Fast Iterative Image Restoration Algorithm

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Abstract

In the present paper we propose two new improved iterative restoration algorithms. One is to accelerate convergence of the steepest descent method using the improved search directions, while the other accelerates convergence by using preconditioners. It is also shown that the proposed preconditioned algorithm can accelerate iteration-adaptive iterative image restoration algorithm. The preconditioner in the proposed algorithm can be implemented by using the FIR filter structure, so it can be applied to practical application with manageable amount of computation. Experimental results of the proposed methods show good perfomance improvement in the sense of both convergence speed and quality of the restored image. Although the proposed methods cannot be directly included in spatially-adaptive restoration, they can be used as pre-processing for iteration-adaptive algorithms.

I. Introduction

Various image processing methods, which restore noisy-blurred images obtained from a non-ideal imaging system, have been introduced in the literature [1, 2]. Among them, iterative cost function. Despite of many advantages, iterative methods are limited in pratical applications because of slow convergence [3]. By this reason, many efforts have been made for accelerating convergence of iterative image restoration algorithms [3, 4].

This paper consists of six sections. After introduction, background of iterative image restoration is briefly reviewed in Sec. II. In Sec. III we propose a modified steepest descent method using the improved search directions with line search. In Sec. IV we propose a fast iterative algorithm using preconditioners. Experimental results are given in Sec. V, then we conclude the paper.

II. Iterative Image Restoration Basics

In many cases, the image degradation process can be modeled by a linear space-invariant system with additive white Gaussian noise process, that is,

$$y = Hx + \eta \tag{1}$$

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where the vectors $y, x,$ and η respectively represent the lexicographically ordered observed, the original, and noise images. The block Toeplitz matrix H represents the appropriately ordered samples in the point spread function of the imaging system [2].

According to the degradation model in (1), the regularized image restoration problem is to minimize the functional

$$f(x) = \frac{1}{2} x^T T x - b^T x, \tag{2}$$

whose minimum occurs at the solution of the following linear equation

$$Tx = b, \tag{3}$$

where $T = H^T H + \lambda C^T C$, $b = H^T y$, C is a highpass filter, and λ represents the regularization parameter which controls the fidelity to the original image and smoothness of the restored image [4]. For solving (3), the following steepest descent iterative method has been used in [3].

$$x_k = x_{k-1} + \beta(b - Tx_{k-1}) = \beta b + (I - \beta T) x_{k-1}, \tag{4}$$

where β determines the size of each step, and $b - Tx$ represents $-\nabla f(x)$, which means the direction of minimizing $f(x)$. In (4), β controls the convergence speed, because it determines the spectral radius of the iteration matrix $(I - \beta T)$. One simple iterative method is using a constant β. But this method cannot reach the minimized point in each iteration step along the direction vector $(b - Tx)$. An improved version of the iteration given in (4) is the steepest descent method with line search, which computes the optimal step length by using

$$\beta_k = r_{k-1}^T r_{k-1} / r_{k-1}^T T r_{k-1}, \quad (5)$$

where $r_k = b - Tx_k$ represents the residual vector in each step. However, in this case the search direction of solution traverses back and forth across the valley rather than down to the valley. To avoid this pitfalls of the steepest descent method, the successive minimization of $f(x)$ along a set of general search derections $\{p_1, p_2, \cdots\}$ that do not neccssarily correspond to the $\{r_0, r_1, \cdots\}$ is used. In order to minimize $f(x_{k-1} + \beta p_k)$ with respect to β,

$$\beta = \beta_k = p_k^T r_{k-1} / p_k^T T p_k \quad (6)$$

is to be used.
To ensure reduction of $f(x)$ in each iteration step, p_k must not be orthogonalto r_{k-1}.

For iteration-adaptive restoration, Kang proposed an adaptive restoration by adaptively changing the regularization parameter at each iteration [5], such as,

$$\lambda^i = \frac{\|y - Hx^i\|^2}{\frac{1}{\gamma} - \|Cx^i\|^2}. \quad (7)$$

where x_i represents the i-th iteration of restored image.
To guarantee convergence, he proved that $\frac{1}{\gamma} \geq 2\|y\|^2$ in linear regularization functional case, and that $\frac{1}{\gamma} \geq \frac{4}{3}\|y\|^2$ in quadratic regularization functional counterpart.

