

# Analysis of Phase Velocity Matching in Coupled Microstrip Lines with Dielectric Overlay

Yong K. Lee, Seung Y. Rhee, Nam Kim, and Han K. Park

## Abstract

This paper describes a concrete method for computing characteristic impedances and effective dielectric constants of the microstrip coupled lines without and with a dielectric overlay. The frequency-independent spectral domain method is used for the analysis of these lines. This method is a powerful, accurate, and numerically efficient approach for planar transmission line structure.

For designing the optimal directional coupler, the velocities of even and odd mode must be equal but velocities of these two modes are different in the conventional coupled line which is inhomogeneous. The results show that these two velocities can be almost same according to variations of structural and material parameters in terms of the overlay(superstrate).

## I. Introduction

The coupled line circuits are used for many applications including filters, couplers, and impedance matching networks. A "coupled line" consists of two transmission lines placed parallel to each other and in close proximity. In such a case, there is a continuous coupling between the electromagnetic fields induced by two lines.

Coupled lines can support two different propagation modes and these modes have different characteristic impedances. The velocity of propagation of these two modes is equal when the lines are imbedded in a homogeneous dielectric medium. However, for transmission lines such as coupled microstrip lines, the dielectric medium is not in general homogeneous, so the velocity of propagation of these two modes are not equal. When the even- and odd-mode propagation constants are equal, the isolation of a directional coupler is theoretically infinite. However, this condition is not always satisfied in an inhomogeneous structure such as a microstrip[1].

This coupled line structure can be designed by the impedance, admittance, chain and matrix parameters

characterizing the coupled line four port network. And these parameters can be obtained in terms of impedances or admittances of the coupled line. As a matter of fact, many publications have analyzed structures of the inhomogeneous coupled line to overcome this limit. Especially the paper published in reference of [2] suggested that a dielectric overlay is one way to improve the isolation of a microstrip coupler, by which the difference in even- and odd-phase velocities can be greatly reduced and even equalized. In that paper, a full wave analysis that includes frequency dependent behavior of coupled microstrip lines with overlay is formulated based on the spectral-domain method. But this method is very complex and requires a lot of the computing time[3,4,5].

Therefore in this paper, the coupled mode approach and frequency-independent spectral domain method(quasi-static approximation in spectral domain)[6,7] can be applied to evaluate the characteristics of coupled microstrip lines with or without overlay. The results show that the characteristics of coupled microstrip lines with overlay is less dependent on frequency than without overlay. It is seen that there exists particular structures for which even- and odd-mode phase velocities are almost equal.

## II. Formulation

Figure 1 shows the cross section of the coupled lines with dielectric overlay, which is to be analyzed in this paper. Note that they have symmetric structure because the widths are

Manuscript received May 1, 1996; accepted June 18, 1996.

Yong K. Lee is with the Department of Electronics Engineering, Yonsei University, Seoul, Korea.

Seung Y. Rhee is with the Department of Electronics Communication Engineering, Yosu Nat'l Fisheries University, Yosu, Korea.

Nam Kim is with the Department of Computer & Communication Engineering, Chungbuk Nat'l University, Cheongju, Korea.

Han K. Park is with the Department of Radio Engineering, Yonsei University, Seoul, Korea.

same( $w_1=w_2$ ). The structure is enclosed in a perfect conductor for the discrete Fourier transform analysis and assumed to be uniform and infinite in the  $z$ -direction. We also assume that the ground planes and coupled strips are perfectly conducting and infinitesimally thin, and the dielectric substrates are lossless. This symmetric parallel coupled line supports two propagation modes, namely, the even-mode and odd-mode. We will apply quasistatic SDA[2] to obtain the capacitances per unit length for the even- and odd-symmetric excitations for symmetric coupled lines shown in Fig. 1, both with and without overlay.

