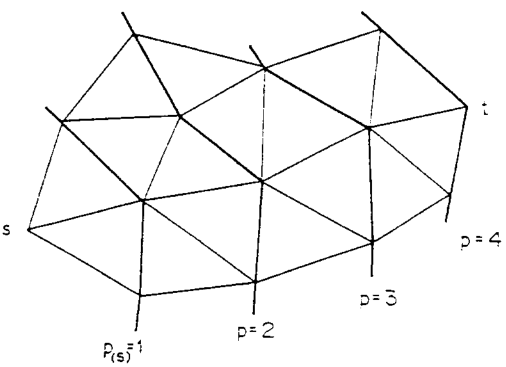


Fig 4 Equi-depth lines of neighbor level Nk(s)

(1) Determine the starting node to begin sequential numbering.

Now we are going to take a node s on the domain boundary to construct its neighbor level system $N_k(s)$ and to examine the depth $p(s)$ of its descendants $D(s)$. The direction from lower neighbor levels to higher ones is considered as the longitudinal direction of the mesh system. Long longitude means narrow latitude for a given domain of mesh. Then maximal depth $p(s)$ of the node s provides the maximum number of members in neighbor level $N_k(s)$ to be minimal, which is desired situation The following steps are the algorithm to find a starting node of new numbering.

i) Among the nodes on the boundary delineating the domain, find a node s such that $dgr(s)$ is minimal.

ii) Construct its neighbor level $N_k(s)$ for k=1 to p(s) and its descendants D(s).

iii) Set CHECK(s)=$N_p(s)$.

iv) Find a node t in CHECK(s) of which dgr(t) is minimal and construct D(t) and get p(t).

v) Test while CHECK(s) is not empty.

```
{if p(t) > p(s) then
    let s=t
    p(s)=p(t)
    N_k(s)=N_k(t), k=1,...,p
    goto iii)
else
    exclude node t from CHECK(s)
    goto iv)
endif}
```

The result of the present process gives two nodes s and t of extreme locations on the boundary of the domain as seen in Figure 4. The two nodes constitute pseudo-diameter encompassing the meshed physical domain and node

t is found to be a member of the last neighbor level $N_p(s)$ in the descendants $D(s)$ and its $dgr(t)$ is minimal on $N_p(s)$.

(2) Assign new node numbers

After constructing descendant levels from the both ends, nodes s and t, of pseudo-diameter, the equi-depth lines of neighbor level $N_k(s)$ and $N_q(t)$ to which each and every node belongs are drawn. The over all algorithms are summarized as follow. We define new node number array NEW(r) for old node number r and set new node number n as

NEW(r) = n

i) If dgr(s) 〈 dgr(t) then exchange the role of the nodes s and t, hence

$N_{p-i+1}(s) = N_i(t)$ for i = 1 to p

ii) Set n = 1 for node s then

NEW(s) = 1

$N_0(s) = s$

iii) For k=0 to p−1

iii-1) Find the lowest new node number u in the neighbor level Nk(s) one level lower than the level $N_{k+1}(s)$ of which nodes are not renumbered yet.

iii-2) Construct utility set B such that

$B = \{v \in N_1(u) \cap N_{k+1}(s)\}$

iii-3) Do while B is not empty.

{Find node w of minimal dgr(w) in B.

n = n + 1

NEW(w) = n

B = B − w}

End of loop.

When the loop is finished, the value of n equals to NX.

5.2 Element Numbering

When frontal method is used, assembling equations and elinminating variables are executed at the same time. As the solution front advances static condensation is carried out whenever possible. Hence it is desirable that element numbers be assigned in such a way that the solution front isolates variables(nodes) more frequently. Since new node numbers are so labeled in a sense, we let the element ordering scheme run parallel to the node ordering.

We define new element number array NEWE(j) for old element number E_j and set new element number num as

NEWE(j) = num.

i) num = 0

ii) Set NEWE(i) = 0 for all i

iii) For I = 1 to NX

Do while there is $E_j \in T_k$ such that $I \in E_j$

{if NEWE(j) = 0 then

num = num + 1

NEWE(j) = num

else

Ej is already checked and assigned new element number end if}

End of loop

When the loop is finished, the value of num equals to NEL.

5.3 Renumbering Examples

The present algorithm is very efficient in reducing the profile of a system of matrices. To illustrate the performance of the method we show several examples. The total profile is the sum of each column height from the diagonal of a symmetric stiffness matrix. and the average pro-file is the quotient of the total profile by the number of the columns. The half bandwidth is the maximum column height of the matrix. Table 1 shows the result of the examples for 3-node triangular meshes. Here mesh 1 is the mesh of Figure 1, mesh 2 that of Figure 5(a), mesh 3 that of Figure 5(b), and mesh 4 that of Figure 5(c), respectively. The half bandwidth is reduced by 1/8 to 1/11 of those of before renumbering. Rest of the values also are reduced significantly.