III. Improved Search Direction for Faster Convergence

In this section we propose a new algorithm to find improved search directions for minimizing (2). The proposed algorithm is the same to the ordinary steepst descent method with line search for first two iteration steps, while it uses the sum of two previous search direction vectors at the third step instead of using the normal steepst descent direction. Above mentioned three iteration steps, using two normal steepest descent and one improved search directions, are repeated until the iteration convergences.

Algorithm 1 *A fast algorithm using the improved search directions with line search*

1. $k=0$; $x_0 = b$; $r_0 = b - Tx_0$.
2. $k = k+1$
3. if(k mod 3) < 3 *then*
   $\beta_k = r_{k-1}^T r_{k-1} / r_{k-1}^T T r_{k-1}$
   $x_k = x_{k-1} + \beta_k r_{k-1}$
   $r_k = b - Tx_k$
   elseif then
   $p_k = r_{k-2} + r_{k-1}$
   $\beta_k = p_k^T r_{k-1} / p_k^T T p_k$
   $x_k = x_{k-1} + \beta_k p_k$
   $r_k = b - Tx_k$
   endif
4. *go to step 2*

In general, convergence of the steepest descent method is slow when the smallest and the lagrest eigenvalues of the matrix T are significantly different, because the solution oscillates back and forth in the neighborhood of the solution. In other words, one direction is close to the eigenvector which corresponds to the largest eigenvalue, and the other is close to the eigenvector which corresponds to the smallest eigenvalue. By intuition, it is anticipated that the sum of two previous steepest descent directions helps the solution be deviated from the trace of the repeated two steepest descent directions. The proposed algorithm is illustrated in Fig. 1.

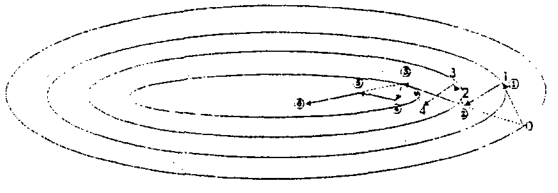


Fig 1. steepest descent search directions and the proposed search direction.

IV. Fast Iterative Algorithm Using Preconditioner

In this section, we propose the fast iterative algorithm based on precontioner.

1. Fast iterative algorithms using preconditioners for constant length of the search direction

In this subsection, a different approach for accelerating convergence of the iterative algorithm with constant length of the search direction is proposed. In order to analyze convergence, the spectral radius of the iteration matrix is used.

First, the iteration in (4) can be rewritten as

$$x^+ = \beta b + (I - \beta T)x = \beta b + Gx, \quad (8)$$

where $G = I - \beta T$ is called the "iteration matrix." In analyzing iterative algorithms by splitting, spectral radius, which represents the maximum absolute eigenvalue of the iteration matrix, should be less than unity for guaranteeing convergence [6]. In the iteration given in (8) the convergence tends to be accelerated as T becomes closer to the identity matrix when $\beta=1$. In order to do that, the preconditioned equivalent system to (3) is the following [6]

$$\tilde{T}\tilde{x} = \tilde{b} \tag{9}$$

where

$$\tilde{T}=PTP,\ \tilde{x}=P^{-1}x,\ \text{and}\ \tilde{b}=Pb. \tag{10}$$

In order to solve (9), the following iteration

$$\tilde{x}^{+} = \tilde{x}+\beta(\tilde{b}-\tilde{T}\tilde{x}) \tag{11}$$

can be used. By substituting (10) into (11), we have the preconditioned version of the iteration as

$$x^{+} = x+(P^2b-P^2Tx)=P^2b+(I-P^2T)x. \tag{12}$$

It is easily found that the ideal preconditioning matrix is

$$P=T^{-\frac{1}{2}}, \tag{13}$$

which minimizes the spectral redius of the iteration matrix $I-P^2T$. If we approximate the block-toeplitz matrix T to block-circulant, 2D DFT can diagonalize T. Based on that approximation, we propose an algorithm to obtain a preconditioning matrix P^2 as following.