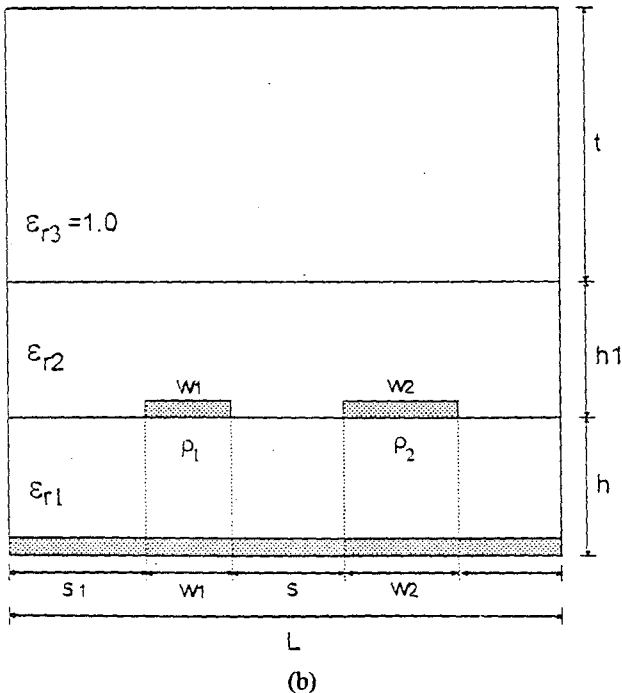
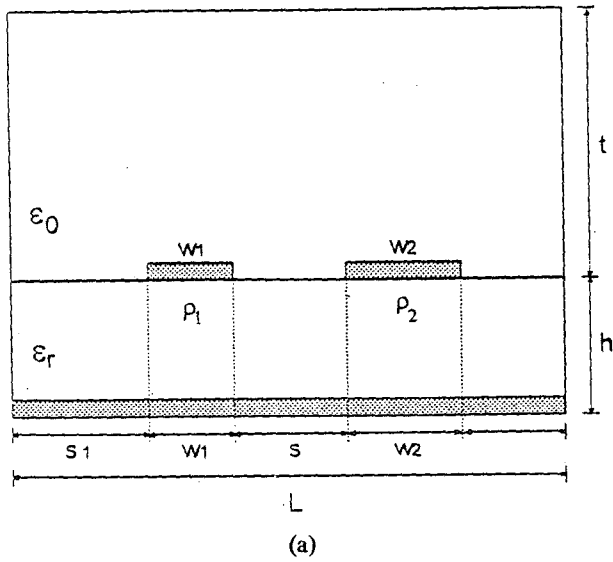


Fig. 1. Cross section of the coupled lines (a) without overlay (b) with overlay

We begin the analysis by defining the Fourier transform [4] with respect to  $x$  as

$$\tilde{f}(a_n, y) = \frac{2}{L} \int_0^L f(x, y) \sin a_n x \, dx$$

where the tilde( $\sim$ ) specifies the Fourier-transformed quantity, and  $a_n = n\pi/L, n=1, 2, 3 \dots$  is the Fourier-transform variable. The unknown potentials are the solutions of the Fourier-transformed Laplace equation. Imposing the boundary conditions then yields the following system of coupled linear algebraic equations according to whether there exists an overlay or not:

(A) without overlay

$$\tilde{C}(n) (\tilde{\rho}_1(n) + \tilde{\rho}_2(n)) = \tilde{\phi}_{v_1} + \tilde{\phi}_{v_2} + \tilde{\phi}_o \quad (1)$$

where

$$\tilde{C}(n) = \frac{1}{\epsilon_{\alpha n} (\coth a_n t + \epsilon_r \coth a_n h)}$$

and  $\tilde{\rho}_1(n), \tilde{\rho}_2(n)$  are the unknown transformed charge distributions on strip 1 and strip 2,  $\tilde{\phi}_{v_1}, \tilde{\phi}_{v_2}$  are the transformed quantities of the given strip potentials, and  $\tilde{\phi}_o$  is the unknown transformed potential in the respective interface.