Table 1 Initial profiles and final profiles of 3-node mesh example

mesh	1	2	3	4
elements	381	187	377	449
nodes	219	118	217	267
total profile-old	56071	17943	60436	94485
average profile	49.06	31.48	53.46	70.15
half bandwidth	215	117	197	250
total profile-new	15481	3593	10707	12853
average profile	13.54	6.4	9.47	9.54
half bandwidth	30	12	20	21

The result of renumbering for 6-node mesh is examined for the same domain of Figure 5 (b). Total number of elements of 96 with 225 nodes are used for 6-node triangular mesh system. Initially the total profile of 108189, the average profile of 73. 60, and the half bandwidth of 208 are reduced to 18991, 12. 92 and 39, respectively. The reduction ratio of half bandwidth for 6-node mesh is approximately half of that of 3-node mesh.

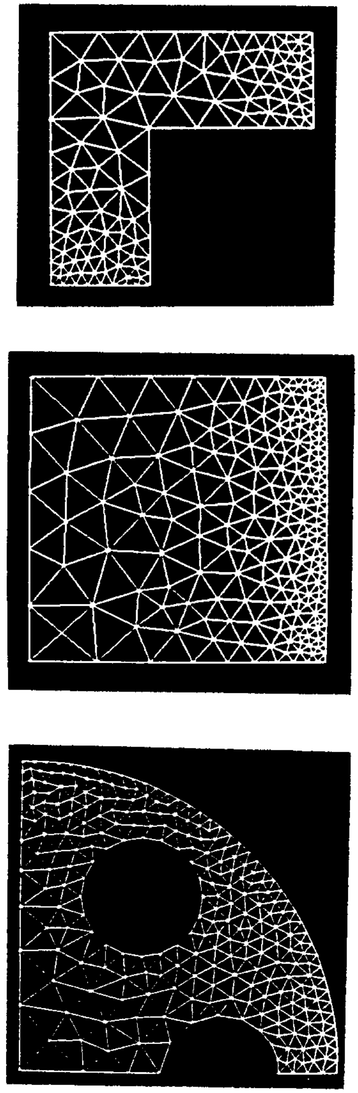


Fig 5 Node renumbering test examples

6. Implemetation of Quadratic Triangular Elements in Metal Formimg

When metal deforms plastically during the manufacturing process, the shape of its boundary changes continuosly. Such deformation often concentrates locally at sharp corners or rapid shape transition regions of dies. Thus in order to simulate the metal forming process one needs a proper algorithm having a remeshing capability locally as well as globally. Many studies of simulating metal-forming have been concerned and tried to develop methods such as various rezoning techiniques to rediscretize the domain when the mesh system is deformed(mostly locally) too much to continue the simulation. Delaunayi algorithm presented in this paper executes nicely in such a practice.

Following is the descriptions of the mechanics of plastic deformation for rigid-plastic materials from the work of Kobayashi et al.[31] ;

Equilibrium equations : $\frac{\partial \sigma_{ij}}{\partial x_j} = 0$ (8a)

Yield criterion : $\bar{\sigma} = \sqrt{\frac{3}{2}}(\sigma'_{ij}\sigma'_{ij})^{\frac{1}{2}}$ (8b)

Constitutive equations : $\dot{\varepsilon}_{ij} = \frac{3}{2}\frac{\dot{\bar{\varepsilon}}}{\bar{\sigma}}\sigma'_{ij}$

where $\dot{\bar{\varepsilon}} = (\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{\frac{1}{2}}$ (8c)

Compatibility conditions $\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ (8d)

where σ_{ij} Cauchy stress tensors, $\dot{\varepsilon}_{ij}$ strain-rate tensors, u_i velocity component, respectively. Barred simbol stands for effective stress and effective strain-rate of plastic deformation, and primed simbol is the deviatoric parts of stress and strain rate tensors. Again following the finite element formulation from the work of Kobayashi et al.[31], a simple upsetting of circular cylinder in Figure 6 was simulated.

During upsetting the material on the inteface of the upper die flows upwards and finally folds over at step 23, which causes further simulation of upsetting to fail since the element at the corner, distorted too much, has the negative Jacobian. Using Delaunay algorithm, remeshing can be performed at the step so that the simulation could continue beyond the failed step. Figure 7 is the original mesh and Figure 8 shows the sequence of plastic deformation which will occur similarly during forming process of the metallic material. Severe deformation of the upper right hand corner of the cylinder is nicely represented by six node triangular elements.

Dimensions : h = 1. and r = 1.
Die speed : v = 0.1
Friction factor : m = 0.5
Flow stress : $10.0(\dot{\bar{\varepsilon}})^{0.1}$
Total reduction in height : 40%
Incremental step size : 2%
Number of nodes in the initial mesh system = 85
Number of elements in the initial mesh system = 34