Algorithm 2 $P^2\simeq T^{-1}$ *(preconditioner for 1D motion blur)*

1. *Compute the DFT of the first column of the upper left block of the symmetric, positive definite block circulant matrix T.*
2. *Compute the reciprocal of the real DFT coefficients.*
3. *Compute the inverse DFT of the reciprocal of the DFT coefficients obtained in step 2.*
4. *Truncate step 3's coefficients to desired length with raised cosine window.*
5. *Make the block circulant matrix $P^2=T^{-1}$ by placing the IDFT values obtained in step 4 at the first column of each diagonal blocks.*

Algorithm 3 $P^2\simeq T^{-1}$ (preconditioner for 2D uniform blur)

1. *Place T's circulant coefficients symmetric to DC axis in frequeny domain. And, apply 2D DFT.*
2. *Compute the reciprocal of the real DFT coefficients.*
3. *Compute the inverse 2D DFT of the reciprocal of the DFT coefficients obtaned in step 2.*
4. *Truncate step 3's coefficients to desired length with raised cosine window.*
5. *Make the block circulant matrix $P^2=T^{-1}$ by placing the IDFT values obtained in step 4.*

As mentioned earlier, these algorithms cannot be applied if β varys in each step. In the following, we give explanation that P^2 obtained by Algo. 1 without truncation is the same to T^{-1}.

A. 1D-CASE

If we assume that T is block circulant, T is diagonalized by the DFT as mentioned above.
And the corresponding diagonal elements are eigenvalues of T. Since T is positive definite, all its eigenvalues are positive. If W is Fourier transform matrix, T_F^{-1} is obtained as follows.

$$DFT\ :\ WTW^T=T_D=diag\{e_1,e_2,\cdots,e_N\},\quad e_1,e_2,\cdots,e_N>0$$

$$T_D^{-1}=diag\{\frac{1}{e_1},\frac{1}{e_2},\cdots,\frac{1}{e_N}\}$$

$$IDFT\ :\ T_F^{-1}=W^TT_D^{-1}W$$

To show that T_F^{-1} inverse of T, we use $WW^T=W^TW=I$

$$\begin{aligned} TT_F^{-1} &= T(W^TT_D^{-1}W)=TW^Tdiag\{\frac{1}{e_1},\frac{1}{e_2},\cdots,\frac{1}{e_N}\}W \\ &= (W^TW)TW^Tdiag\{\frac{1}{e_1},\frac{1}{e_2},\cdots,\frac{1}{e_N}\}W \\ &= W^Tdiag\{e_1,e_2,\cdots,e_N\}\,diag\{\frac{1}{e_1},\frac{1}{e_2},\cdots,\frac{1}{e_N}\}W \\ &= I \end{aligned}$$

and

$$\begin{aligned} T_F^{-1}T &= (W^TT_D^{-1}W)T=W^Tdiag\{\frac{1}{e_1},\frac{1}{e_2},\cdots,\frac{1}{e_N}\}WT \\ &= W^Tdiag\{\frac{1}{e_1},\frac{1}{e_2},\cdots,\frac{1}{e_N}\}WT(W^TW) \\ &= W^Tdiag\{\frac{1}{e_1},\frac{1}{e_2},\cdots,\frac{1}{e_N}\}\,diag\{e_1,e_2,\cdots,e_N\}W \\ &= I \end{aligned}$$

Therefore, $T_F^{-1}=T^{-1}$.

B. 2D-CASE

The relation for 1D-CASE also can be applied to 2D-CASE [7].

2. *A fast algorithm based on preconditioner for iteration-adaptive restoration.*

In this section, we apply the nonadaptive version of preconditioning technique to iteraition-adaptive iterative method.

* Proposed algorithm

1. Determine λ^i by (7) and make $T_i=H^TH+\lambda^iC^TC$ at each step.
2. Update restored image by, where p_i is a preconditioner of T_i.
3. Repeat steps 1 and 2 until convergence.