(B) with overlay

$$\tilde{C}_{11}(n) \tilde{\rho}_1(n) + \tilde{C}_{12}(n) \tilde{\rho}_2(n) = \tilde{\phi}_o \quad (2)$$

$$\tilde{C}_{21}(n) \tilde{\rho}_1(n) + \tilde{C}_{22}(n) \tilde{\rho}_2(n) = \tilde{\phi}_{v_1} + \tilde{\phi}_{v_2} + \tilde{\phi}_o$$

where

$$\tilde{C}_{11}(n) = \tilde{C}_{12}(n) = \frac{1}{\Delta} \frac{\epsilon_r \tanh a_n h}{\sinh a_n h_1}$$

$$\tilde{C}_{21}(n) = \tilde{C}_{22}(n) = \frac{1}{\Delta} (\epsilon_r \coth a_n h_1 + \epsilon_r \coth a_n t) \tanh a_n h$$

$$\Delta = \epsilon_r \coth a_n t (\epsilon_r + \epsilon_r \coth a_n h_1 \tanh a_n h) + \epsilon_r (\epsilon_r \tanh a_n h + \epsilon_r \coth a_n h)$$

In order to solve the equations (1) and (2) using the Galerkin's method, the unknown charge distributions can be expanded into series in terms of the known basis functions [4]. The following basis functions which are defined only over the strips are used.

$$\rho_{1m}(x) = \frac{2}{w_1} \frac{\cos[(m-1)\pi \frac{x-s_1}{w_1}]}{\sqrt{(\frac{w_1}{2})^2 - [x - (s_1 + \frac{w_1}{2})]^2}} \quad (3)$$

$$\rho_{2m}(x) = \frac{2}{w_2} \frac{\cos[(m-1)\pi \frac{x-(s_1+w_1+s)}{w_2}]}{\sqrt{(\frac{w_2}{2})^2 - [x - (s_1 + w_1 + s + \frac{w_2}{2})]^2}} \quad (4)$$

In case of symmetrical coupled microstrip line,  $w_1$  is equal to  $w_2$ .

### III. Numerical Results and Discussion

We first transform the charge distributions in (3) and (4), substitute the series into (1) and (2) and take the inner products of the resulting equations with the individual basis functions.

Data in Fig. 2 are obtained with frequency-dependent full wave analysis in spectral domain for symmetrical coupled microstrip lines with or without overlay. Figure 2 shows that the impedances and effective dielectric constants of the overlay structure are much less dependent on frequency than in case of without overlay. Therefore the frequency-independent analysis(quasi-static analysis) is available for analyzing the coupled microstrip lines with overlay in order to reduce the computing time and complexity. So the following results are calculated using quasi-static method in spectral domain.