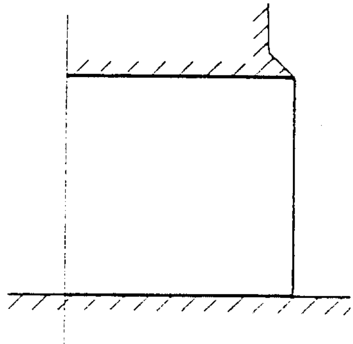


Fig 6 Simple upsetting of a cylinder

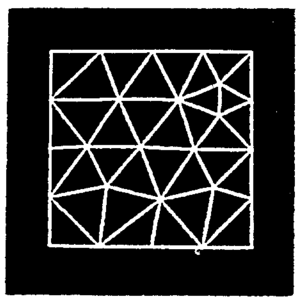


Fig 7 Original mesh of the upsetting problem

The analysis program used in this study was not capable of transferring data of various state variables for restart analysis. This prevents us from performing remeshing procedure which is an essential part in observing continued deformation of forming process. Remeshing after 20th step with 6-node triangular elements by Delaunay triangulation is shown in Figure 9. Firstly 3-node triangular mesh is generated, then 6-node triangular mesh is regenerated bas-

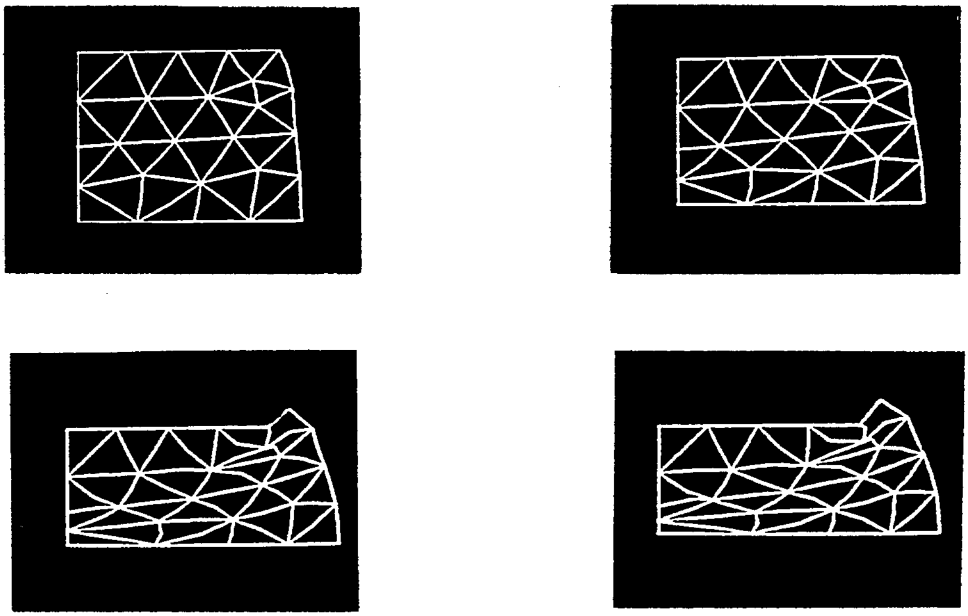


Fig 8 Sequence of plastic deformation of example problem

ed on the 3-node element mesh. The sequence is shown in Figure 10(a) and (b) enlarged locally at the right upper corner of the body. Smooth boundary fitting is observed by higher order triangular elements

7. Conclusion

The adaptive ability and boundary approximation of Delaunay triangulation is greatly enhanced by adding generation algorithm of 6-node triangular elements. Thus simulating the problems of large deformation with severe local distortion and varying boundary and varying domain can be carried out not causing any biased mesh environment. The felxibility of the Delaunay method is so effectve that the argument over preference on the options between adaptive refinement and adaptive remeshing methods be insignificant. Delaunay triangulation does not differentiate the adaptive refinement and the adaptive remeshing but encompasses all the processes of them at the same time. One weakness of generating low order element is resolved by adding a module for 6-node triangular elements. The other one of causing large a half bandwidth of stiffness matrix is also resolved by providing renumbering algorithm of node and element ordering. The extension to 3-dimensional problems should not be too difficult.

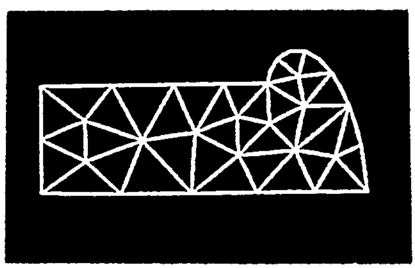
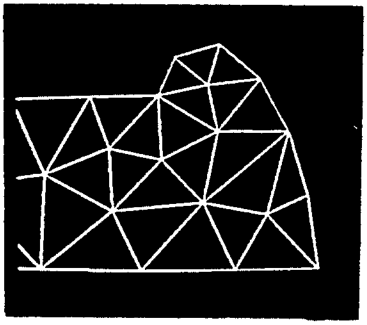
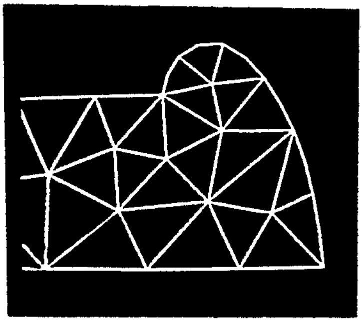


Fig 9 Remeshing for restart



(a) 3-node base net



(b) 6-node mesh

Fig 10 Boundary fitting by 6-node elements

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