V. Experimental Results

In analyzing the convergence of general iterative algorithms, the normalized step length, such as,

$$\frac{\|x_k - x_{k-1}\|^2}{\|x_k\|^2} \tag{14}$$

is used.
In comparing different iterative algorithms, however, the smaller normalized step length does not always coincides with faster convergence to the desired solution. In order to compensate for this pitfall, we also use the improvement in signal-to-noise ratio (ISNR), which is defined as

$$\text{ISNR} = 10\log_{10}\frac{\|y-x\|^2}{\|\hat{x}-x\|^2}\ [\text{dB}], \tag{15}$$

where x, y, and $\hat{x}$ respectively represent the original, the degraded, and the restored images.

The 256×256 original Lena image is used and shown in Fig. 2. As a non-ideal imaging system, 1×11 motion blur and 7×7 uniform blur both with 40[dB] additive noise are simulated as shown in Fig. 3.

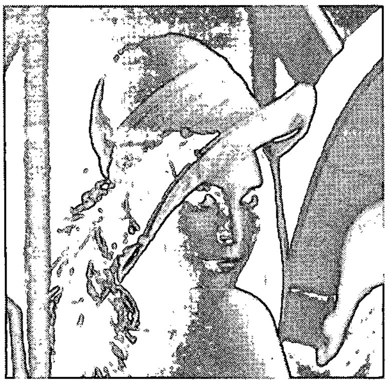
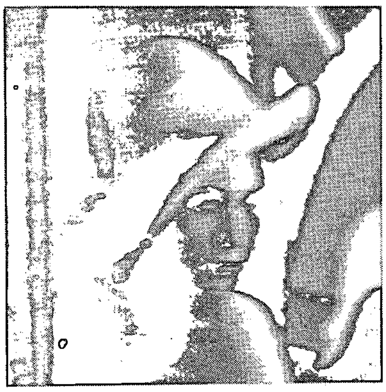
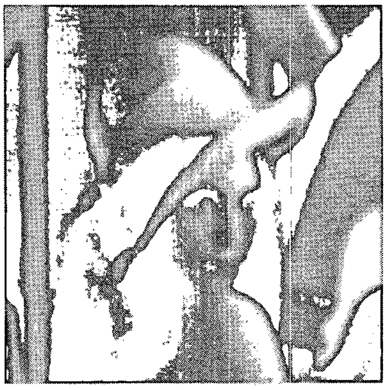


Fig 2. The 256×256 original Lena image



(a)



(b)

Fig 3. The degraded images by (a) 1×11 motion blur, and (b) 7×7 uniform blur, both with 40[dB] additive noise.

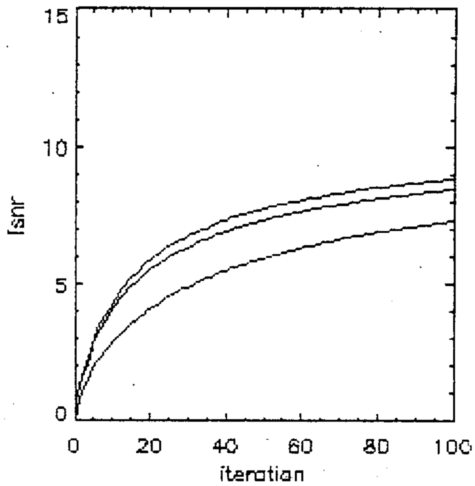


Fig 4. Plot of ISNR values of the steepest descent method with constant step length, β=1.0(lower), the steepest descent method with line search (middle), and the proposed algorithm (upper) versus the number of iterations.

A. The proposed iterative algorithm with improved search directions

In order to restore the degraded image shown in Fig. 3a, the steepest descent method with constant step length, the steepest descent method with line search, and the proposed algorithm with improved search directions are used. Fig. 4 shows the corresponding ISNR values versus the number of iterations for those three methods. Corresponding three differently restored images are shown in Fig. 5a, 5b, and 5c. And the corresponding ISNR values after 100 iterations are given in Table 1. As a result, the proposed iteration using the improved search directions outperforms other two steepest descent methods in the sense of ISNR.