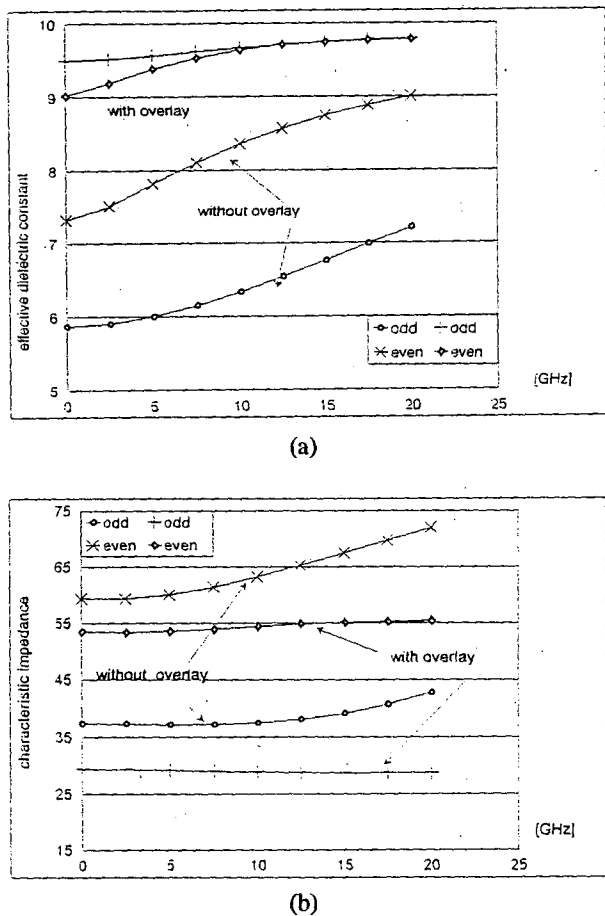


Fig. 2. Characteristic impedance and effective dielectric constants as a function of frequency with  $h=h_1=1.27\text{mm}$ ,  $W=1.27\text{mm}$ ,  $s=0.635\text{mm}$ . For coupled microstrip line with overlay,  $\epsilon_1=\epsilon_2=10.0$ . For coupled microstrip line without overlay,  $\epsilon_1=10$ ,  $\epsilon_2=1$ . Note that the structure is symmetric.

Figure 3 presents the even- and odd- mode characteristic impedances and effective dielectric constants of the coupled microstrip lines with overlay versus the normalized strip width of the substrate. It turned out that the difference between these two modes( $\Delta\epsilon_{eff}$ ) with overlay is less than without overlay.

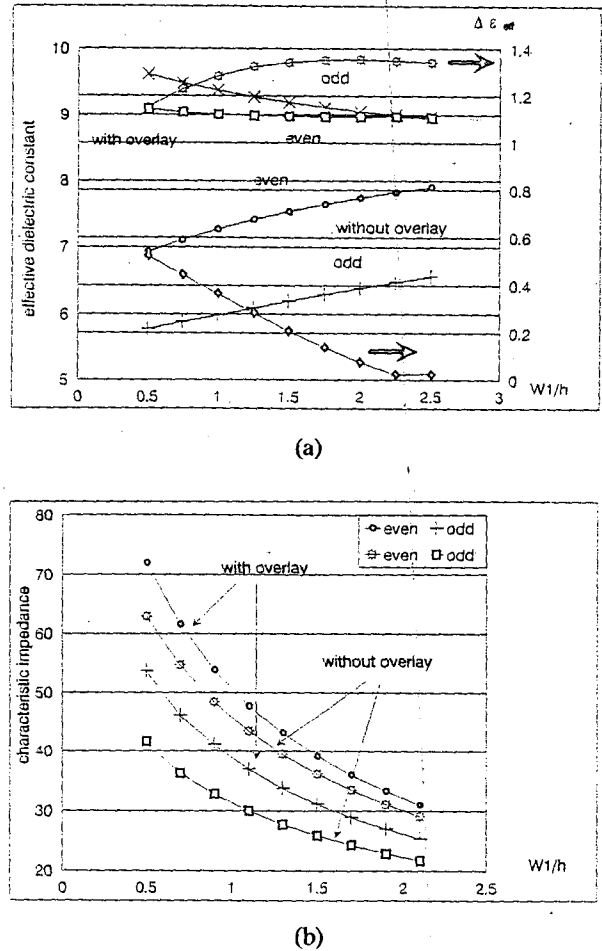
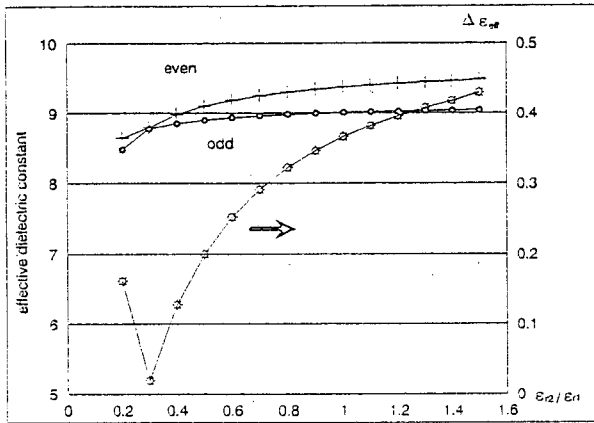
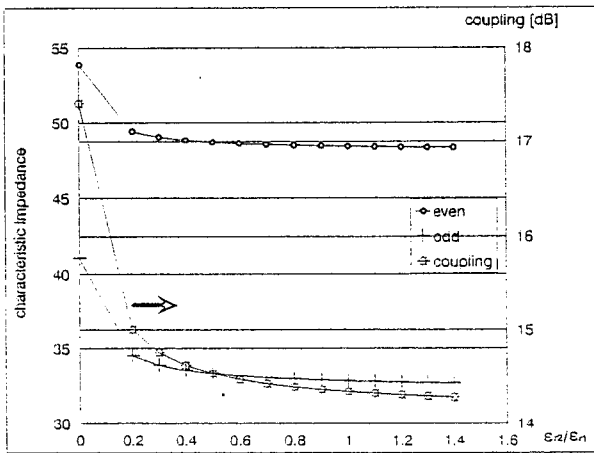


Fig. 3. Characteristic impedance, effective dielectric constants and difference between two modes as a function of width of microstrip line with  $h=h_1=1.27\text{mm}$ . For coupled microstrip line with overlay,  $\epsilon_1=\epsilon_2=10.0$ . For coupled microstrip line without overlay,  $\epsilon_1=10$ ,  $\epsilon_2=1$ .

Figure 4 shows variations of two mode characteristic impedances and effective dielectric constants versus the dielectric constant of the overlay for symmetric coupled lines with overlay. The dielectric constant of a substrate is fixed to be 10.0 and that of a overlay is made to vary from 1.0 to 15.0 in step of 2.0. The characteristic impedances turned out to be essentially unchanged, whereas the even-mode effective dielectric constant seems to increase significantly comparing with the odd-mode effective dielectric constant as the dielectric constant of the overlay increases.



(a)



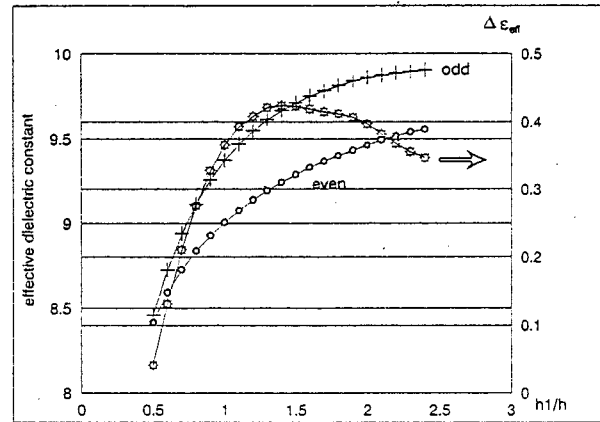
(b)

Fig. 4. Characteristic impedance, effective dielectric constants and difference between two modes as a function of dielectric constant of overlay with  $h=h_1=1.27\text{mm}$ ,  $W_1=W_2=0.635\text{mm}$ ,  $\epsilon_1=10.0$ .

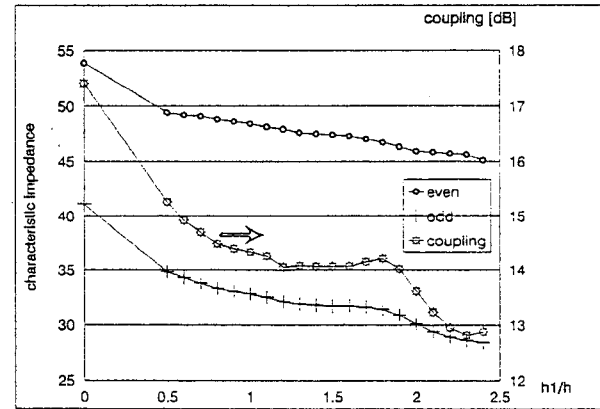
Figure 5 shows variations of two mode characteristic impedances and effective dielectric constants versus the height of the dielectric overlay. The characteristic impedances are slightly changed but both of two mode effective dielectric constants increases significantly as the height of an overlay increases.

From these two figures, we observe the following : (1) the coupled line with a dielectric overlay has lower characteristic impedance than without an overlay; (2) for symmetric coupled microstrip lines, specifically, the difference of characteristic impedance between even-mode and odd-mode with overlay is larger than that without overlay, and hence it turned out that the overlay increases the coupling between two lines compared with lines without overlay; (3) the effective dielectric constants with overlay is larger than that without overlay; and finally (4) the difference of effective

dielectric constants between two modes with overlay becomes smaller than that without overlay.