(a) (b) (c)

Fig 5. The restored images obtained by (a) The steepest descent method with $\beta=1.0$, (b) The steepest descent method with line search, and (c) The proposed method using the improved search directions.

Table 1. ISNR values for three different methods after 100 iteration.

	constant β	line search	proposed method
ISNR	6.94	8.43	8.82

B. Preconditioning approach - 1D case

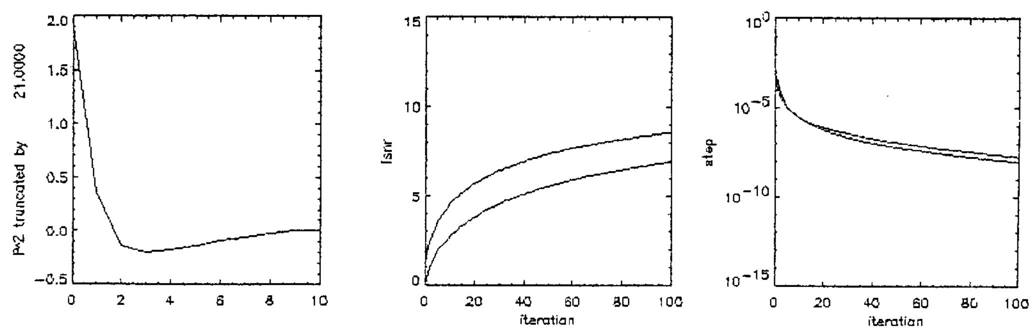
Since each diagonal block in the preconditioning matrices obtained by using Algorithm 2 and 3 has the same dimension to the processed image, we need to truncate it to a finite length for realization in the FIR structure at the cost of convergence speed. For that purpose, three types of windows, such as rectangular, triangular, and raised cosine, is considered. Truncation of P^2 with rectangular and triangular windows gives quite successful results for a certain length, but, in general, they are very unstable. On the other hand, truncation with the raised cosine window gives stable results. It is also noted that convergence speed as well as the quality of the restored images becomes higher as the number of filter coefficients increased.

The preconditioner truncated by 21 coefficients is shown in Fig. 6a. Both the ISNR and step length at each iteration with and without preconditioning are shown in Fig. 6b and c, respectively, when restoring the 1×11 motion blurred image shown in Fig. 3a.

The same results with 101 coefficients are also shown in Fig. 7. Four restored images without preconditioning, with the full-length preconditioner, with the 21 tap preconditioner, and with the 101 tap preconditioner are shown in Fig. 8. ISNR values and the step lengths of the four different restorations are summarized in Table 2.

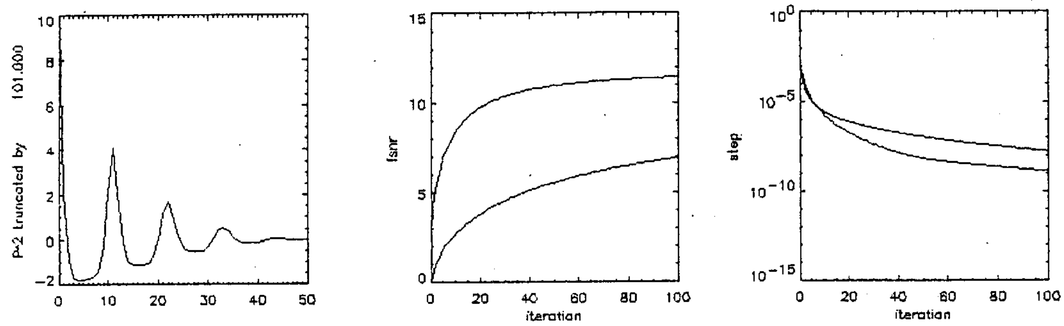
C. Preconditioning approach - 2D case

In restoring the 7×7 uniformly blurred image shown in Fig. 3b, the ISNR and step length at each iterations with and without preconditioning are shown in Fig. 9a and 9b. The similar results using the 101×101 preconditioner are shown in Fig. 10.