(a)



(b)

Fig. 5. Characteristic impedance, effective dielectric constants and difference between two modes as a function of height of dielectric overlay with  $h=1.27\text{mm}$ ,  $W_1=W_2=0.635\text{mm}$ ,  $\epsilon_1=\epsilon_2=10.0$ .

#### IV. Conclusions

In this paper, frequency-independent SDA(quasi-static analysis) is used to compare its various characteristics of the symmetric coupled microstrip lines with and without dielectric overlay. And we obtain some concrete design parameters for the directional couplers and filters, according to variations of the dielectric constant and the height of a dielectric overlay. We believe that these two velocities can be almost same according to variations of structural and material parameters in terms of the overlay(superstrate) and the overlay in general improves the characteristics of the microstrip coupled lines from the results we obtained.

## References

- [1] K. C. Gupta, R. Garg and I. J. Bahl, *Microstrip lines and slotlines*, Artech House Inc., 1979.
- [2] L. Su, T. Itoh and J. Riviera, "Design of an overlay directional coupler a full-wave analysis," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-31, No. 12, pp. 1017-1022, Dec. 1983.
- [3] T. Itoh and R. Mittra, "Dispersion characteristics of the slot lines," *Electron Lett.*, Vol.7, No.13, pp. 364-365, July 1971.
- [4] T. Itoh and A. S. Hevert, "A generalized spectral domain analysis for coupled suspended microstriplines with tuning septums," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-26, No. 10, pp. 820-826, Oct. 1978.
- [5] T. Itoh, *Numerical Techniques for microwave and millimeter-wave passive structures*, John Wiley and Sons, Inc., New York, 1989
- [6] V. K. Tripathi, "On the analysis of symmetrical three-line microstrip circuits," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-25, No. 9, Sep. 1977.
- [7] C. Nguyen, "Broadside-coupled coplanar waveguides and their end-coupled band-pass filter applications," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-40, No.12, Dec. 1992.



**Yong Kook Lee** received the B.S. and M.S. degrees in 1982 and 1986 in Electronics from Yonsei University, respectively and is currently pursuing the Ph.D degree at Yonsei University, Seoul, Korea. He is currently an associate professor of computer science department at Shin-Gu College. He is a

member of KITE and KICS. His research interests include microwave devices, antenna and propagation, and numerical analysis for the solutions of electromagnetic radiation problems.



**Nam Kim** received the B.S, M.S., and Ph. D. degrees in 1981, 1983, and 1988 respectively, all in electronics from Yonsei University, Seoul, Korea. Since 1989, he is currently an associate professor of computer and communication department at Chungbuk National University. He is a member of KITE,

KICS, KEES, OSK, IEEE and SPIE. During 1992-1993, he joined the Dr. Goodman's group as a visiting professor in electrical engineering of Stanford University. His research interests are adaptive array antenna, numerical methods for electromagnetic radiation and optical information processings.



**Seung Yup Rhee** received the B.S degree in 1986, the M.S in 1988, and the Ph. D. degree in 1993, all from Yonsei University, Seoul, Korea. He is currently an associate professor of electronic communication department at Yosu National Fisheries University. He is a member of KITE, KICS, and

KEES. His research interests include microwave devices, antenna and antenna arrays, numerical methods for the solution of electromagnetic radiation problem, and optical controlled microwave devices.



**Han Kyu Park** received the B.S. and M.S. degrees from Yonsei University, Seoul, Korea, in 1964 and 1968, respectively, and the Ph. D. degree from Paris VI University, France, in 1975, all in electrical engineering. From 1979 to 1980, he was a visiting professor of electrical engineering at

Stanford University. From 1976 to 1992, he was a professor of electronic engineering at Yonsei University. Since 1992 He has been a professor of radio science and engineering at same University. He is a president of Korea Institute of Communication Sciences. His research interests are in the area of microwave and optical devices, antenna theory and design, numerical modelling of electromagnetics, radar signal processing, propagation measurements, and mobile communication system.