(a) (b) (c)

Fig 6. (a) Truncated coefficients (half of 21 taps) of the preconditioner, (b) ISNR without preconditioning (lower curve) and ISNR with preconditioning (upper curve), and (c) step sizes with and without preconditiong.

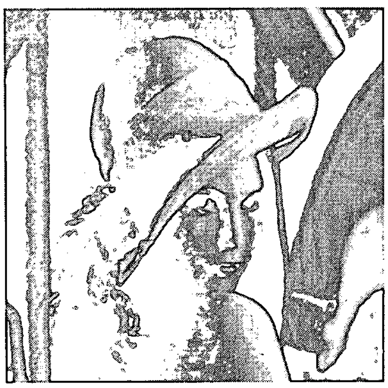


(a) (b) (c)

Fig 7. (a) Truncated coefficitnts (half of 101 taps) of the preconditioner, (b) ISNR without preconditioning(lower curve) and ISNR with preconditioning (upper curve), and (c) step sizes with and without preconditiong.

Table 2. ISNRs of the four different restorations after 100 iterations, and the number of iteration required for the step to be less than 10-7.

	without preconditioning	with preconditioning		
		21 tap	101 tap	full-length
ISNR	6.94	8.85	11.78	12.49
# of iteration	53	42	25	1



(a) ISNR = 6.94[db]

(b) ISNR = 12.49[dB]

(c) ISNR = 8.85[dB]

(d) ISNR = 11.78[dB]

Fig 8. The restored images (a) without preconditioning, (b) with the full-length preconditioner, (c) with the 21 tap preconditioner, and (d) with the 101 tap preconditioner.

Four restored images without preconditioning, with the full-length preconditioner, with the 21×21 tap preconditioner, and with the 101×101 tap preconditioner are shown in Fig.

11. ISNR values and the step lengths of the four different restorations are summarized in Table. 3.

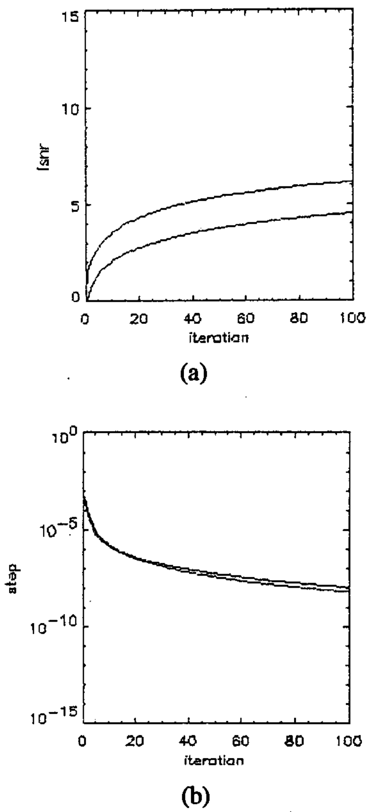


Fig 9. (a) ISNR without preconditioning (lower curve) and ISNR with the 21×21 preconditioner (upper curve), and (b) the corresponding step sizes with and without preconditioning.

As shown in the experimental results, the proposed algorithm converges faster than the non preconditioned counterpart in the sense of both ISNR and the step length.

Table 3. ISNRs of the four different restorations after 100 iterations, and the number of iteration required for the step to be less than 10^{-7}.

	without preconditioning	with preconditioning		
		21×21 tap	101×101 tap	full-length
ISNR	4.54	6.16	7.58	7.75
# of iteration	40	34	25	1

D. A fast algorithm based on preconditioner for iteration-adaptive restoration

The 256×256 lena image degraded by 7×7 uniform blur with 40[dB] additive noise is used for experiments. Fig. 12a shows the ISNR values obtained by Kang's method and the proposed method using 21×21 preconditioner. The corresponding step sizes are shown in Fig. 12b. In Fig. 13, the same results are shown for the proposed method with 101×101 preconditioner. Fig. 14 shows the restored image of Kang's method. And Fig. 15 shows restored images with 21×21 and with 101×101 preconditioners.

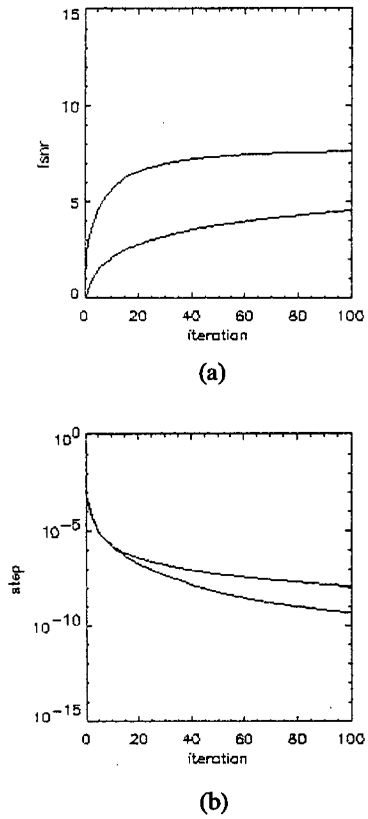
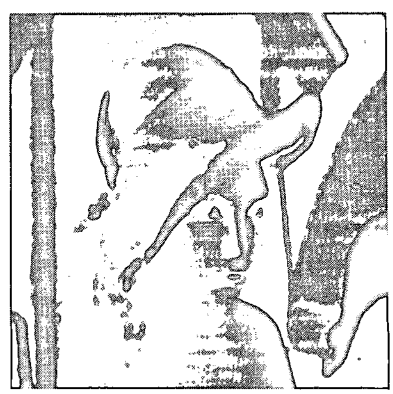
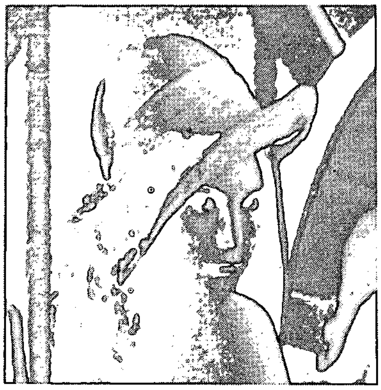


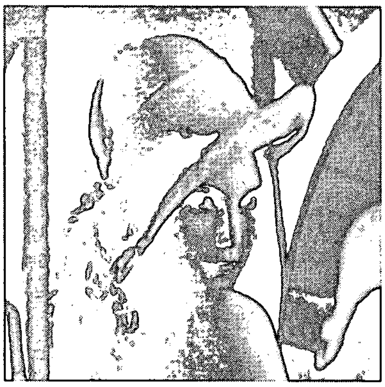
Fig 10. (a) ISNR without preconditioning (lower case) and ISNR with the 101×101 preconditioner, and (b) the corresponding step sizes with and without preconditioning.



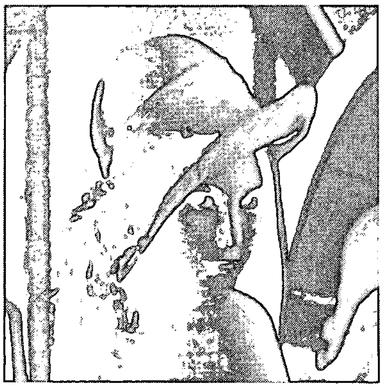
(a) ISNR = 4.54[dB]



(b) ISNR = 7.75[dB]



(c) ISNR = 6.16[dB]

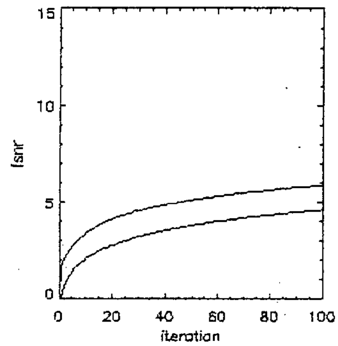


(d) ISNR = 7.58[dB]

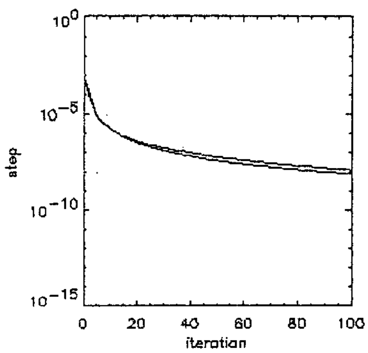
Fig 11. The restored images (a) without preconditioning, (b) with the full-length preconditioner, (c) with the 21×21 tap preconditioner, and (d) with the 101×101 tap preconditioner.

VI. Conclusions

In the present paper, two improved iterative image restoration algorithms are proposed. The one is an improved version of the steepest descent method with improved search directions with line search. The other is to accelerate the convergence of the steepest descent method by using preconditioners. A method for truncating the preconditioner is also proposed for efficient FIR structure. Experimental results shows that when the same number of iterations number is performed, proposed algorithms provide the better results in both convergence speed and the quality of restored image.

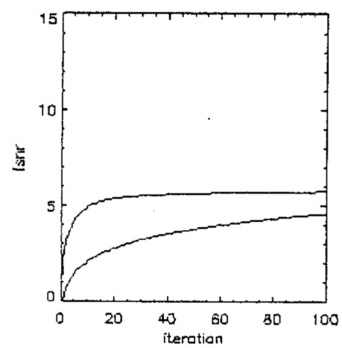


(a)

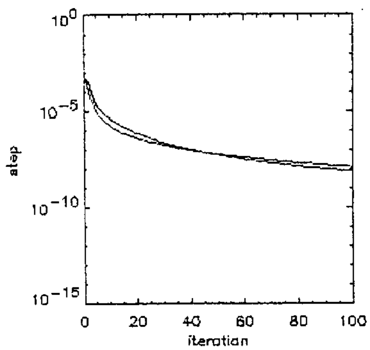


(b)

Fig 12. (a) ISNR of Kang's method and proposed method with 21×21 preconditioner, (b) step size of Kang's method and proposed method with 21×21 precondotioner.



(a)



(b)

Fig 13. (a) ISNR of Kang's method and proposed method with 101×101 preconditioner, (b) step size of Kang's method and proposed method with 101×101 precondotioner.

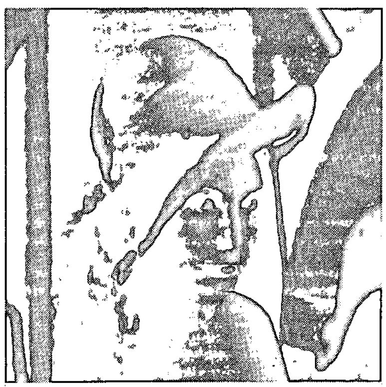
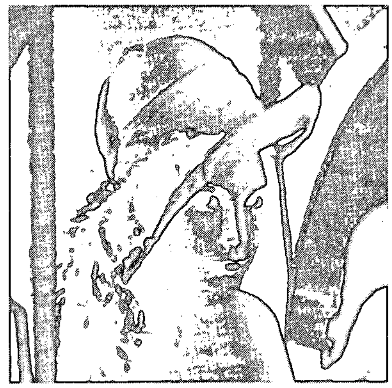
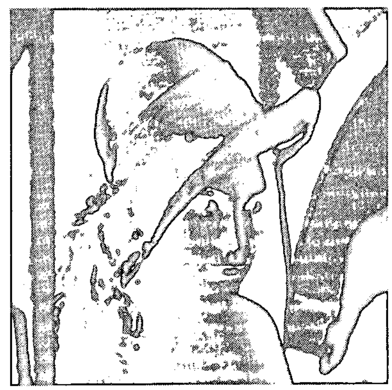


Fig 14. The restored image of Kang's method.



(a)



(b)

Fig 15. (a) The restored image with 21×21 preconditioner, (b) The restored image with 101×101 preconditioner.

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